

AN EASY
INTRODUCTION
TO
FORTIFICATION
AND
PRACTICAL GUNNERY.

CONTAINING,

- | | |
|---|--|
| <p>I. Decimal Arithmetic, Extraction of Roots.</p> <p>II. The Laws of Motion ; of Gravity. Descent of heavy Bodies. Vibration of Pendulums.</p> <p>III. Geometrical Problems, and some of the most useful Theorems in Geometry demonstrated in a very plain and easy Way.</p> <p>IV. The measuring of Superficies.</p> <p>V. The measuring of Solids. Of the Strength of Beams of Timber, with Rules for cutting Scantlings for Buildings, from Mathematical Principles.</p> <p>VI. The Computation of Balls and Shells.</p> <p>VII. Logarithmical Arithmetic.</p> <p>VIII. PLANE TRIGONOMETRY, the Proportion of Sines, Co-sines, Tangents, &c. With many useful</p> | <p>Problems for finding Heights and Distances, &c. copiously handled.</p> <p>IX. FORTIFICATION, explaining the Terms, with Rules for fortifying any regular Polygon, from the Square to the Decagon, and all the Sines and Angles computed by Trigonometry, with the Operation at large.</p> <p>X. GUNNERY, where the Cases are solved by Addition and Subtraction only, with numeral Examples ; the Rules in Words at length, whether the Projections be made on horizontal, or on ascending or descending Planes. The Theory of Projectiles. Tables of Experiments of Cannons and Mortars, with some Observations. The Solution of a Problem to find the Velocity of a Bullet shot from any Piece of Ordnance ; with the necessary Tables.</p> |
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The SECOND EDITION, corrected and very much enlarged.

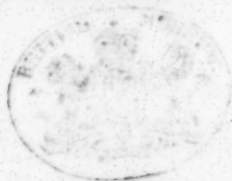
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L O N D O N,

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MDCCLXXIV.

THE CRITICAL REVIEW for *November, 1756*, gave the following account of the First Edition of this work, viz.—“Upon the whole, we think, that the authors quoted in this work, are well chosen; the practical rules inserted are clearly delivered, and the observations and examples annexed to them are pertinent and familiar. The design is certainly laudable, being intended, at this critical conjuncture, not only to excite in the public a desire of attaining some knowledge in this necessary science, but also to assist them therein, at an easy expence.”



44.
9. 7.
15.

T H E
P R E F A C E.

A GREAT part of the ensuing treatise was some years ago drawn up for my own use, in teaching some young gentlemen, who had not a conveniency or opportunity to search into different authors that have treated on this subject: the particulars of the following sheets are as follow;

I. *Decimal Arithmetic, Extraction of the Square and Cube Roots*, being necessary to all those who are inclined to proceed regularly in attaining this useful branch of knowledge.

II. *Mensuration of Planes and Solids*, and the method of computing the strength of timber joists, whether oak, elm, beech, or fir; and it must be here supposed that the timber is homogeneous, and of the same goodness as those pieces Mr. Emerson made his experiments with, or else a proper allowance must be made for the defect.

III. The usual rules for finding the *Weight and Diameter of Bullets, whether Lead or Iron*; the requisite charges of powder for any piece of artillery of any given dimensions, and for estimating the quantity of powder for bomb shells.

IV. The necessary *Problems and Theorems in the Elements of Geometry*, demonstrated in a very easy and short way, as may be comprehended by any person, without the help of a master.

V. *Logarithmical Arithmetic*, shewing the nature and use of Logarithms; with *Plane Trigonometry* solved in all cases, illustrated with proper examples; the usefulness whereof is so great in all parts of the mathematics, that the most important and useful parts of knowledge would be quite lost, if men were destitute of it; for the engineer would make but a despicable figure when he came to take the measures of heights and distances, whether accessible or inaccessible, or for laying down plans of fortification on the ground, computing the capitals, curtains, flanks, faces, lines of defence, &c. if he was ignorant of *Trigonometry*.

VI. The reader will find several examples for *measuring heights and distances*, and at one station, all done by *Plane Trigonometry*.

VII. *The Theory of Practical Gunnery*, wherein the doctrine of projectiles is illustrated in a multitude of the most useful problems that can occur, with numeral examples to each; and to make it more plain and useful to persons of but an ordinary capacity, I have given the rules to every problem in words at length, whether the projections be horizontal, or those made on ascending or descending planes; all which are designed more for those persons who have not skill and abilities in the mathematics, than for those who are furnished with mathematical knowledge sufficient to understand the learned tracts on this subject, and made farther advances in the elements of the higher geometry. Yet,

VIII. For the sake of my more ingenious readers, I have added what the celebrated Mr. *Cotes* has wrote on projectiles, done into *English* from his *Harmonia Mensurarum*, where they will find this subject elegantly handled by this author; to which I have annexed two general *scholia* on projectiles, of the learned mathematician Mr. *Emerson*, taken from his *Mechanics* with his own consent, and illustrated them with examples. I took this method, because many of the problems in gunnery were wrote before his *Principles of Mechanics* were published, and by these two *scholia* the solution of every problem was tried, and the answers very exactly agreed; so that I am in no concern what some persons may say of this performance.

IX. The next part treats of experiments ranged into tables; these experiments were made with cannon and mortars, and calculations were made thereon from the principles of the immortal Sir *Isaac Newton*, by *D. Bernoulli* and *M. Gunther*: this memoir, quoted by Mr. *Robins* in his *New Principles of Gunnery*, is printed in *Latin*; a particular account of which is inserted at the conclusion of this work.

X. Having a little spare room, I have given a fluxionary solution to a difficult problem, for finding the velocity of a bullet shot from any piece of artillery; though this is of no material use in practical gunnery, yet it perhaps may please some of my readers, as the conclusion comes out so simple.

XI. I have also inserted the necessary tables of gunnery, &c.

What is said of the flight of bombs in the following sheets, is equally applicable to cannon balls; the method for directing

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in^d cannons, and placing them at any elevation, is by means of proper instruments for that purpose. And here I would not be understood to mean, that experience does always exactly agree with the rules given for throwing bombs to hit an object, for there often happens many unavoidable accidents which will cause some difference; and yet these rules are not for that reason to be carelessly attended to, for they furnish the gunner with the readiest and shortest methods of perfecting his practice, and will answer the end proposed. Indeed a careful attention must be given to those persons who are daily employed in the exercise of artillery, and have founded their methods upon long experience; for this will always be of more weight than the most learned precepts. But yet, such a person will be better able to judge by the help of these rules, joined to his own experience, than he can by his experience alone.

As I have conformed to the method of other writers, it being the most easy and best adapted for beginners; so I expect to have objections raised against me, because I have made no allowance for the air's resistance; now to anticipate these objections, What engineer computes this for *Practical Gunnery*? The problem would then become very complicate, and of a very operose kind. The task is not only extreme difficult, but useless; for to shew how far a projectile deviates from a parabolic track by the resistance of the air, is one of the most intricate problems in all the doctrine of fluxions; and which is worse still, it can be of no manner of use; for the motion of a projectile in a resisting medium can be reduced to no rules, so as to be of any use in the art of gunnery. I thought, therefore, by laying down short and plain rules for practice, it would be more serviceable to the young engineer, than to puzzle him with tedious demonstrations and perplexed calculations in fluxions, which would create an endless deal of trouble for no real advantage.

It perhaps may be said, I might have added some problems by supposing the shot to move in a right line to a point blank distance from the mouth of the piece, and then to describe the curve of the parabola; but I do not apprehend it can be of any use in gunnery by putting a right line and parabola together; for the curve described by a ball on this hypothesis, will be nothing like the curve described in the air, and therefore I have purposely omitted it, for the ball never describes a right line.

But if any one would make a computation from the resistance of the air, he will find the problem elegantly solved by fluxions at page 84, Vol. II. of my *Miscellanea Curiosa Mathematica*, where it is said, that the description of the true curve is so perplex, that it is scarcely of any use; and if, according to the second corollary, the resistance be small, as is commonly experienced by an iron ball projected through the air with a sufficient velocity, the curve which the globe describes in a resisting medium, does not differ much from a conic parabola; yet because the resistance diminishes the velocity of the projection, the ordinate to the trajectory in a resisting medium will be a little less than the ordinate to the conic parabola; but notwithstanding this, the problem, by reason of its difficulty, is of no use in gunnery.

The *Art of Gunnery* is a science at all times useful, and much more so amongst the *English*, who I am sure have as great an inclination to arms and arts as any nation in *Europe*; their matchless valour and bravery, their singular prudence and conduct, both by sea and land, is so universally known, that many powerful states have been constrained to yield to them by the dint of their sword; and as to arts and ingenious literature, they justly claim a true title to the empire of human knowledge.

These sciences, it is known to all, are the peculiar care of princes, that design to raise the force and power of their countries; and these are none of the least arts, whereby the *French King* hath brought his subjects to make that figure which they at this time do; I mean, the care that he takes for educating those appointed for this service: he orders that there be professors to teach these sciences publicly in several parts of the kingdom, that the teachers must know designing, and to teach it to their pupils, in order to lay down the appearances of things in their real form and situation; they are to keep their schools open, and to read four times a week to their scholars, where they must have books and instruments necessary to teach their art, who have handsome salaries from the government for that service, and to teach *gratis*. The directors of hospitals are obliged to send to these academies every year several of their boys, to be taught and furnished with books and instruments, explained with a vast variety of experiments, and thereby practice and theory go on hand in hand, and receive mutual assistance from each other; and that nothing can exceed the order of these schools, the officers placed at the head of them are of the greatest ability

lity and knowledge in the management of artillery, which they teach with as much method as grammar and accounts are taught in our schools ; and hence it is that *France* is well provided with so great a number of able and sufficient engineers.

During the time of the late unnatural rebellion, some mathematical lectures were read in *London* to instruct officers in the art of engineering and fortification, as knowing very well the importance thereof, and this continued some time after the peace ; and it is worthy the consideration of the legislature, whether the restoring and continuing this even in times of peace, be not expedient for the bringing up engineers, who are so useful and valuable in the military way, and so difficult to be had in time of war, and so little dangerous in time of peace.

The arts of war and trade require much the assistance of these sciences, an officer that understands fortification and engineering, will, *cæteris paribus*, much better defend his post, as knowing wherein its strength consists, or make use of his advantage to his enemies ruin, than he that does not : he knows when he leads never so small a party, what his advantage or disadvantage in defending and attacking are, and hereby can do more service than another of as much courage, who for want of such knowledge, it may be, throws away himself, and a number of brave fellows under his command, and it is well if the mischief reaches no further.

Fortification is so necessary an art, that it greatly deserves more encouragement ; for, excepting one or two academies founded on royal authority, I cannot learn that it is properly taught in any parts of this kingdom ; for most of those gentlemen that make geometry their peculiar province in teaching it, apply it often to useless speculations. Now fortification is the chief art wherein geometry is used, and to it only is owing the great success and improvement of military architecture ; indeed, where the ground is regular, it admits but of small variety ; yet where the ground is made up of natural strengths and weaknesses, it affords some scope for thinking and contrivance ; and what contributes much towards its strength, with regard to its force or resistance, are the due measures and proportions of its lines and angles ; as it is now arrived to such a degree of perfection, that in all likelihood it cannot receive any essential addition ; so it is the more easily to be comprehended ; because, it seems im-

possible to find any thing that is not already invented for the making attacks upon places. Earth is made use of for trenches, banks, and retrenchments; water, for sluices and inundations; and fire is applied various ways; below, by the contrivance of mines and their galleries, whose havoc is dreadful, and destruction rapid, in a moment bursting open the bowels of the earth, rending in pieces the strongest walls, the firmest towers, and blowing up whole troops of men; above, by bombs, carcasses, and granadoes, directly by cannons, mortars, &c. "Let the reader imagine to himself (says an ingenious writer) a great number of mortars playing continually from the batteries round a besieged town, and bombs like one uninterrupted tempest of fire falling at once on all its quarters, forcing their passage irresistibly, demolishing buildings, and burying multitudes under the ruins, tearing up pavements, sweeping whole ranks of men away, and tossing their mangled limbs into the air, penetrating with the violence of lightning even subterraneous vaults, overthrowing, rending or setting on fire whatever they touch, their execution is experienced as fatal and general, as their effects are terrible and destructive." The cunning of mankind never exerting itself so much as in their arts of destroying one another, therefore methods of defence have been invented to prevent in a great measure these effects, and this is the proper business of fortification; and unless mankind find out some other elements, there is not much probability that they can invent new methods, either offensive or defensive.

It were to be wished that proper encouragement were allowed for experienced mathematicians and engineers to make all the necessary experiments on pieces of artillery, for this art now makes one of the most considerable branches of the military science, and it is absolutely necessary to have it understood by many, in order to support the honour of our arms, and to maintain the public safety of this nation; and I am sure never more so than at this present conjuncture, where it is said from public authority, that *Great Britain* or *Ireland* is daily threatened to be invaded by one of the most powerful and formidable fleets and armies that *France* ever had; whose armies are in continual motion on all sides, their coasts towards the ocean swarm with troops, their roads to *Flanders*, *Normandy*, and *Brittany*, are continually covered with carriages laden with cannon, warlike stores, arms of all kinds, provisions, &c. all the apparatus of some great enterprise,

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prize, whose injustice as well as insolence is circumscribed by no bounds ; who now fully shew their ambitious projects, ill faith, notorious breach of treaty, and manifest aggravation against the best of **KINGS** that ever sat on the *British* throne. This attempt of invading these kingdoms in revenge for his Majesty's generous and steady conduct in maintaining the rights and possessions of his crown and subjects, against the unprovoked aggressions and hostilities first begun on the part of *France*, is so unjust, daring and insolent, as not to be paralleled in any history. But—

To conclude, as the practice of *Sea Gunnery* is extremely easy, their marks being mostly point blank or very near it, and that ascents and descents bear no considerable share here, I have taken no notice of it, and have omitted the demonstrations of the rules in gunnery, in order to render the whole as concise, and deliver it with as much plainness as my own abilities and the nature of the subject would permit me ; and I hope the candid and impartial reader will excuse what is amiss, and amend those errors which unavoidably attend things of this nature, or may happen by reason of the author's great distance from the press ; for he has no other design in publishing this book, than to be serviceable to his own countrymen, for whose benefit he heartily wishes it was much more complete than it is.

Sept. 4,
1756.

F. HOLLIDAY.

P. S. From the favourable reception the former edition of this work has met with, and the opinion which some good judges of the subject have been pleased to form of it, the author thinks himself authorized to consider his plan as right. This, joined with his desire to oblige those who wish to see the subject more complete, induced him with the utmost care to revise and improve it. In this second edition are made many alterations and additions in several parts of the book, as the laws of motion, the laws of gravity, descent of bodies, the vibration of pendulums. To the geometry are prefixed the definitions, which were omitted in the former edition. At the desire of some friends, I have enlarged the number of propositions in the mensuration of planes and solids ; and have treated largely of the strength of beams of timber, with rules for cutting the proper dimensions of scantlings, founded on
mathe-

mathematical principles, delivered in such a manner as to explain them to the meanest capacity. To the trigonometrical part, I have given the proportion of sines, co-sines, tangents, &c. and several other useful and curious propositions, too long to be mentioned here. And farther, to render the book as useful as I could, there is now added a short introduction to fortification, the definitions are according to the best French writers, whom I consulted; also I have given the operation at large, for computing all the lines and angles of regular fortified polygons, as the *square*, *pentagon*, *hexagon*, and *octagon*, from trigonometry, as laid down in this treatise. In gunnery, or the doctrine of projectiles, the reader will meet here and there inserted farther improvements, and more examples for illustrating the learned Mr. *Emerson's* two general *scholia*, on the motion of projectiles, with a table of theorems for horizontal projections; and it may here be observed, that Propositions I. II. VII. and XI. are the most useful for practice, which are few and very easy.

These, it is presumed, will be acceptable to gentlemen in the army, which are peculiarly adapted to assist those who desire to be acquainted with so useful a science. Though men in private life may be allowed to neglect these sciences, yet persons of eminent stations, who have the conduct of societies and states, may by no means be admitted to do it; for not only military gentlemen, but all others who would speak properly on this subject, and understand what they read in the public news-papers, should be acquainted with this necessary branch of science.

If that which is here delivered prove useful, it will be a pleasure to him, who, upon all occasions, is glad of promoting real and useful knowledge.

May 4, 1772.

F. H.

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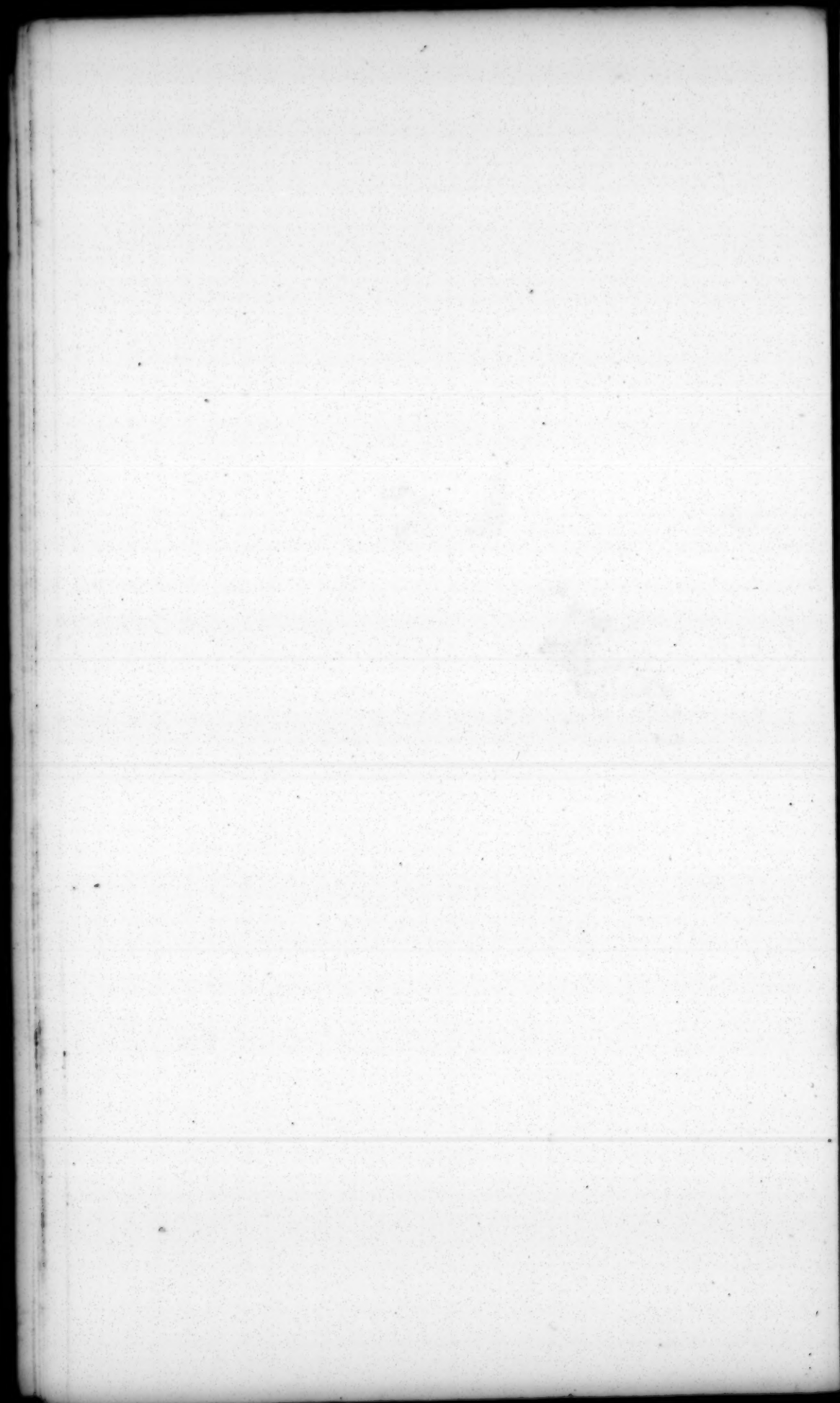
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An explanation of the SYMBOLS used in this treatise.

- + signifies addition, as $A+B$ denotes B added to A.
- signifies subtraction, as $A-B$ denotes B to be taken from A.
- $\times$ signifies multiplication, as $A \times B$ denotes A multiplied by B.
- = signifies equality, as $A=A$ or A is equal to A.
- $\sqrt{}$ signifies the square root, as $\sqrt{A}$ the square root of A, or $\sqrt{144}$ the square root of 144 or 12.
- Log. signifies the logarithm of.
- $::$ signifies proportion, or rule of three, thus $2:4::6:12$, that is, as 2 is to 4 so is 6 to 12.
- Δ signifies triangle, as ΔABC denotes the triangle ABC.
- $\angle$ signifies angle, as $\angle A$ signifies the angle A.
- s. signifies sine, as s. ABC denotes the sine of the angle B.
- t. signifies tangent, as t. ABC denotes the tangent of angle B.
- $\square^2$ signifies the square, as $\overline{CD}^2$ denotes CD squared.
- $\square$ signifies parallelogram, as $\square DB$ the parallelogram DB.
- $^\circ$ signifies degrees, as 24° signifies 24 degrees.
- ' signifies minutes, as $35'$ denotes 35 minutes.



PART I.

OF

DECIMAL ARITHMETIC.

DEFINITIONS.

A FRACTION is called *vulgar* or *decimal*, according as it is divided, as $\frac{3}{5}$ is called a *vulgar fraction*, the uppermost number is called the *numerator*, and the undermost the *denominator*.

A *decimal* fraction supposes *one* of any thing whatever to be divided into 10 parts, 100 parts, 1000 parts, &c. and hath always a dot set before it as .5 is called 5 tenths, and in vulgar fractions $\frac{5}{10}$, but as decimals are the easiest, I shall say nothing of vulgar fractions here.

As places in whole numbers increase by tens, so do places in decimals decrease by tens; for as the first, second, third, &c. place above that of units is tens, hundreds, thousands, &c. so the first, second, third, &c. place below that of units is tenths, hundredths, thousandths, decreasing in a tenfold proportion, as is manifest from the following table.

7	Millions.	2	Tenth parts.
6	Hundreds of thousands.	3	Hundredth parts.
5	Tens of thousands.	4	Thousandths.
4	Thousands.	5	Ten thousandths.
3	Hundreds.	6	Hundred thousandths.
2	Tens.	7	Millionths.
1	Unit.		

B

So

So $\left\{ \begin{array}{l} .5 \\ .25 \\ .175 \end{array} \right\}$ signifies $\left\{ \begin{array}{l} 5 \text{ Tenths.} \\ 25 \text{ Hundredths.} \\ 175 \text{ Thousandths.} \end{array} \right.$

Cyphers set after decimals do not alter their value, as .8 and .8000 are the same, viz. only 8 tenths: but when cyphers are set before decimals, they remove them farther from the decimal point, and therefore decrease them, as .08 is less than .8, so .008 is less than .08, consequently, each removal of a figure into a place forward makes it ten times less than it was before.

To reduce Vulgar Fractions into Decimals.

R U L E.

Add as many cyphers to the numerator as you intend decimals, and divide by the denominator, the quotient is the decimal required.

Examples.

1. Reduce $\frac{1}{4}$ of any thing into a decimal.

4) 1.00 (.25 the decimal.

$$\begin{array}{r} 8 \\ \hline 20 \\ 20 \\ \hline \end{array}$$

2. Reduce $\frac{1}{2}$ of any thing into a decimal.

2) 1.0 (.5 the decimal.

$$\begin{array}{r} 10 \\ \hline \end{array}$$

3. Reduce $\frac{3}{4}$ of any thing into a decimal.

4) 3.00 (.75, the decimal.

$$\begin{array}{r} 28 \\ \hline 20 \\ 20 \\ \hline \end{array}$$

So the quarter, half, and three quarters of any thing are .25; .5; and .75 decimally.

4. Reduce

4. Reduce $\frac{2}{7}$ into a decimal.

7) 2.00000 (.28571 the decimal of $\frac{2}{7}$.

$$\begin{array}{r}
 14 \\
 \hline
 60 \\
 56 \\
 \hline
 40 \\
 35 \\
 \hline
 50 \\
 49 \\
 \hline
 10 \\
 7 \\
 \hline
 3 \\
 \hline
 \end{array}$$

5. Reduce $\frac{1}{32}$ into a decimal.

32) 1.00000 (.03125

$$\begin{array}{r}
 96 \\
 \hline
 40 \\
 32 \\
 \hline
 80 \\
 64 \\
 \hline
 160 \\
 160 \\
 \hline
 \end{array}$$

Note. Here I annexed five cyphers to 1 the given numerator, as I intended to have five places in decimals, and there arise but four in the quotient, I must supply such defect by prefixing as many cyphers on the left-hand of the first figure in the quotient as there want places; as in the example above there is prefixed a cypher to make the quotient compleat, because I annexed five cyphers to the numerator.

6. Reduce 6s. 8d. into the decimal of a pound.

Thus, 6s. 8d. is = 80d. and a pound = 240d.

Then 240) 80.00000 (.33333 is the decimal.

B 2

7. What

7. What is the decimal of 5 cwt. 1 qr. 11 lb. the integer being a tun weight ?

Thus, a tun weight = 2240 lb. and 5 cwt. 1 qr. 11 lb. = 599 lb.

Then $2240 \overline{) 599.0000}$ (.2674, the decimal.

To find the Value of any Decimal Fraction.

R U L E.

Multiply your given decimal by the parts of the next inferior denomination that is equal to the integer, the decimal dotted off gives the part thereof ; and if there be any remainder, multiply it by the next inferior denomination, &c. till all is done.

Examples.

1. What is the value of .875 of a pound Sterling ?

Thus $\begin{array}{r} \text{£} .875 \\ 20 \end{array}$

$\begin{array}{r} \text{s. } 17.500 \\ 12 \end{array}$

$\begin{array}{r} \text{d. } 6.000 \end{array}$ Answer 17 s. 6d.

2. What is the value £ .9874 ?

Thus $\begin{array}{r} \text{£} .9874 \\ 20 \end{array}$

$\begin{array}{r} 19.7480 \\ 12 \end{array}$

$\begin{array}{r} 8.9760 \\ 4 \end{array}$

Answer 19 s. 8 $\frac{1}{4}$ d. $\begin{array}{r} 39040 \end{array}$

3. What is the value of .5875 of a tun weight ?

Thus $\begin{array}{r} .5875 \text{ Tun.} \\ 20 \end{array}$

$\begin{array}{r} 11.7500 \\ 4 \end{array}$

Answer 11 cwt. 3 qrs. $\begin{array}{r} 30000 \end{array}$

4. What

4 What is the value of .3712 of a hundred weight?
Thus .3712 cwt.

$$\begin{array}{r} 4 \\ \hline 1.4848 \\ 28 \\ \hline 38784 \\ 9696 \\ \hline 13.5714 \\ 16 \\ \hline 34464 \\ 5744 \\ \hline 9.1904 \end{array}$$

Answer.
1 qr. 13 lb. 9.1924 oz.

ADDITION of DECIMALS.

R U L E.

LET the numbers be placed according to their value, as units under units, tens under tens, &c. in whole numbers; and in decimals tenths under tenths, hundredths under hundredths, &c. then work as in whole numbers, always remembering to make as many places of decimals in the sum, total, as the greatest number thereof were in your example.

Examples.

$$\begin{array}{r} \text{£} \\ .583 \\ .372 \\ .281 \\ .846 \\ .75 \\ .6 \\ \hline \text{£ } 3.432 \end{array}$$

$$\begin{array}{r} \text{Yards.} \\ 48\ 94 \\ 21\ 648 \\ 6.5 \\ 19.8168 \\ 72\ 1108 \\ 16.614 \\ \hline 185.6295 \end{array}$$

$$\begin{array}{r} £818.4 \\ 6.97 \\ 381 \\ .54 \\ .61 \\ 1.82 \\ 3.25 \\ \hline 835.40 \end{array}$$

Or £3: 8: 7 $\frac{1}{2}$.72 Or 185 yds. 2 qrs. 2 na. &c. Or £835: 8
B 3 S U B-

SUBTRACTION.

Place your numbers as directed in addition, then work as in whole numbers.

Examples.

From	.591	.6198	1.	54.
Take	<u>.397</u>	<u>.1499</u>	<u>0.987</u>	<u>6.9843</u>
Remains	<u>.194</u>	<u>.4699</u>	<u>0.013</u>	<u>47.0157</u>

MULTIPLICATION.

Work in multiplication as in whole numbers, always remembering to set off as many decimals in the product as there are decimals both in your multiplicand and multiplier, and if you have not a sufficient number in your product, then prefix as many cyphers as you want.

Examples.

<u>.3416</u>	<u>.4918</u>
<u>.37</u>	<u>37</u>
23912	34426
10248	14754
<u>.126392</u>	<u>18.1966</u>
<u>37.241</u>	<u>.131461</u>
<u>12.23</u>	<u>.2132</u>
111723	262922
74482	394383
74482	131461
37241	262922
<u>455.45743</u>	<u>.0280274852</u>

DIV.

DIVISION.

Divide as if all were whole numbers, annexing cyphers to the dividend, if need be; and when you have finished the operation, point off for decimals in the quotient, so many places as the decimal parts used in the dividend exceed those of the divisor, and those to the left, if any, are whole numbers.

If the places in the quotient are not as many as the above rule requires, then supply the defect with cyphers on the left hand of the quotient.

Examples in all the Varieties.

$$246 \overline{) 160.884} (.654 \text{ quotient.})$$

$$\begin{array}{r} 1476 \\ \hline 1328 \\ 1230 \\ \hline 984 \\ 984 \\ \hline \end{array}$$

$$246 \overline{) 160884} (.0654$$

$$\begin{array}{r} 1476 \\ \hline 1328 \\ 1230 \\ \hline 984 \\ 984 \\ \hline \end{array}$$

$$2.46 \overline{) 1608.84} (.654$$

$$\begin{array}{r} 1476 \\ \hline 1328 \\ 1230 \\ \hline 984 \\ 984 \\ \hline \end{array}$$

$$2.46 \overline{) 16.0884} (6.54$$

$$\underline{1476}$$

$$1328$$

$$\underline{1230}$$

$$984$$

$$\underline{984}$$

$$2.46 \overline{) 16.0884.00} (6.5400$$

$$\underline{1476}$$

$$1328$$

$$\underline{1230}$$

$$984$$

$$\underline{984}$$

$$\underline{00}$$

$$.0246 \overline{) 160.8840} (6540$$

$$\underline{1476}$$

$$1328$$

$$\underline{1230}$$

$$984$$

$$\underline{984}$$

$$\underline{0}$$

$$.0246 \overline{) .160884} (6.54$$

$$\underline{1476}$$

$$1328$$

$$\underline{1230}$$

$$984$$

$$\underline{984}$$

The Rule of Three in Decimals.

State your question as in whole numbers, and work as
aforesaid,

Examples.

If a pound and half of butter cost a groat, what will two
pounds cost?

Thus

lb.	d.	lb.
If 1.5	4	2
		4
		d.
		1.5) 8.000 (5.33
		75 4
		50 1.32
		45
		50
		45
		5

Answer $5\frac{1}{4} \cdot 32$ d.

Two men bartered; A had 40.7 yards of shalloon, for
which B gave him 25.6 ells of Holland, at 4 s. 6 d. per ell,
what is the shalloon a yard?

Ell.	s.	Ells.
If 1	4.5	25 6
		4 5
		1280
		1024
		s. 115.20

If

Yards. s. Yard.
If 40.7 ——— 115.2 ——— 1

$$\begin{array}{r}
 \text{40.7) } 115.200(\\
 \underline{814} \\
 3380 \\
 \underline{3256} \\
 1240 \\
 \underline{1221} \\
 19
 \end{array}
 \qquad
 \begin{array}{r}
 \text{s.} \\
 2.83 \\
 \underline{12} \\
 \text{d. } 9.96 \\
 \underline{4} \\
 3.84
 \end{array}$$

Answer 2 s. $9\frac{1}{4}$.8 d. per yard. 19

A owes B £296 : 17s. but not being able to pay the whole, he compounds for 7 s. 6 d. in the pound, what must B receive for his debt?

First, the decimal of 17 s. is £.85

£ s. £
If 1 ——— 75 ——— 296.85

$$\begin{array}{r}
 7.5 \\
 \underline{148425} \\
 207795 \\
 \text{s. } 2226.375 \\
 \underline{12} \\
 4.500 \\
 \underline{4} \\
 2.000
 \end{array}$$

Answer 2226 s. $4\frac{1}{2}$.6 d
or £111 : 6 : $4\frac{1}{2}$.

Note, When more requires less, that is, when the third term is greater than the first, and requires the fourth term to be less than the second, or when less requires more, that is, when the third term is less than the first, and requires the fourth term to be greater than the second, then the proportion is *inverse*; and to find the fourth term,

Multiply the first and second terms together, and divide that product by the third term, the quotient is the fourth term, or answer to the question.

Examples.

Examples.

There is provision enough in a castle to serve a garrison of 200 soldiers for a year, and it was reported that an enemy were coming to attack it, the garrison was augmented to 500 men more, how many days will the provision serve these 500 men?

Men.		Days.		Men,
If 200		365		500
		200		
		5100		
		73000		
		145 days, answer.		

A certain garrison, containing 4000 men, hath provision sufficient for six weeks and four days, but being about to be besieged by the enemy, and no assistance to be got to relieve it in less than two months; how many men must the governor discharge out of the said garrison, that the said provision may serve the remainder four months two weeks?

First, the decimal of four days, a week the integer, is .57

For 7) 4.00 (.57

Weeks.		Men.		Weeks.
If 6.57		4000		16
		4000		

16) 26280 (1642 will serve.

$$\begin{array}{r}
 16 \\
 \hline
 102 \\
 96 \\
 \hline
 68 \\
 64 \\
 \hline
 40 \\
 32 \\
 \hline
 8
 \end{array}$$

From	4000
Take	1642

2358 men to be discharged.

Com-

Compound Rule of Three Direct.

In the compound rule of three direct there are five numbers given to find a sixth, which is to the third as the product of the two last to the product of the two first.

Multiply the first and second together for a divisor, and the fourth, fifth, and third terms together for a dividend the quotient arising by dividing the dividend by the divisor is the answer.

Example.

If 12 men in 8 days dig 48 cubic fathoms of earth to make an entrenchment, how many fathoms at the same rate will 28 men dig in 4 days?

Men.	Days.	Fath.	Men.	Days.
If 12	8	48	28	4
8			4	
96 divisor			112	
			48	
			896	
			448	
			96) 5376	(56 fathoms. Ans.
			480	
			576	
			576	

Compound Inverse Rule of Three,

Is to find a sixth number from five given numbers, which is to the third as the product of the two first is to the product of the two last. To find the sixth, multiply the product of the two first by the third, and this product divide by the product of the fourth and fifth, the quotient is the sixth, or answer.

Example.

Example.

In a garrison containing 3000 men there was, at the time of their being besieged, a provision of bread in the magazine sufficient to serve the full complement, viz. 4000 men, just three months, at 20 ounces daily allowance per man: but the governor being determined to defend the garrison six months, desires to know how much bread each man may be allowed a day for that purpose?

Men.	Months.	Ounces.	Men.	Months.
If 4000	— 3 —	20 —	3000	— 6 —
3			6	
12000			18000	
20				
18 00) 240 000 (13 $\frac{1}{3}$ ounces for each man a day.				
18				
60				
54				
6				

Extraction of the Square Root.

ANY number multiplied into itself is said to be squared, or the square of that number; thus 6 multiplied into itself, or $6 \times 6 = 36$, the square of 6; whence the following table:

Roots	1.	2.	3.	4.	5.	6.	7.	8.	9.
Squares	1.	4.	9.	16.	25.	36.	49.	64.	81.

R U L E.

When any number is given to be extracted, point it off in every other place, beginning at unity; then find the first figure in the root, whose square shall be equal to, or nearest less than the figure or figures to the first point, and then subtract that square from the first point, and to the remainder bring down the next point, and call that the resolvend. Then double the quotient or root, and set it for a divisor on the left-

left-hand of the resolvend, and seek how oft the divisor is contained in the resolvend, reserving always the units place, and put the answer in the quotient or root, and also in the units place of the divisor; then multiply the divisor by the figure last put in the quotient or root, and subtract the product from the resolvend, as in common division, and bring down the next point to the remainder, and this is the resolvend; and proceed as before till the operation is finished.

Examples.

What is the square root of 116964?

First let it be pointed thus, 11⁶9⁶4; then find the nearest square number to 11 the first period, as 9, &c. Thus:

$ \begin{array}{r} 11\overset{6}{6}9\overset{6}{6}4 \text{ (342, root.} \\ \underline{9} \\ 64) 269 \text{ resolvend.} \\ \underline{256} \\ 682) 1364 \text{ resolvend} \\ \underline{1364} \\ \hline 0 \end{array} $	$ \begin{array}{r} \text{For } 342 \\ \text{by } 342 \\ \hline 648 \\ 1368 \\ 1026 \\ \hline 116964 \text{ proof.} \end{array} $
---	---

What is the square root of 13169641?

Thus, 13¹6⁹6⁴1 (3629 root.

$$\begin{array}{r}
 9 \\
 \hline
 66) 416 \\
 \underline{396} \\
 722) 2096 \\
 \underline{1444} \\
 724) 65241 \\
 \underline{65241} \\
 \hline
 0
 \end{array}$$

When any whole number is proposed to extract the square root, and there be a remainder, the root may be found to any degree of exactness, by adding two cyphers to the remainder, and proceeding as before.

What

What is the square root of 37468?

37468 (193.566, &c.

I

29) 274
261

38 3) 168
149

3865) 21900
19325

38706) 257500
232236

387126) 2526400
2322756

203644, &c.

When a mixt number or decimal is to be extracted, point every other figure as before, beginning at the units place in whole numbers, but in the decimal always begin in the second or hundreds place, and if you have not a full period then add a cypher to compleat it, and when you have not as many figures in the root as dots or periods, then prefix a cypher or cyphers to answer them.

What is the square root of 523.176?

523.1760) 22.87, &c.

4

42) 123
84

448) 3917
3584

4567) 33360
31969

1391, &c.

What

What is the square root of .143219?

.143219 (.3784, &c.

9

67) 532
469

748) 6319
5984

7564) 33500
30256

3244, &c.

What is the square root of 7?

7.000000 (2.645, &c.

4

46) 300
276

524) 2400
2096

5285) 30400
26425

3975, &c.

What is the square root of .0007612816?

.0007612816 (.02759

4

47) 361
329

545) 3228
2725

5509) 50316
49581

735

T.

To find the square root of a vulgar fraction, reduce it into a decimal, and extract the square root of that decimal, and the thing is done.

What is the square root of $\frac{4}{9}$ or $\frac{1}{2}$?

Thus $\frac{4}{9}$ or $\frac{1}{2}$ in decimals is .5 then .5000 (.707, &c.

$$\begin{array}{r}
 49 \\
 \hline
 1407 \overline{) 10000} \\
 \underline{9849} \\
 151, \text{ \&c.} \\
 \hline
 \end{array}$$

To extract the Cube Root.

R U L E.

First point off from the unity place every *third* figure, find the first figure in the root by the following table, subtract it from the first period, augment the remainder by the first figure in the next point, this call the dividend; divide this dividend by the triple square of the whole root last found, the quotient figure found annex to the root; cube the root thus found, and subtract it from as many points as you have brought down; to the remainder bring down the first figure of the next point for a new dividend, divide this new dividend by the triple square of the root, the quotient figure annex to the root again, cube the whole root, and subtract it from as many points as you have brought down, and proceed in this manner till the work is ended.

The TABLE of POWERS.

Roots	1	2	3	4	5	6	7	8	9
Squares	1	4	9	16	25	36	49	64	81
Cubes	1	8	27	64	125	216	343	512	729

C

Thus

Thus to extract the cube root of 13312053, the number is first to be pointed after this manner, viz. 13312053, then you are to write in the root the figure 2, whose cube 8 is the next less cube to the period 13, which is not a perfect cube number, and having subtracted that cube according to the rule, there will remain 5, which being augmented by the next figure of the resolvend 3, and divided by the triple square of the root 2 or 12, by seeking how oft 12 is contained in 53, it gives 4 for the second figure of the root, but since the cube of 24, viz. 13824 would come out too great to be subtracted from 13312 that precedes the second point, there must only 3 be wrote in the root, then 23 cubed is 12167, which taken from 13312 will leave 1145, which augmented by the next figure of the resolvend 0, and divided by the triple square of the quotient 23, viz. 1587, it gives 7 for the third figure of the root; then 237 cubed and subtracted from the three periods or the whole resolvend, there remains 0. Whence the true root is 237. See Sir Isaac Newton's *Arithmetica Universalis*.

Examples.

$$\begin{array}{r} 13312053 \text{ (237 Root.} \\ 8 \end{array}$$

$$2 \times 2 = 4 \times 3 = 12 \overline{) 53} (4 \text{ or } 3.$$

Subtract cube 12167.

$$23 \times 2 = 529 \times 3 = 1587 \overline{) 11450} (7$$

Subtract cube 13312053

Remains 0

What is the cube root of 27054036008?

$$\begin{array}{r} 27054036008 \text{ (3002 Root.} \\ 27 \end{array}$$

$$3 \times 3 = 9 \times 3 = 27 \overline{) 0542} (2$$

3002 cubed is 27 054036008

0

What

What is the cube root of 122615327232?

122615327232 (4968 Root.
64

$4 \times 16 \times 3 = 48$) 586 (9

49 cubed, subtract 117641

$49 \times 49 = 2401 + 3 = 7203$) 49743 (6

Subtract 496 cubed, viz. 122023936

$496 \times 496 = 246016 \times 3 = 738048$) 5913912 (8

Subtract the cube of 4968, viz. 122615327232

Remains 0

When the resolvend has a remainder, you may carry on the root to any degree of exactness by adding three cyphers for each new period you intend to carry the root to in places of decimals; and always remember to point off every third figure from the tenths place in decimals, the whole numbers as before.

What is the root of 61218.00121?

61218.001210 (39.41 root
27

$3 \times 3 = 9 \times 3 = 27$) 342 (9

39 cubed 59311

$39 \times 39 = 1521 \times 3 = 4563$) 19070 (4

394 cubed 61162984

$394 \text{ sqd} = 155236 \times 3 = 465648$) 550172 (1

3941 cubed, is 61209566621

Remainder 8434589

C 2

What

What is the cube root of .0069761218?

.006976121800000 (.19107

I

$$1 \times 1 = 1 \times 3 = 3 \quad 59 \quad (9$$

$$19 \text{ cubed} \quad 6859$$

$$19 \times 19 = 361 \times 3 = 1083 \quad 1171 \quad (1$$

$$191 \text{ cubed} \quad 6967871$$

$$191 \times 191 = 36481 \times 3 = 109443 \quad 82508 \quad (0$$

$$1910 \text{ cubed} \quad 6967871000$$

$$1910 \text{ fqd.} = 3648100 \times 3 = 10944300 \quad 82508000 \quad (7$$

$$19107 \text{ cubed, is} \quad 6975534818043$$

$$\text{Remiander} \quad 586981957$$

P A R T II.

T H E

L A W S O F M O T I O N.

D E F I N I T I O N S.

1. **T** H E *place* of a body is that part of space, which a body occupies.

2. *Motion* is a continual change of place.

In motion there are three things to be considered, the body which is moved, the space which is passed over or described, and the time in which it is described.

3. The *direction* of motion is a right line which a body in motion describes, or endeavours to describe.

4. *Equable* or *uniform* motion is that, by which a body in motion describes or passes over equal spaces in equal times.

5. *Accelerated* motion is that, by which a body in motion describes continually greater spaces in equal times; or which is continually increased.

6. *Retarded* motion is that, which continually decreases in equal times; and if the increase or decrease of motion be equal in equal times, the motion is then said to be equally accelerated, or retarded.

7. *Velocity* or *celerity* is that affection of motion, whereby a body in motion passes over, or describes a given space in a given time.

Let V express the velocity, T the time, and S the space; when it is said, the velocity is directly as the space, and inversely as the time; or the velocity is as the space directly, and time reciprocally, it is expressed thus $V : \frac{S}{T}$: so also the time $T : \frac{S}{V}$, that is, the time is as the space directly and velocity inversely. Hence theorems may be determined for the time, velocity, space passed over, in equable and accelerated motion, as follows.

Let Q and q express the quantities of matter in two bodies.

V and v , their respective velocities.

M and m , their *momenta*, or quantity of motion.

S and s , the spaces described or passed over in the times T and t respectively. Then

I.

If the velocities are equal, the *momenta* will be as the quantity of matter in the moving bodies, that is

If $V = v$

Then $M : m :: Q : q$

Whence $Mq = mQ$

$$\begin{array}{ll} \text{And 1.} & M = \frac{mQ}{q} \\ & 2. \quad q = \frac{mQ}{M} \\ & 3. \quad m = \frac{Q}{\frac{Mq}{Q}} \\ & 4. \quad Q = \frac{Mq}{m} \end{array}$$

II.

If the quantities of matter in the moving bodies are equal, the *momenta* will be as their respective velocities. That is

If $Q = q$

Then $M : m :: V : v$.

III.

The ratio of the *momenta* is in complicate ratio of the quantities of the matter and the velocities. That is

$$M : m :: Q \times V : q \times v.$$

IV.

The ratio of the velocities is composed of the direct ratio of the *momenta*, and the inverse ratio of the quantities of matter. That is

$$V : v :: q \times M : Q \times m.$$

V.

If the *momenta* are equal, the velocities will be inversely as the quantities of matter in the moving bodies. That is

If $M = m$

Then $V : v :: q : Q$

VI.

VI.

If the times are equal wherein bodies move, the spaces described will be as the velocities with which they move
That is

$$\begin{aligned} &\text{If } T = t \\ &\text{Then } S : s :: V : v. \end{aligned}$$

VII.

If the velocities are equal, the spaces described by the moving bodies, will be directly as the times. That is

$$\begin{aligned} &\text{If } V = v \\ &\text{Then } S : s :: T : t \end{aligned}$$

VIII.

In all cases, the ratio of the spaces described is in the compound ratio of the times and velocities conjointly. That is

$$S : s :: T \times V : t \times v.$$

IX.

If the spaces described are equal, the velocities are in the reciprocal ratio of the times. That is

$$\begin{aligned} &\text{If } S = s \\ &\text{Then } V : v :: t : T. \end{aligned}$$

X.

The ratio of the times is compounded of the direct ratio of the spaces described, and the reciprocal ratio of the velocities. That is

$$T : t :: S \times v : s \times V.$$

XI.

The ratio of the velocities is compounded of the direct ratio of the spaces described, and the reciprocal ratio of the times. That is

$$V : v :: t \times S : T \times s.$$

XII.

The ratio of the times is compounded of the direct ratio of the spaces described, divided by the velocities.

$$T : t :: \frac{s}{V} : \frac{v}{S}.$$

C 4

These

These are the laws of motion deduced from those general laws of Sir *Isaac Newton*, in his *Principia*, and are elegantly demonstrated in Mr *Emerson's* *Mechanics*; whence equations may be deduced, as given in the first.

SCHOLIUM.

MOTION is measured by the *velocity and quantity of matter*. Therefore if a ball of iron, and a ball of wood of the same bigness be projected with the same velocity, there will be more motion in the ball of iron, than in that of wood; so likewise, if two equal leaden balls, the one solid, the other hollow and empty, be moved with the same velocity; the solid ball will have more motion than the hollow one, and will strike a body against which it is thrown with greater force; and the quantity of matter, which is properly contained in any body, is to be determined by its weight: wherefore the quantity of motion is not to be measured by the velocity and bigness, but by the velocity and weight of the body in motion, which is carefully to be observed; and all bodies resist in proportion to their *density*, that is, to the quantity of matter contained in them.

Whence we have the following conclusions.

1. Velocity = $\frac{\text{space}}{\text{time}}$.
 2. Space = velocity multiplied by the time.
 3. Time = $\frac{\text{space}}{\text{velocity}} = \sqrt{\frac{\text{space}}{16\frac{1}{2}}}$ of describing any space.
- Momentum, or the quantity of motion, is compounded of the velocity and quantity of matter, or weight. Whence
4. Momentum = velocity multiplied by the weight.
 5. Weight = $\frac{\text{momentum}}{\text{velocity}}$.
 6. Velocity = $\frac{\text{momentum}}{\text{weight}}$.

Of Gravity, Descent of heavy Bodies.

The velocities of descending heavy bodies, are proportional to the times from the beginning of their fall. This follows, because the action of gravity being continual, in every space of time, the following body receives a new impulse, equal to what it had before in the same space of time received from the first

P. II. DESCENT of Heavy BODIES. 25

first power, viz. In the first second of time, a body hath acquired a velocity of $32 \frac{1}{2}$, for an heavy body let fall from any height near the surface of our earth, descends in a second of time $16 \frac{1}{2}$ feet, and were there no new force, it would continue to descend at that rate with an *equable motion*: but in the next second of time, the same power of gravity continually acting thereon, superadds a *new velocity* equal to the former, so that at the end of two seconds, the velocity is double to what it was at the end of the first, and after the same manner, may it be proved to the triple at the end of the third second, and soon; wherefore the velocities of falling bodies are proportional to the times of their falls.

Cor. 1. The spaces described by the fall of a body are as the squares of the times from the beginning.

Cor. 2. The velocities acquired by bodies falling are in proportion to the squares of the times in which they fall; therefore the velocity is as the square root of the height fallen.

Cor. 3. All bodies descending or ascending gain or lose equal velocities in equal times, and whatever a velocity a falling body gains in any time; if it be projected directly upwards, it will lose as much in an equal time; therefore if a body be projected upwards with the velocity it acquired by falling in any time, it will in the same time lose all its motion; hence also bodies projected upwards lose equal velocities in equal times.

Cor. 4. If a body be projected upwards with the velocity it acquired in falling, it will in the same time ascend to the height it fell from; and describe equal spaces in equal times, both in ascending and descending, but in an inverse order; and will have the same velocity at every point of the line described; for gravity acts incessantly and at every instant of time from the very beginning to the end of the fall.

Cor. 5. If bodies be projected upwards with any velocities, the heights of their ascents will be as the squares of the times of their ascending.

All these things would be true, if it was not for the resistance of the air, which is very great in swift motions, and has a great effect in destroying the motions of bodies.

Cor. 6. The forces requisite to throw equal bodies to different heights, are to each other, as the square roots of heights.

Therefore any distance or depth may be found by this.

P R O.

PROPOSITION. I.

Given the time in seconds of a body descending to find the space it has fallen through.

R U L E.

Multiply the square of the seconds, the body took in falling by $16\frac{1}{2}$ feet, and the product gives the space it has described.

Example.

Suppose a bullet was 4 seconds in falling from the top of a steeple, how high is it?

"	"	sq. time	feet
4	×	4	= 16
			×
			$16\frac{1}{2}$
			= 257 $\frac{1}{2}$

Thence may be determined the height of any ball that is projected perpendicular to the horizon.

For half the time of the ball's continuance in the air, is the time of its descent.

Example.

Suppose a cannon placed perpendicular to the horizon, and then fired off in that direction, it was found by observation, that the ball staid in the air 23 seconds, how high did the ball ascend?

$$\begin{array}{r} 2 \overline{) 23 \text{ seconds}} \\ 11.5 \end{array}$$

$11.5 \times 11.5 = 132.25 + 16\frac{1}{2} = 2127.02$
feet the height the ball ascended: but because of the resistance of the air it will retard the motion.

PROPOSITION II.

Given the height of any place, to determine in what time a heavy body let fall from the top will reach the bottom.

R U L E.

Divide the given height in feet by $16\frac{1}{2}$ or 16.09 and the quotient is the square of the time, whose square root is the number of seconds required.

Example.

Admit there is a precipice 2127.02 feet high, in what time would a bullet be falling to the bottom?

$$16.09 \overline{) 2127.02} \quad 132.19 \quad 11.5 \text{ seconds.}$$

It is easy to demonstrate, that the time of the least vibration of a pendulum is to the time of the *fall* of a body from the height of half the length of the pendulum, as the circumference of a circle, to its diameter very near. Whence it follows,

As the square of the diameter, is to the square of the circumference; so is half the length of a pendulum vibrating seconds, to the space described by the fall of a body in the first second of time. The length of a pendulum vibrating seconds is found to be 39.13 inches, and the descent in that second will be found to be $16\frac{1}{12}$ feet.

Now $\frac{2}{3.1416} =$ time of falling through 2×39.13 . Whence by Cor. 1. we have $\frac{4}{3.1416^2} : 2 \times 39.13 :: 1^2 : \frac{39.13}{2} \times \frac{1}{3.1416^2} = 193.096 \text{ inches} = 16.083 \text{ feet.}$

Of P E N D U L U M S.

The lengths of pendulums are always accounted from the point of suspension to the center of ball or bob, but if a large ball, then from the point of suspension to the center of oscillation; and their lengths are to each other as the squares of the times of vibration performed in one and the same time.

PROPOSITION I. PROB.

To find the length of a pendulum which shall make any assigned number of vibrations in a minute.

R U L E.

Say, as the square of the assigned number of vibrations, is to the square of 60, the seconds in a minute; so is 39.13, the length of a royal pendulum, to the length of the pendulum required.

Example.

What is the length of a pendulum that vibrates half-seconds, or 120 times in a minute;

$$120 \times 120 = 14400$$

$$60 \times 60 = 3600$$

Then as 14400 : 3600 :: 39.13 : 9.78, &c. inches the length required.

Or the length of a pendulum may be found thus.

Divide 140848 by the square of the number of vibrations assigned, and the quotient gives the length of the pendulum required.

Exam-

Example.

What is the length of a pendulum that swings double seconds, or vibrates 30 times in a minute?

The square of 30 = 900

900)140868(156.52 inches.

PROPOSITION II. PROB.

Given the length of a pendulum, to find the number of vibrations such a pendulum shall make in a minute.

R U L E.

As the given length in inches : to 39.13 :: so is the square of the time given : to the square of the number of vibrations, whose square root is the number sought.

Examples.

How many times does a pendulum 9.78 inches long vibrate in a minute?

inch. inch.

As 9.78 : 39.13 :: 3600 : 14400, whose square root is 120, the number sought.

Or divide 140868 by the length of the pendulum, the quotient is the square of the number of vibrations sought.

Example.

How many vibrations will a pendulum make in a minute, that is 156.52 inches long?

inches

156.52)140868(900(30 long.

P R O B L E M.

Holding in my hand a string and bullet, whose length from my finger, the point of suspension, to the center of the bullet, was 33 $\frac{1}{2}$ inches; I observed the flash of a cannon, the same instant I let the bullet a swinging and it made 11 vibrations between the time of seeing the flash and hearing the report, how far was the cannon from me?

inch inch

As 33.5 : 39.13 :: 3600 : 4205, whose square root is 64.8 the number of vibrations the bullet made in a minute, then

vib. vib. seconds
as 64.8 : 60 :: 11 : 10.18.

feet

But sound flies 1142 feet in a second, then 1142 $\times$ 10.18 = 11625.56 feet = 2.2 miles.

P A R T

P A R T III.

FIG.

O F

G E O M E T R Y.

D E F I N I T I O N S.

1. **G**EOMETRY is a science of whatever is extended, viz. of lines, superficies, and solids. It is a science of enquiring, inventing, and demonstrating all the affections of magnitude.
2. A *point* hath neither parts nor dimensions, and incapable of being divided.
3. A *line* is length without breadth, and is made by the flowing of a point.
4. A *right line* is the nearest distance between two points as A B. 1.
5. A *superficies* has length and breadth, but without thickness.
6. A *solid* hath length, breadth, and depth or thickness.
7. An *angle* is the inclination of two lines the one to the other, meeting in a point called the angular point, as A B, A C, in A. 2.
8. If lines that form the angle be right lines, it is called a *right lined angle*.
9. When a right line C D standing upon another right line A B, makes on both sides thereof the angles C D A, C D B, equal one to the other, then both those angles are *right angles*; and the right line C D, which stands upon the other is called a *perpendicular*. 3.
- Cor.* When an angle is mentioned it is noted by three letters, the middle letter always denotes the angular point, as C D A, D is the angle denoted.
10. An *obtuse angle* is greater than a right angle, as E D B; for C D B is a right angle. 4.

11. An

FIG. 11. An *acute angle* is less than a right angle, as ADE ; for
4. ADC is a right angle: acute and obtuse angles are called
oblique angles.

Cor. The lines that form an angle, are called its *legs*.

12. An *equilateral triangle* hath three equal sides, as the triangle A .

13. An *isosceles triangle* hath only two sides equal, as the triangle B .

14. A right angled triangle hath one right angle as at n ,
 db is called the *hypothenuise*, bn the *perpendicular*, dn the
base, as the triangle C .

15. The triangle D is called an *obtuse angled triangle*,
for m is an obtuse angle.

5. 16. The *altitude* or *height* of any triangle is the perpendi-
6. cular CD , let fall from the vertex C to the base AB ; and
it may fall either within or without the triangle.

17. *Parallel lines* are such, if infinitely produced, would
7. never meet, as AB, CD .

8. 18. A *square* $ABCD$ has four equal sides, and is right
angled, and therefore equi-angled.

9. 19. A *parallelogram* $ABCD$ hath its opposite sides parallel,
and consequently equal.

10. 20. A *trapezium* is a four-sided figure, whose opposite sides
are not parallel; and if a line be drawn across it from corner
to corner, it is called a *diagonal line*, as AB .

11. 21. A *circle* is a plane figure contained under one line
only, called its *circumference* or *periphery*. AB is the *diameter*,
 AD or CD the *radius*.

Cor. The circumference of every circle is divided into 360
degrees, and is the most capacious of any figure.

22. All figures that have more than four sides are called
polygons, and are named from the number of their sides, and
when their sides are equal, they are called *regular polygons*.
As if it be of

Five sides	- -	is - -	a pentagon.
Six sides	- - - -	- -	an hexagon.
Seven sides	- - - -	- -	an heptagon.
Eight sides	- - - -	- -	an octagon.
Nine sides	- - - -	- -	a nonagon.
Ten sides	- - - -	- -	a decagon.
Eleven sides	- - - -	- -	endecagon.
Twelve sides	- - - -	- -	dodecagon.

23. A *problem* is a *proposition* which proposes something to be done. FIG.

24. A *theorem* is when something is proposed to be demonstrated.

25. A *corollary* is a consequence drawn from something that has been already demonstrated.

26. A *scholium* is a remark on any proposition or corollary.

27. An *axiom* is such a common, plain, self-evident and received notion, that it cannot be made more plain and evident by demonstration, because itself is better known than any thing that can be brought to prove it.

P R O B. I.

To divide a given right line into two equal parts, that is, to bisect a right line.

Let the given line be A B, then with one foot of the compasses in A, opened to any distance greater than half the line A B, describe an arch, then with the same opening of your compasses set one foot in B, describe another arch, and where these two arches cut one another, as in *b* and *d*, if the line *b d*, be drawn, that will bisect the given line A B. 12.

P R O B. II.

From a point in a given line to raise a perpendicular.

Let A B to be the given line, and C the given point; take two points equally distant from C, as D, E, and setting the foot of the compasses in D and E, opened to any distance greater than A C, strike two arches, and where they intersect one another as in *b*, there draw *b c*, which shall be the perpendicular required. 13

P R O B. III.

At the end of a given line to erect a perpendicular, as suppose at B.

At any convenient distance out of the given line as at E, set one foot of the compasses, and extend the other to the given point B, with this distance draw a circle, and from the intersection in the given line as at D draw a line through the point E to intersect the arch above in C, then C B drawn will be the perpendicular. 14.

N. B. By this problem may as many perpendiculars as you please be raised from given points on a right line, as well as from the end.

P R O B.

FIG.

P R O B. IV.

- To let fall any perpendicular on any given line AB, from a given point above that line, as suppose from C.*

From the point C describe an arch as cd , cutting the line AB, then with the same distance of the compasses and one foot in c and d ; describe two arches below, and where they intersect as at e , draw Ce , it will be the perpendicular required.

P R O B. V.

With a given right line to make an angle of any number of degrees.

Example 1.

16. Let it be required to lay down an angle of 30 degrees 30 minutes on AB from the point A.

Take the distance of 60 degrees with your compasses from the line of chords, then setting one foot in A, with that distance, describe an arch as CD, then take $30^{\circ} 30'$ from the line of chords, and set it off from C to D, then a line drawn from A through D, the angle DAC is the angle required.

Example 2.

- To lay down an angle above 90 degrees.
17. From A, on a given line AB, it is required to make an angle of 145 degrees. With the distance of 60 from the line of chords sweep an arch, then since the line of chords extends only to 90 degrees, set it off on the arch at twice; as first take 90° , then 55° to C, and the angle is made by drawing AC.

P R O B. VI.

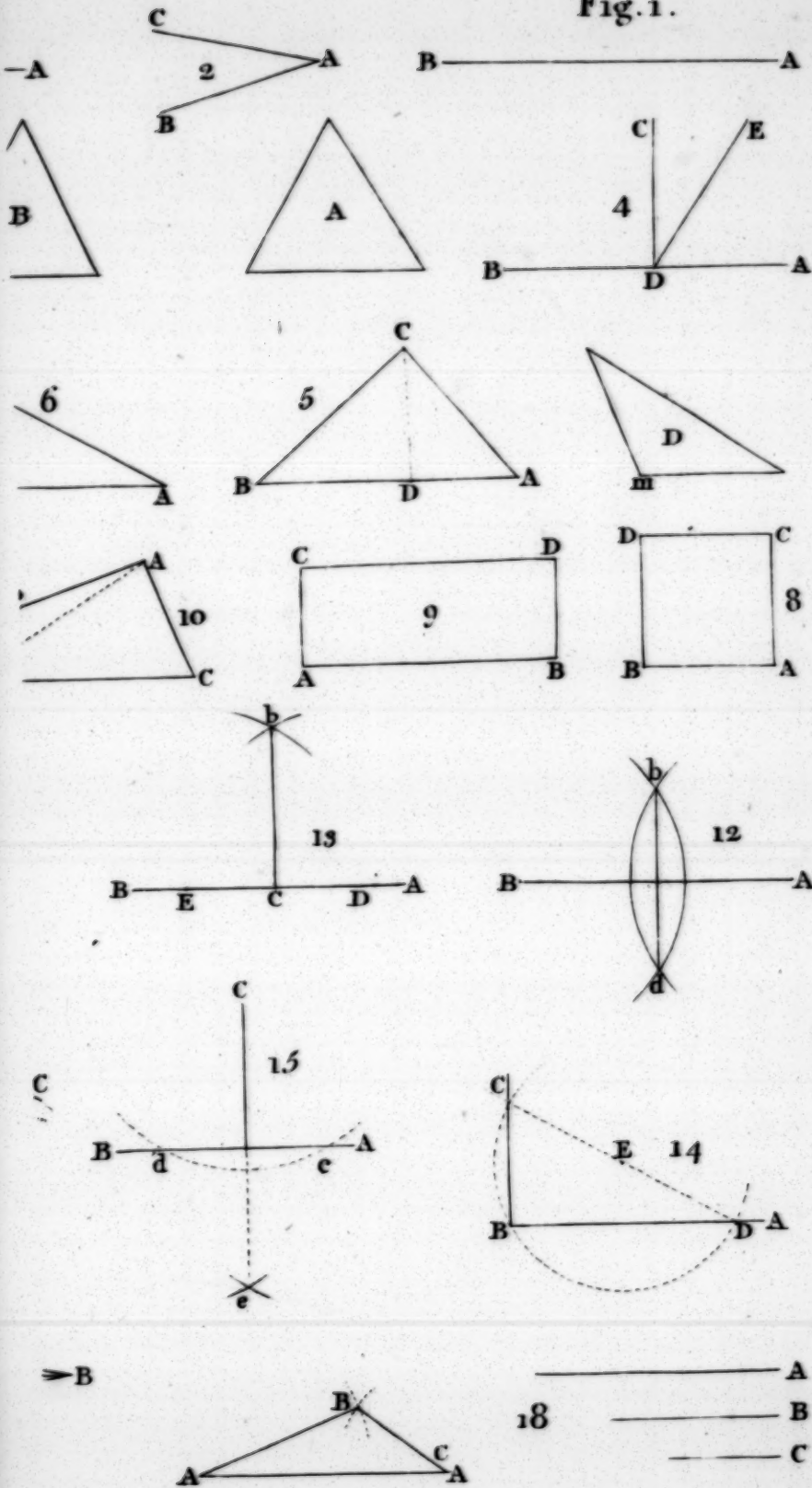
Three lines being given to make thereof a triangle.

18. Let the lines be A, B and C, whose lengths are 12, 9 and 6 respectively.

Make AC equal to the line A, or 12 from a scale of equal parts, take the length of the line B or 9, and with one foot of the compasses in A, describe an arch, then with the compasses take the extent of the line C, or 6 equal parts from the same scale, and with one foot in C describe another arch, and where they cut one another as in B, join AB and BC, and the triangle is constructed.

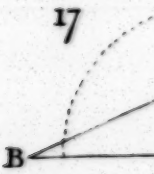
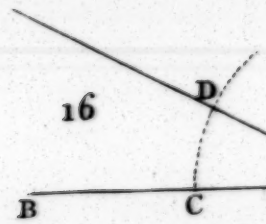
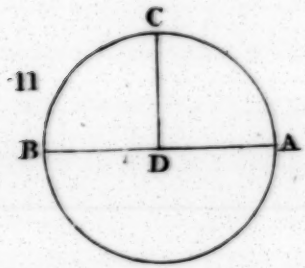
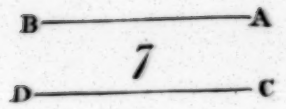
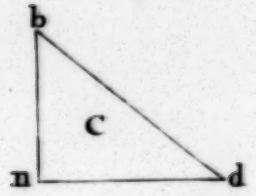
P R O B.

Fig. 1.



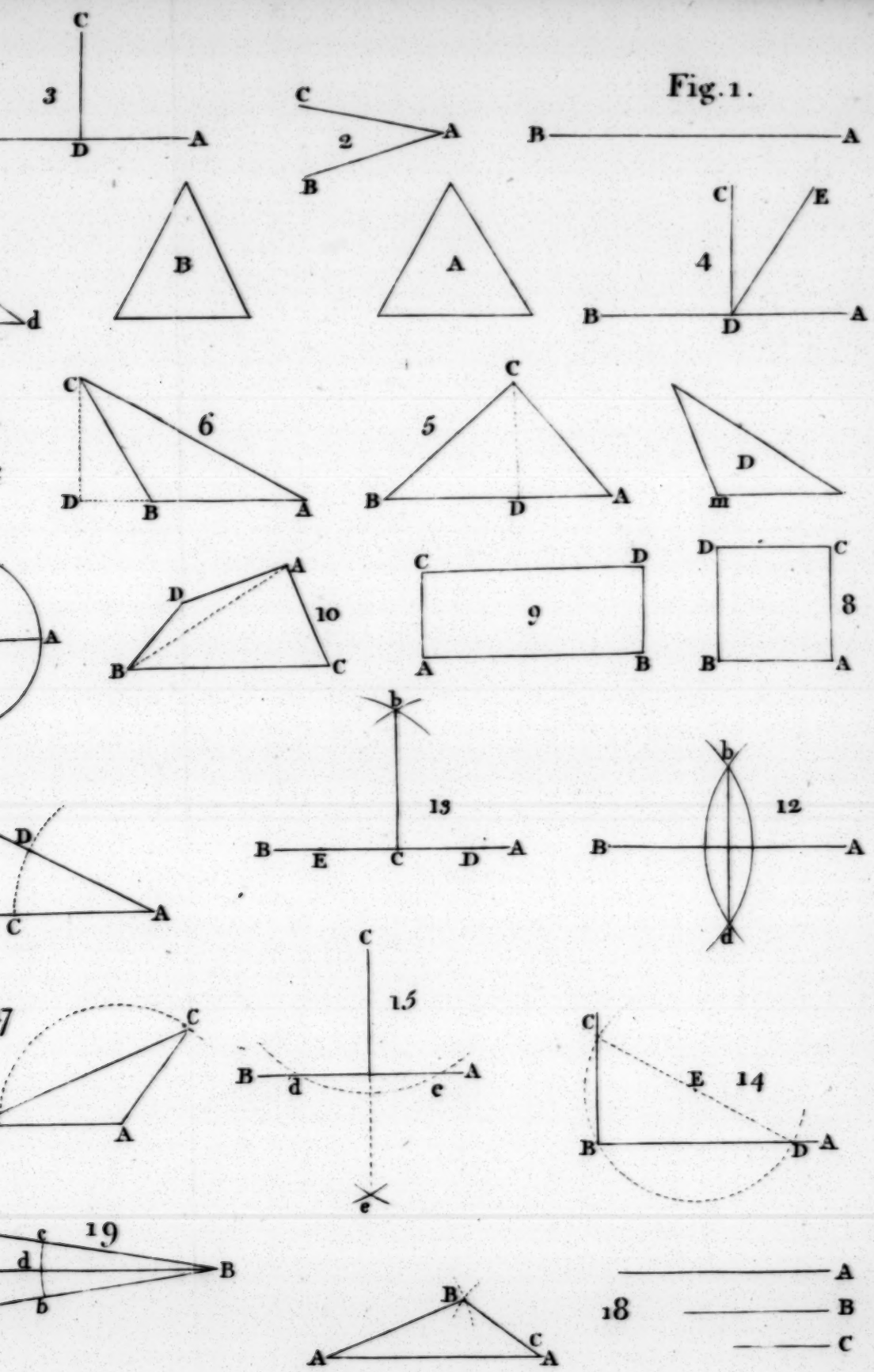
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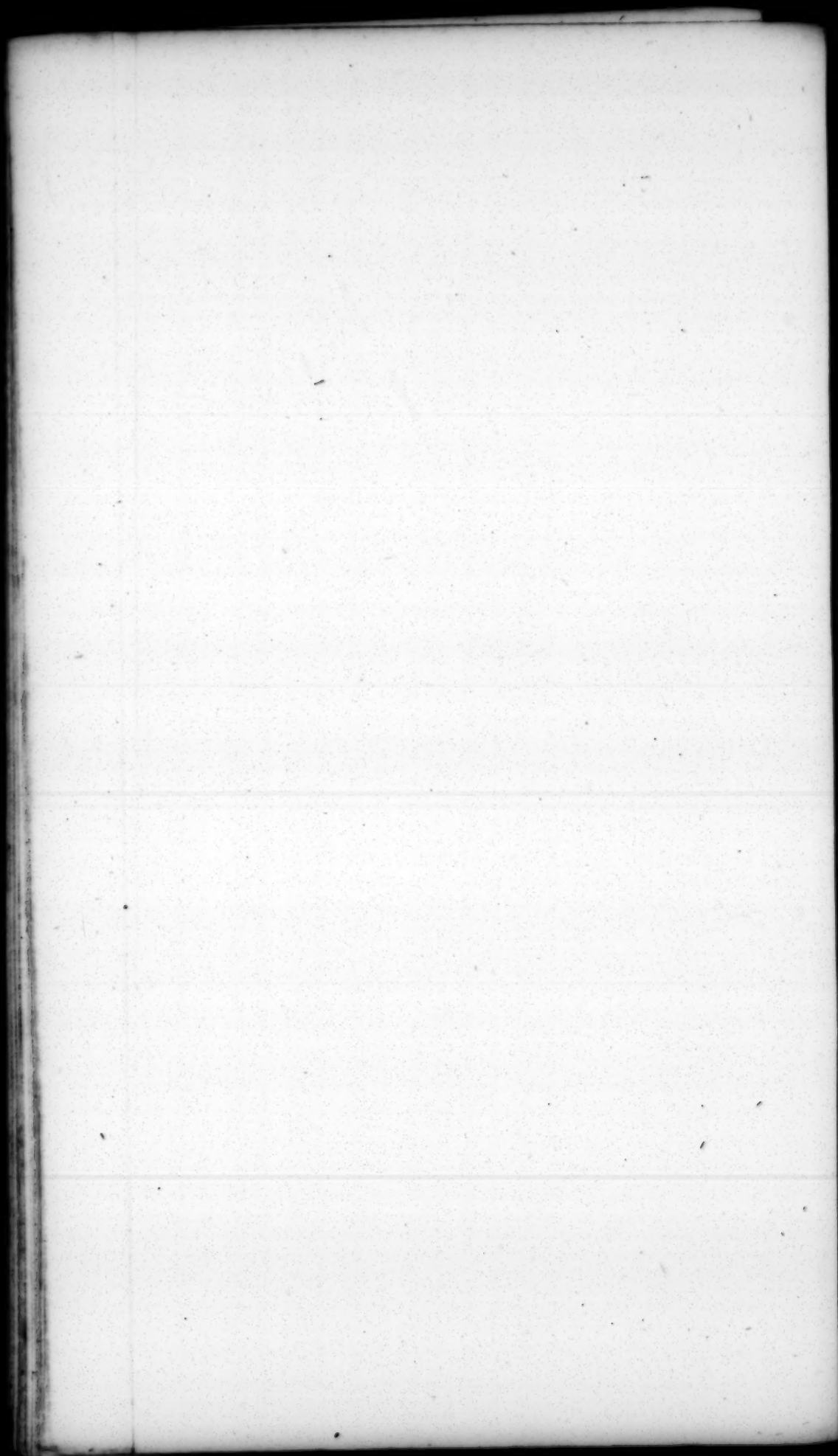
B



Geometry

Fig. 1.





P R O B. VII.

To bisect or divide an angle ABC into two equal parts.

Set one foot of the compasses in B, and describe an arch 19. *b c* at any extent, take *b d* equal to half *b c*, then through *d*, draw the line BD, and the angle B will be divided into two equal parts.

P R O B. VIII.

Through a given point C to draw a parallel line to a given right line A B.

Take any point in the given line A B, as F, and with the 20. distance F C, describe a semicircle; take the arch E D equal to the arch B C, then through D draw the line C D, and it will be parallel to A B.

P R O B. IX.

To draw a circle through three points, as ABC, not lying in a straight line.

Join the points A B, and A C; bisect A B and B C (by 21. prob. 1.) then draw the bisecting lines E and F, and where they meet, as in D, then the point D is the center of the circle required.

Cor. By having the segment of any circle given, the whole circle may be completed by taking any three points in the segment's arch, and proceeding as above.

S C O L I U M.

By this problem may be known what kind of ordnance an enemy makes use of by collecting the segments or pieces of the shell shot from their artillery; for the circle may be perfected by drawing two lines A B and B C, within the given arch, and bisecting them.

P R O B. X.

To inscribe in a given circle the side of a pentagon.

Draw the diameters A G and B H at right angles to one 22. another, bisect the radius B C in F, and draw A F; make F E equal to F C, and through the point E, draw B D; then B D is the side of the pentagon required.

D

P R O B.

FIG.

P R O B. XI.

To inscribe a circle EFG in a given triangle ABC.

23. Bisect the angles B and C with the right lines BD, CD, meeting in point D; and draw the perpendiculars DE, DF, DG, from the center D; describe a circle through E, and it will pass through F and G, and touch the sides of the angle.

Cor. For the angle DBE = DBF by construction, and the angle DEB = DFB, and DB common, therefore DE = DF = DG.

S C H O L I U M.

Hence the sides of a triangle being known, their segments which are made by the touchings of the circle inscribed may be thus found.

Let $AB=12$; $AC=18$; $BC=16$.

1. $AB+BC-AC=BE+BF=10$, and $BE=BF=5$. And $AC=AE+FC$: also $FC=CG$.

Therefore $BC-BF=FC=CG=11$; and $BA-BE=AE=AG$.

P R O B. XII.

To describe a segment of a circle upon a given right line AB, that shall contain an angle equal to a given angle F.

24. Make an angle BAE equal to the given angle F. From G, the middle of AB, raise GC perpendicular to AB, and draw AC perpendicular to AE, cutting GC in C; with the radius CA from C, as a center, describe a circle; then will the segment ADB contain an angle ADB equal to the given angle.

P R O B. XIII.

Upon a given right line AB, to describe any regular polygon.

Divide 360 degrees in a circumference, by the number of sides of the required polygon, and the quotient is the angle at the center; and the angle at the center taken from 180 degrees, leaves the angle at the circumference.

Hence is constructed the following

T A B L E.

TABLE.

Polygons names.	Equal sides	Angles at the center.		Angles at the circumference.	
Trigon	3	120°	00'	60°	00'
Square	4	90	00	90	00
Pentagon	5	72	00	108	00
Hexagon	6	60	00	120	00
Heptagon	7	51	25 $\frac{1}{2}$	128	34 $\frac{3}{4}$
Octagon	8	45	00	135	00
Nonagon	9	40	00	140	00
Decagon	10	36	00	144	00
Endecagon	11	32	43 $\frac{1}{4}$	147	16 $\frac{1}{4}$
Dodecagon	12	30	00	150	00

The use of the above table is to construct any of the regular polygons upon a given right line, as follows.

Example 1.

Upon a given line AB to describe a hexagon.

At A and B make the angles each equal to 120 degrees, 25, as in the table, then draw AC and BD each equal to AB; then at C and D, make angles each equal to 120 degrees as before, and make CE, DF, each equal to AB, draw EF and the hexagon is constructed.

Example 2.

Upon a given line AB to construct a pentagon.

Make the angles A and B each equal to 108 degrees as in 26. the table, then draw the lines AC and BD each equal to the given line AB; on the point C and D, with the compasses opened to the distance AB, describe arches crossing each other in E, and the pentagon is constructed.

By the sector.

But such like questions as these relating to regular polygons may be easily answered by the scales of polygons on the inner side of the sector marked *pol.* these scales begin at 4 and figured backwards to 12 sides; the transverse distance from

D 2

6 to

FIG. 6 to 6, (which may be opened to any distance) is equal to the radius of the circle circumscribing the polygon, the legs of the sector being kept open at the required distance; then the interval between 5 and 5, is the side of a pentagon; between 6 and 6, is the side of a hexagon; between 7 and 7, is the side of a heptagon; between 8 and 8, is the side of an octagon, &c.

Or if you would have the radius of a circle circumscribing a regular polygon, open the sector till the transverse distance be equal to the side of the given polygon, the sector being kept at that opening, the interval between 6 and 6 is the radius of the circle, in which that polygon may be inscribed; this being so plain, examples would be needless.

Some of the most useful THEOREMS in plane Geometry demonstrated.

IT will be proper to premise first the following *axioms*, being self-evident principles.

I. Things that are equal to one and the same thing, are equal to one another.

II. If equal things be added to equal thing, the sum will be equal.

III. If equal things be subtracted from equal things, the remainder will be equal.

IV. If equal things be multiplied by equal things, the product will be equal.

V. If equal things be divided by equal things, the quotient will be equal.

VI. Things which being laid upon one another do agree, or meet in all their parts, are equal to one another.

VII. Every whole is equal to all its parts taken together; and therefore every whole is greater than its part.

VIII. All right angles are equal the one to the other.

THEOREM I.

If a right line stands upon another right line, the angles which it makes with it will either be two right angles, or together equal to two right angles.

DEMONSTRATION.

27. Let the lines be AB and CD meeting in the point C, upon which point describe any circle at pleasure.

Then

Then the arch AD is the measure of the $\angle ACD$, and the FIG. arch DB of the $\angle DCB$; but the arches $AD + DB = 180$ degrees, since they complete a semicircle; consequently $\angle ACD + DCB = 180^\circ =$ to two right angles. *Q. E. D.*

Corollary 1. Hence it follows, that if $\angle ACD = 90^\circ$, then must $\angle DCB = 90^\circ$; but if $\angle ACD$ be obtuse, then the $\angle DCB$ will be acute.

Cor. 2. And if ever so many right lines stand upon a right line at one and the same point, all the angles taken together will be equal to two right angles.

T H E O R E M II.

If two lines cut or cross each other, the two opposite angles will be equal.

D E M O N S T R A T I O N.

Let the two right lines be AC, DE, crossing each other 28 in B.

Then by *theo. 1.* $\angle ABD + \angle DBC = 180^\circ = \angle DBC + \angle CBE$; hence by axiom 3. $\angle ABD = \angle CBE$. In like manner it is proved that $\angle DBC = \angle ABE$. *Q. E. D.*

Cor. Hence it is evident that if two or more right lines intersect each other, in one and the same point, they will make twice as many angles as there are lines; and all the angles that are capable of being formed about one point, are equal to a whole circle, or 360 degrees.

T H E O R E M III.

Any right line crossing two parallels makes the alternate angles on the same side equal to one another, i. e. $\angle AHF = \angle CGF$.

D E M O N S T R A T I O N.

Let the two parallel right lines AB, CD be crossed by 29 the line EF.

Since parallel lines are such right lines, in some plane, which if infinitely produced will never meet, and Th. parallel lines must have the same inclination to any right line that crosseth them, and consequently $\angle AHF = \angle CGF$. *Q. E. D.*

T H E O R E M IV.

Any right line crossing two parallels (see the last figure) makes the alternate angles on different sides equal, i. e. $\angle AHG = \angle HGD$.

Fig.

DEMONSTRATION.

By th. 3. | 1 | $\angle AGH = \angle CGF$.
 Th. 2. | 2 | $\angle CGF = \angle HGD$.
 By Ax. 1. $1 = 2$. | 3 | $\angle AGH = \angle HGD$. *Q. E. D.*

THEOREM V.

If a right line cross two parallels, the two internal angles are equal to two right angles, i. e. $\angle AHG + \angle CGH = 180^\circ$.

DEMONSTRATION.

By theo. 4. $\angle AHG = \angle HGD$, and by ax. 2. adding $\angle CGH$ to both sides we have $\angle AHG + \angle CGH = \angle HGD + \angle CGH$, but $\angle HGD + \angle CGH = 180^\circ$, and therefore by ax. 1. $\angle AHG + \angle CGH = 180^\circ$. *Q. E. D.*

THEOREM VI.

The three angles of every plane triangle are equal to two right angles.

DEMONSTRATION.

30. Draw the right line DE through the angular point C , and parallel to the side AB . Then by theo. 4. $\angle A = \angle ACD$ and $\angle B = \angle BCE$; hence by ax. 2. $\angle A + \angle B = \angle ACD + \angle BCE$, to which add the $\angle ACB$, and we have $\angle A + \angle B + \angle ACB = \angle ACD + \angle BCE + \angle ACB$; but $\angle ACD + \angle BCE + \angle ACB = 180^\circ$ by cor. 2 to theo. 1. hence by ax. 1. $\angle A + \angle B + \angle ACB = 180^\circ =$ two right angles. *Q. E. D.*

Corollary. Hence if the sum of any two angles be subtracted from 180° , the remainder will be the other angle; and if one angle of a plane triangle be 90° , the sum of the other two acute angles will be equal to 90° .

THEOREM VII.

The sum and difference of any two quantities being given, the quantities themselves are thereby given.

For half the sum added to half the difference gives the greater, and half the difference subtracted from half the sum gives the lesser quantity.

DEMONSTRATION.

31. Let AB represent the less, and BC the greater quantity, make $DC = AB$, bisect BD in E , then will $AE = EC =$ half the sum, and $BE = ED =$ half the difference; hence it

P. III. GEOMETRY.

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is plain that $BE + EC = BC$, the greater, and $AE - BE = AB$, the less quantity. *Q. E. D.* Fig.

THEOREM VIII.

If one side of any plane triangle be produced, the outward angle will be equal to the two inward opposite angles.

DEMONSTRATION.

By theo. 1. $\angle DAC + \angle CAB = 180^\circ$, and by theo. 6 $\angle B + \angle C + \angle CAB = 180^\circ$, hence by ax. 1. $\angle DAC + \angle CAB = \angle B + \angle C + \angle CAB$, and subtracting $\angle CAB$ from both sides there will remain $\angle B + \angle C = \angle DAC$. *Q. E. D.*

THEOREM IX.

In every plane triangle, equal sides subtend or are opposite to equal angles; and consequently equal angles in the same triangle are subtended by equal sides.

DEMONSTRATION.

Let the triangle ABC be isosceles, that is, let $AC = BC$, then I say, $\angle A = \angle B$: for let AB be bisected in D, and CD joined; then since $AC = BC$, $AD = DB$, and CD common; if the triangle CDB be placed upon ACD, so that CB lie upon its equal AC, then must the side DB fall upon its equal AD, and the line CD upon itself being common; and Th. the triangle CDB will coincide in all its parts with the triangle ADC, hence by ax. 6. $\angle A = \angle B$.

Corollary. Hence it follows that the three angles of every equilateral triangle are equal to one another, and therefore each equal to 60 degrees.

THEOREM X.

In every triangle the greatest side subtends the greatest angle; that is, if AC be greater than AB, then will $\angle ABC$ be greater than the $\angle BCA$.

DEMONSTRATION.

Make $AD = AB$, and join the BD. Then by theo. 1. $\angle ABD = \angle ADB$, and by theo. 8. $\angle C + \angle DBC = \angle ADB$. Therefore by ax. 1. $\angle C + \angle DBC = \angle ABD$. But $\angle ABC$ is greater than its part ABD by ax. 7. Wherefore $\angle C + \angle DBC$ must be less than $\angle ABC$, and consequently $\angle C$ is less than $\angle ABC$. *Q. E. D.*

FIG.

THEOREM XI.

In any right angled triangle BPH, the square HH, described upon the hypotenuse H, is equal to the sum of the squares BB, PP, described on the two other sides B, P.

DEMONSTRATION.

35. Make a square whose side is equal to $B + P$, and let that square be represented by S , so will its area be $BB + 2BP + PP$. Also the sum of the areas of the four triangles included in that square, will by mensuration be $\frac{1}{2}BP \times 4 = 2BP$.

I say, $HH = BB + PP$.

For	1	$S = HH + 2BP$.
and	2	$S = BB + 2BP + PP$.
$1 = 2$	3	$HH + 2BP = BB + 2BP + PP$.
$3 - 2BP$	4	$HH = BB + PP$.

Corollary 1.

For	1	$H = \sqrt{BB + PP}$.
2vv2	2	$HH = BB + PP$.
	3	$H = \sqrt{BB + PP}$.
		2.
For	1	$B = \sqrt{HH - PP}$.
2-PP	2	$BB + PP = HH$.
	3	$BB = HH - PP$.
3vv2	4	$B = \sqrt{HH - PP}$.
		3.
For	1	$P = \sqrt{HH - BB}$.
2-BB	2	$PP + BB = HH$.
	3	$PP = HH - BB$.
3vv2	4	$P = \sqrt{HH - BB}$.

Or this DEMONSTRATION.

36. Let ABC be the right angled triangle, make $\angle CAD = 90^\circ$, and draw AD to meet CB produced in D; then by

Theo. 19. following $BC : BA :: BA : \frac{BA^2}{BC} = DB$.

Then will $DC = \frac{BA^2 + BC^2}{BC} : AC :: AC : BC$, and multiplying extremes and means, we have $AC^2 = AB^2 + BC^2$.

Q. E. D.

THEO.

THEOREM XII.

FIG.

In an obtuse angled triangle ABC, if a perpendicular be let fall 36.
upon the base; or one side adjoining to the obtuse angle B, then the
square of the side opposite to that obtuse angle is equal to the sum of
the squares of the two lesser sides, together with twice the rectangle
of the base and distance of the perpendicular from the obtuse angle.

DEMONSTRATION.

I say $AC^2 = AB^2 + BC^2 + 2CB \times BD$. For $AC^2 = CD^2 +$
 $AD^2 = CB^2 + 2CB \times BD + BD^2 = CB^2 + 2CBD + AB^2$.

Cor. Hence $BD = \frac{AC^2 - CB^2 - AB^2}{2CB}$

THEOREM XIII.

If a perpendicular be let fall upon the base, or side adjoining to 37.
the acute angle B, then,

The square of the side opposite to that acute angle, together with
twice the rectangle, of the base, and the distance of the perpendicular
from the acute angle; is equal to the sum of the squares of the
two other sides: $AC^2 + 2CB \times BD = AB^2 + BC^2$.

DEMONSTRATION.

For $AC^2 = AD^2 + DC^2 = AD^2 + BC^2 + BD^2 - 2BD \times BC$
 $= AB^2 + BC^2 - 2CBD$. And $AC^2 + 2CBD = AB^2 + BC^2$.

Cor. $BD = \frac{AB^2 + BC^2 - AC^2}{2CB}$.

THEOREM XIV.

The diagonal of a parallelogram divides it into two equal triangles. 39.

DEMONSTRATION.

Since AC, BD, and AB, CD, are parallels.

By theo. 4. $\left| \begin{array}{l} 1 \\ 2 \end{array} \right| \angle ACB = \angle CBD$.

And $\left| \begin{array}{l} 2 \\ 1 \end{array} \right| \angle BCD = \angle ABC$.

Therefore $\left| \begin{array}{l} 3 \\ 1 \end{array} \right| \triangle ABC = \triangle CBD$, being equiangular tri-
angles, and AC common, and equal to one another in
every respect. Q. E. D.

THEOREM XV.

Parallelograms having the same base and altitude, are equal.

DEMONSTRATION.

Let the two parallelograms ABFE, CDEF, stand upon 40.
the same base EF, and between the same parallels EF, AD.
Then

Since

FIG Since 1 | $AB=EF=CD$ by construction.

Theref. 2 | $AC=BD$, and

Because 3 | $AE=BF$.

And 4 | $\angle A=\angle B$.

Then 5 | $\triangle ACE=\triangle BFD$, take the triangle BCG from both, and there remains the trapezium $ABGE=CDFG$; add the triangle EFG to both sides, and the sum is

$$\square ABFE = \square CDFE.$$

Q. E. D.

Corollary. Since the diagonal of every parallelogram divides it into two equal triangles by *theorem* 14; therefore triangles constituted upon the same base, and between the same parallels, are equal.

Lemma. A right line is said to be multiplied by another right

A

B

1	2	3	4	5	6	7	8
9	10	11	12	13	14	15	16
17	18	19	20	21	22	23	24
25	26	27	28	29	30	31	32

line, when a right angled parallelogram is made of the two lines; that is, the product of the two numbers which expresse the equal parts

D

C

in the two sides, are equal to what Mathematicians call the superficial content or area of such figures, or they are understood to contain so many little squares, whose sides are each equal to one of the equal parts with which the sides were measured. Thus for instance, let $AB=CD=8$, $AD=BC=4$, then $AD \times AB = 4 \times 8 = 32$, which is the area or number of little squares contained in the parallelogram, as appears by the figure.

T H E O R E M XVI.

Parallelograms, and also triangles, that stand between the same parallels, or which have the same height, are in a direct proportion to one another, as their bases.

DEMONSTRATION.

41. By the above *lemma*. The area of the $\square$ $\begin{cases} BD=BE \times DE. \\ BF=BE \times EF, \end{cases}$

which areas are in the proportion of $DE : EF$.

For $BE \times DE : BE \times EF :: DE : EF$.

That is, $\square BD : \square BF :: DE : EF$.

Again by *theo.* 14. area $\triangle$ $\begin{cases} DBE = \frac{1}{2} BE \times DE. \\ EBF = \frac{1}{2} BE \times EF. \end{cases}$

Then will $\triangle DBE : \triangle EBF :: DE : EF$.

Q. E. D.

T H E O.

T H E O R E M XVII.

If a right line be drawn parallel to any side of a plane triangle, it will cut from the given triangle another less triangle, that will be similar or equiangled to the whole triangle.

D E M O N S T R A T I O N.

By *theo. 3.* $\angle B = \angle C$, $\angle E = \angle D$, and since $\angle A$ is com- 42.
mon, it is evident that the triangle ABE is similar or equi-
angled to the triangle ACD. Q. E. D.

T H E O R E M XVIII.

*If a perpendicular be let fall from the right angle of a right an- 43.
gled triangle upon the hypotenuse, it will divide the triangle into
two right angled ones, similar to the whole and to each other.*

D E M O N S T R A T I O N.

Draw the lines as by the figure where $\angle CDB = \angle BDA =$
 $\angle CBA = 90^\circ$; therefore $\angle CBD + \angle DBA = 90^\circ$, and by *cor.*
theo. 6. $\angle C + \angle CBD = 90^\circ$, hence by *ax. 1.* $\angle CBD + \angle$
 $\angle DBA = \angle C + \angle CBD$, subtract $\angle CBD$ and there remains $\angle$
 $\angle DBA = \angle C$.

In like manner $\angle A + \angle ABD = \angle ABD + \angle DBC = 90^\circ$,
and subtracting $\angle DBA$, we have $\angle A = \angle DBC$: hence it is
proved that $\angle C = \angle DBA$, and $\angle A = \angle DBC$ therefore the
triangles CDB, BDA, CBA are similar. Q. E. D.

S C H O L I U M.

Whence we have the following proportions from the
similar triangles CDB, BDA, CBA.

AC : CB :: AB : BD.
CB : BD :: AB : AD.
CB : BD :: AC : AB.
AB : BD :: CB : DC.
DC : BD :: BD : AD.
AD : AB :: AB : AC.
CD : CB :: CB : AC.
CD : BD :: CB : AB.
AD : BD :: AB : CB.

T H E O.

FIG.

T H E O R E M X I X.

If two triangles be similar, their like sides, or the sides subtending equal angles will be proportional.

D E M O N S T R A T I O N.

44. Let DE be parallel to CB, then will the triangles ABC, AED be equiangular, join DB and CE, then by cor. to

Theo. 15. $\triangle ECD = \triangle DBE$.

By theo. 16. $\triangle ADE : \triangle ABD :: AE : AB$

And $\triangle ADE : \triangle ACE :: AD : AC$.

Since the $\triangle DBE = \triangle ECD$, therefore $\triangle ABD = \triangle ACE$.

Hence $AE : AB :: AD : AC$.

Q. E. D.

T H E O R E M X X.

If any angle of a plane triangle be bisected by a right line, it will cut the opposite side in proportion to the other two sides of the triangle.

D E M O N S T R A T I O N.

45. Produce the side DC till CZ=CB, join ZB, and draw CF parallel to BD, then will the triangles CFZ, DCA be similar, for

By theo. 3. 1 $\angle ZCF = \angle D$.

And th. 8. 2 $\angle Z + \angle ZBC = \angle BCD$.

But 3 $\angle Z = \angle ZBC$.

Because 4 $ZC = BC$.

Therefore 5 $\angle Z = \angle ACD$.

And 6 $\angle A = \angle F$.

Since 7 $\angle BCA = \angle FBC$, the lines CA, ZB, are parallel, and

Thence 8 $BA = FC$, and

Theo. 19. 9 $BA (=FC) : BC (=ZC) :: AD : CD$
or which is the

Same 10 $AB : AD :: BC : CD$.

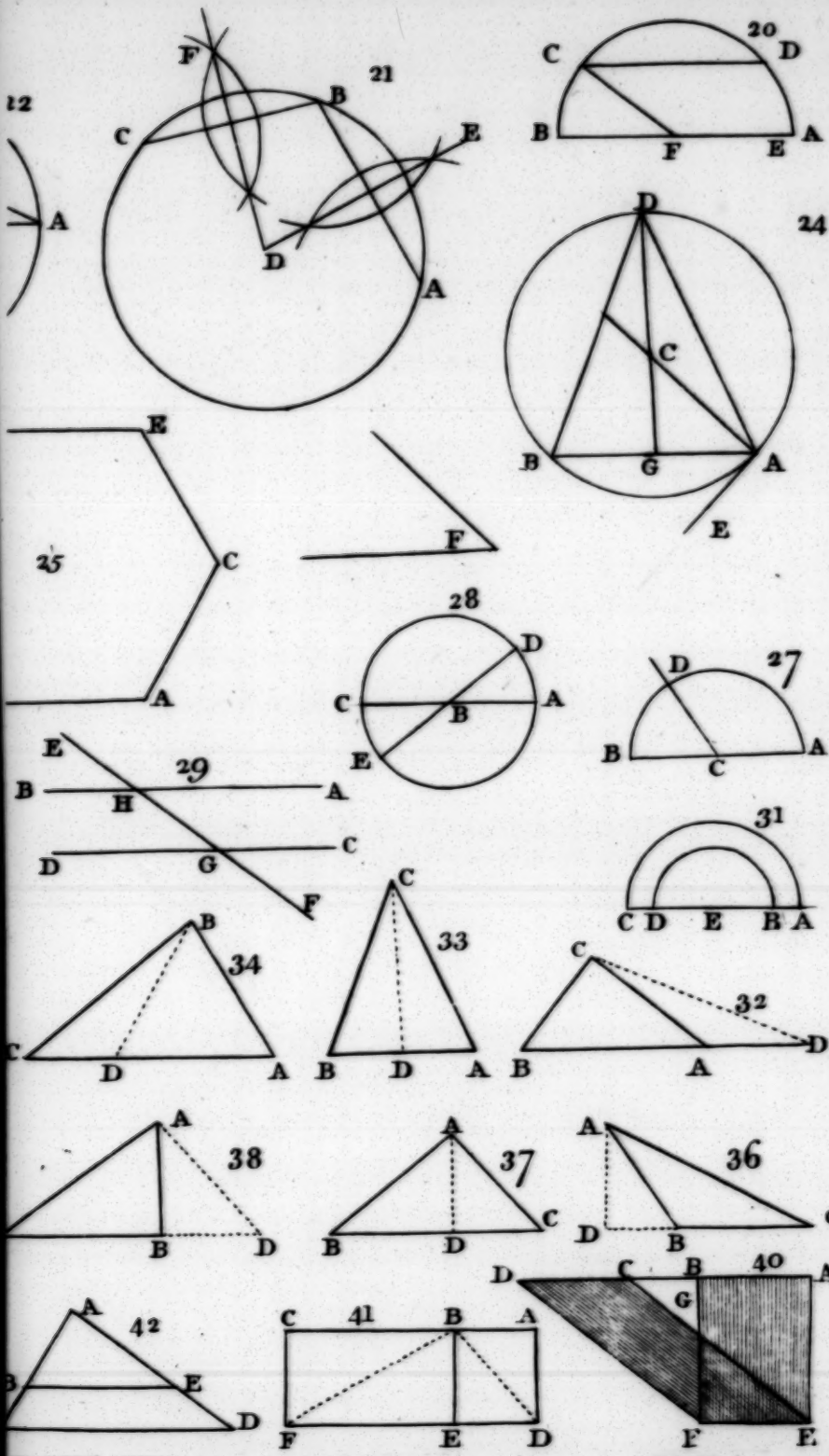
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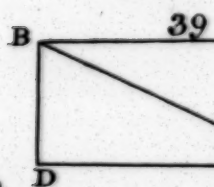
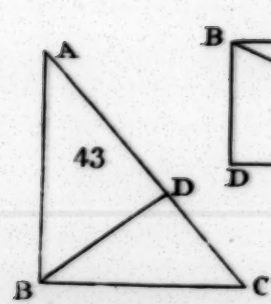
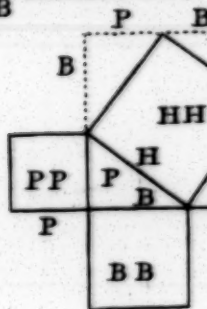
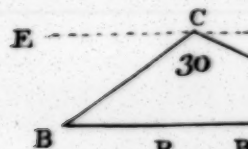
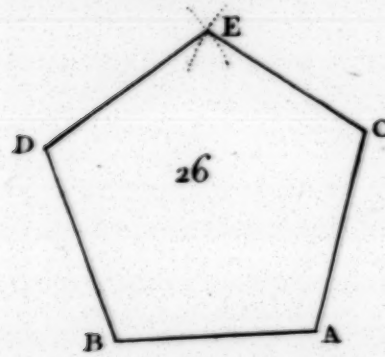
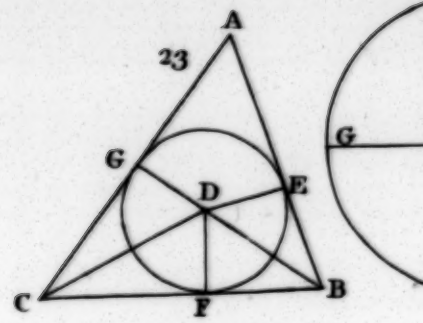
T H E O R E M X X I.

If a right line be divided into two parts or segments, the square of the whole line will be equal to the square of each segment, together with a double rectangle of the same segments.

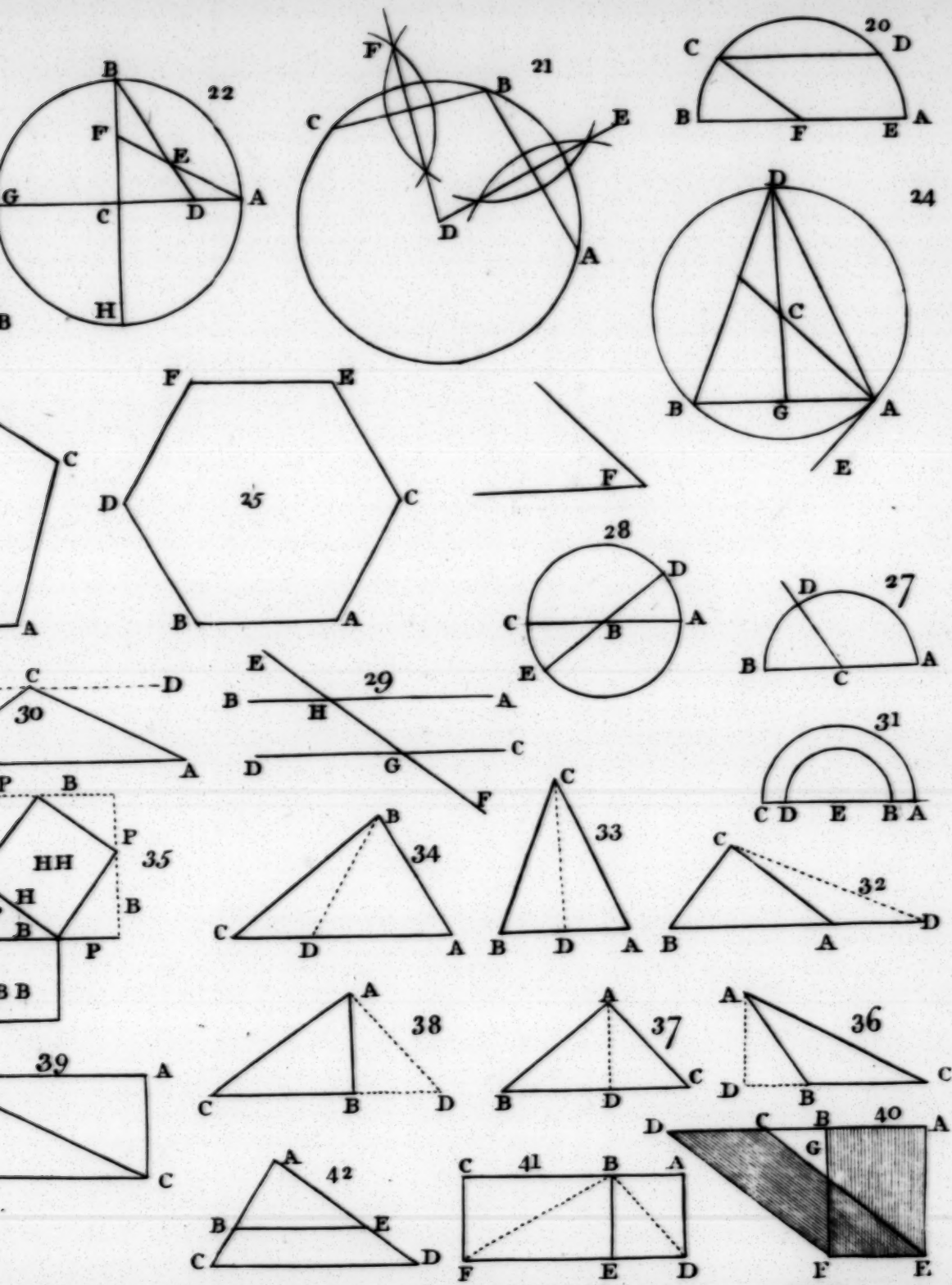
D E M O N S T R A T I O N.

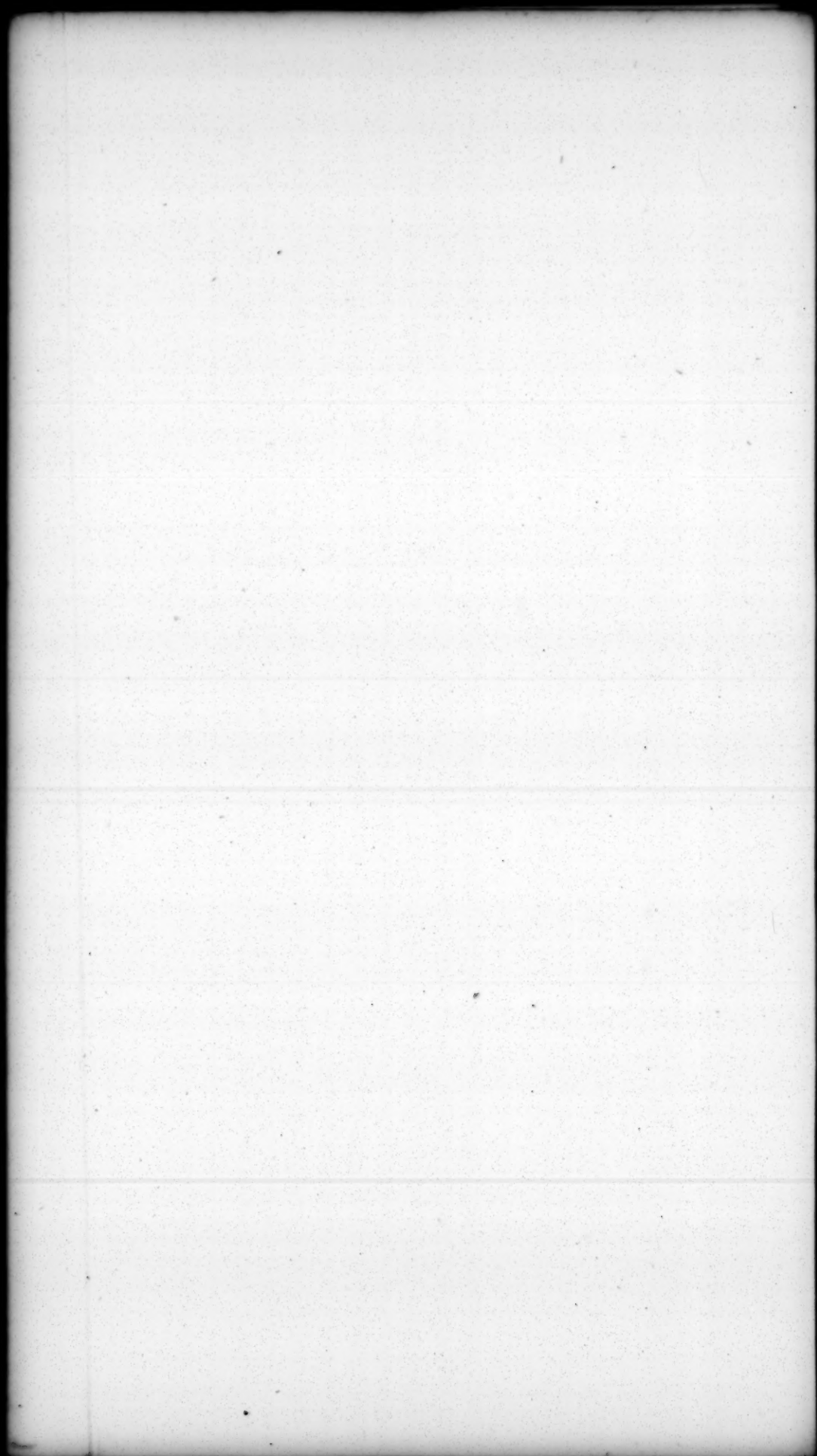
46. Let CG be the square of AC and GE that of AB, and let HE, DE be produced to meet the sides of the square GC in B and





Geometry





B and F, then since $DE=EH$, EC will be the square of BC , FIG. and $AE,=FH$ = a rectangle under $A B, BC$, but $CG=GE+EC+AE+FH$; that is, $\overline{AC}^2=\overline{AB}^2+\overline{BC}^2+2AB \times BC$.

Or thus,

Let the given right line AC be cut any wise as in B , then the square described on the whole line AC is equal to the squares described on the segments AB, BC , and to twice a rectangle made of the segments AB, BC , together.

For $AC \times AB = \overline{AB}^2 + AB \times BC$.

And $AC \times CB = \overline{CB}^2 + CB \times AB$.

Theref. $\overline{AC}^2 = \overline{AB}^2 + \overline{CB}^2 + 2AB \times CB$.

In numbers.

Let $AC=10$, $AB=6$, and $BC=4$,

Then $AC \times AB = 10 \times 6 = 60$,

$\overline{AB}^2 = 36$ } - - - = 60,

$AB \times BC = 24$

Also $AC \times CB = 10 \times 4 = 40$,

$\overline{CB}^2 = 16$ } - - - = 40,

$CB \times AB = 24$

Theref. $\overline{AC}^2 = \overline{AB}^2 + \overline{CB}^2 + 2AB \times BC = 100$.

T H E O R E M XXII.

Q. E. D.

The angle at the center of any circle is double to the angle at the periphery, when both the angles stand upon the same arch.

DEMONSTRATION.

Case I.

Let the diameter AD and line BD form the angle at 47. the periphery, draw the radius CB , then I say, $\angle ACB$ is double $\angle D$.

For $\angle BCA = \angle D + \angle DBC$, by th. 8.

But $\angle D = \angle DBC$, by th. 9. because $DC=CB$.

Theref. $\angle BCA = 2\angle D$.

Case II.

Draw the Diameter AD , then by

48.

Case I. | 1 | $\angle ACB = 2\angle ADB$.

And | 2 | $\angle ACE = 2\angle ADE$.

1 + 2 | 3 | $\angle ACB + \angle ACE = 2\angle ADB + 2\angle ADE$, by

ax. 2.

That is, | 4 | $\angle ECB = 2\angle EDB$.

Case

FIG.

Case III.

49. Draw the diameter AD, then by
- | | | |
|----------|---|---|
| Case I. | 1 | $\angle ACB = 2\angle ADB.$ |
| And | 2 | $\angle ACE = 2\angle ADE.$ |
| 1-2 | 3 | $\angle ACB - \angle ACE = 2\angle ADB - 2\angle ADE,$ by |
| | | <i>ax. 3.</i> |
| That is, | 4 | $\angle ECB = 2\angle EDB.$ |

Corollary. Hence all the angles at the periphery that stand upon the same or equal arches are equal to one another.

T H E O R E M XXIII.

The angle in a semicircle is a right angle.

DEMONSTRATION.

50. Let ABC be the diameter, and draw the radius BD, then by
- | | | |
|---------|---|---|
| Th. 9. | 1 | $\angle A = \angle ADB.$ |
| And | 2 | $\angle C = \angle CDB.$ |
| 1+2 | 3 | $\angle A + \angle C = \angle ADB + \angle CDB,$ by <i>ax. 2.</i> |
| That is | 4 | $\angle A + \angle C = \angle ADC.$ |
| But | 5 | $\angle A + \angle C + \angle ADC = 180^\circ,$ and |
| Theref. | 6 | $\angle ADC = 90^\circ.$ |
- Q. E. D.*

T H E O R E M XXIV.

An angle in a segment less than a semicircle is greater than a right angle, and an angle in a segment greater than a semicircle is less than a right angle.

DEMONSTRATION.

51. Let the $\angle ABC$ be an angle in the segment AEBC greater than a semicircle; and $\angle ADE$ an angle in the segment CDA less than a semicircle. Draw the diameter CE, and join BE, ED, then by *theo. 23.* $\angle EDC = 90^\circ = \angle EBC,$ it is thence evident that $\angle ADC$ is greater than $\angle EDC,$ and also that $\angle ABC$ is less than $\angle EBC.$

T H E O R E M XXV.

If two lines be any how drawn within a circle cutting each other, the rectangle of the segments of one line will be equal to the rectangle of the segments of the other line.

DEMONSTRATION.

52. Join the points $a, b,$ and $c, d,$ then will the triangles $ao b,$ and $co d$ be similar, for $\angle a = \angle c, \angle b = \angle d,$ by *cor. to theo. 20,* therefore by *theo. 19,* $oc : od : ao : ob.$ And by multiplying extremes and means we have $co \times ob = ao \times od.$

Q. E. D.
T H E O-

THEOREM XXVI.

If from a point without a circle two right lines be drawn, to the opposite part of the periphery; the rectangle of one whole line and its part without the circle is equal to the rectangle of the other whole line, and its part without the circle.

DEMONSTRATION.

Draw the lines BC, AD, then the triangles VBC, VAD are similar, for $\angle D = \angle B$ by cor. to theo. 22, and $\angle V$ common, hence $VB : VC : VD : VA$ by the. 11, and multiplying extremes and means $VB \times VA = VC \times VD$.

Q. E. D.

THEOREM XXVII.

The chord of 60° of the arch of any circle is equal to the radius or semidiameter of that circle.

DEMONSTRATION.

Let BC be an arch of 60° , BC its chord, then $\angle COB = 60^\circ$, and $\angle AOC = 120^\circ = \angle C + \angle B$, by theo. 8. but since $OB = OC$ the angle $C = \angle B$, and therefore each $= 60^\circ$, therefore since all the angles are equal the sides must be equal by theo. 9. that is, $BO = OC = BC$ the radius.

Corollary. Hence the reason is evident why 60° is taken from the line of chords on any scale for the radius of a circle suited to that scale.

THEOREM XXVIII.

If any trapezium be inscribed in a circle the two opposite angles taken together are equal to two right angles.

DEMONSTRATION.

Draw the diagonals AD, CB.

Then $\angle ADC = \angle ABC$, and $\angle ACB = \angle ADB$, by cor. to theo. 22. But $\angle CAB + \angle ACB + \angle CBA = 180^\circ$, by theo. 6. And since $\angle CDA + \angle ADB = \angle CDB$, therefore $\angle CAB + \angle CDB = 180^\circ$.

In like manner it will be proved that the angle $ACD + \angle ABD = 180^\circ$.

Q. E. D.

THEOREM XXIX.

If from any angle of a plane triangle inscribed in a circle, there be let fall a perpendicular upon the opposite side; as that perpendicular is in proportion to one of the sides including the angle, so is the other side including the angle to the diameter of the circle.

DEMON-

FIG.

DEMONSTRATION.

56. Draw the diameter BD, and join CD, then the triangles BAM, BCD, are similar.

For | 1 | $\angle A = \angle D$, by *cor. th. 22.*

And | 2 | $\angle M = \angle BCD = 90^\circ$. *th. 23.*

Hence | 3 | $BM : AB :: BC : BD$, *th. 11.* Q. E. D.

T H E O R E M XXX.

If any trapezium be inscribed in a circle, there be drawn two diagonals, their rectangle is equal, to the sum of the rectangles under the opposite sides of the trapezium.

DEMONSTRATION.

57. Make $\angle DAM = \angle CAB$, then the triangles AMD, ABC, are similar, since $\angle MDA = \angle BCA$, *cor. to th. 22.* Hence $AC : BC :: AD : DM$, by *th. 11.* and $AC \times DM = BC \times AD$.

Again, the triangles BAM, ACD are similar.

For | 1 | $\angle ABM = \angle ACD$.

And | 2 | $\angle BAM = \angle CAD$.

Because | 3 | $\angle MAD = \angle BAC$, and $\angle CAM$ common to both triangles.

Theref. | 4 | $AC : CD :: AB : BM$, by *th. 11.*

Hence | 5 | $AC \times BM = CD \times AB$, to which add the equal quantities before found, and we have

| 6 | $CD \times AB + BC \times AD = AC \times BM + AC \times DM = AC \times \overline{BM + DM} = AC \times BD$. Q. E. D.

T H E O R E M XXXI.

The square of the right line bisecting any angle of a triangle and terminating in the opposite side, together with the rectangle under the two segments of that side, is equal to the rectangle of the sides including the proposed angle.

DEMONSTRATION.

58. Circumscribe the triangle ABC with a circle, produce CD to meet the circumference in E, and join AE, then are the triangles ACE, BDE similar,

For | 1 | $\angle E = \angle B$, by *cor. th. 22.*

And | 2 | $\angle ACE = \angle DBC$, by hypothesis.

Hence | 3 | $CE : CB :: AC : CD$, by *th. 11.*

Theref. | 4 | $CE \times CD = CB \times AC$.

But

But

$$5 \quad \left| \begin{array}{l} \text{CD} \times \text{CE} = \text{CD} \times \text{CD} + \text{DE} \\ = \text{CD}^2 + \text{DE} \times \text{CD} = \text{CD}^2 + \text{AD} \times \text{DB, by th.} \end{array} \right.$$

FIG.

Conseq.

$$6 \quad \left| \begin{array}{l} \text{AC} \times \text{CB} = \text{CD}^2 + \text{AD} \times \text{DB.} \end{array} \right. \quad \text{Q. E. D.}$$

T H E O R E M XXXII.

The hypotenuses of any two similar right angled triangles being made the radii of two circles, the perpendiculars, and bases in each will be in proportion to each other, as the sines of their opposite angles answering each radius.

DEMONSTRATION.

Let the two similar right angled triangles be ABC, and *Abc*, making the hypotenuse the radius, then BC is the sine $\angle A$, when AC is made radius; and *bc* is the sine of $\angle A$ when *Ac* is made the radius; then by *theo. 11.*

$$Ab : AB :: bc : BC.$$

Q. E. D.

T H E O R E M XXXIII.

If the bases of any two similar right angled triangles be made the radii of two circles, the perpendiculars and the hypotenuses in each will be in proportion to the bases as the tangents and secants respectively of the angle at the center, answering each corresponding radius.

DEMONSTRATION.

Let the two similar right angled triangles ABC, *Abc* be *proposed*, then by *theo. 11.* it will be,

$$Ab : bc :: AB : BC.$$

$$Ab : Ac :: AB : AC.$$

Q. E. D.

Corollary. Whatever proportion any side of a given right angled triangle that is made the radius or semidiameter of a circle, hath to the assumed radius 10000000000 in the tables of sines, tangents, and secants, the very same proportion hath the other sides of the given triangle.

When they become $\left\{ \begin{array}{c} \text{sines} \\ \text{tangents} \\ \text{secants} \end{array} \right\}$ to the $\left\{ \begin{array}{c} \text{sines} \\ \text{tangents} \\ \text{secants} \end{array} \right\}$ of the same angle in the tables.

T H E O R E M XXXIV.

Or *Axiom 1.* of Plane Trigonometry.

As the word on the side given in any right angled triangle from the table of sines, tangents, and secants, is to the side given

FIG. given, so is the word on the side required from the same tables, to the side sought.

DEMONSTRATION.

61. I. when the hypotenuse is made radius. Let mn be a quadrant of the tabular circle whose radius Ac is 10000000000, and from thence the whole canon of sines, tangents, and secants are constructed and disposed into tables.

$bc = \text{fine } \angle A$
 $Ab = \text{fine } \angle C$ } from the tables $\left\{ \begin{array}{l} AC = \text{radius} \\ BC = \text{fine } \angle A \\ AB = \text{fine } \angle C \end{array} \right\}$ in the triangle ABC .

And since the tabular triangle Abc is similar to the given triangle ABC , we have

$Ac : AC :: bc : BC$, i. e. radius : AC : $\text{fine } \angle A$: BC .

$Ab : AB :: bc : BC$, i. e. $\text{fine } \angle C$: $AB :: \text{fine } \angle A$: BC .

II. Making either leg radius.

Here Abc is the tabular triangle, and ABC is the triangle

62. given to be solved.

Ab the radius
 bc tangent $\angle A$
 Ac secant $\angle A$ } from the tables.
 AB the radius
 BC tangent $\angle A$
 AC secant $\angle A$ } in the triangles given.

And since the triangles ABC , Abc are similar, it will be

$Ac : AC :: Ab : AB$, i. e. secant $\angle A$: $AC ::$ radius : AB .

$Ab : AB :: bc : BC$, i. e. radius : $AB ::$ tangent $\angle A$: BC .

Q. E. D.

T H E O R E M XXXV.

Or *Axiom* 2. of Plane Trigonometry.

The sides of any plane triangle, are in proportion to one another as the sines of their opposite angles.

DEMONSTRATION.

63. Produce the side AC to k , so that $Ak = EC$, and let fall the perpendiculars Cn , km , then will $km = \text{fine } \angle A$, and $Cn = \text{fine } \angle B$. Now the triangle AnC is similar to the triangle Akm .

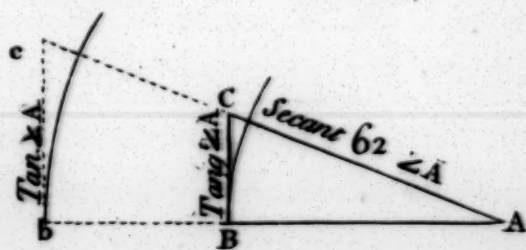
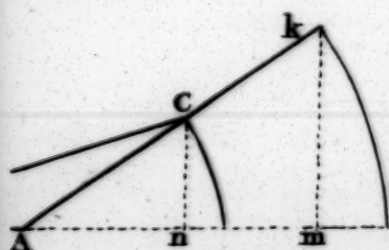
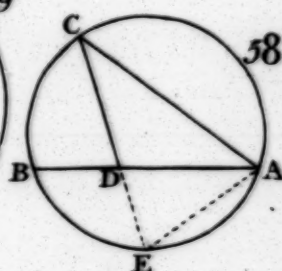
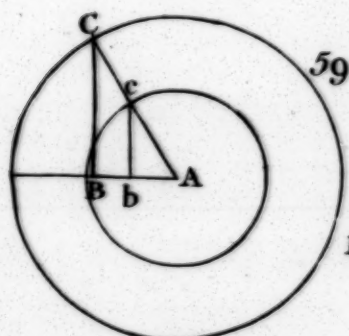
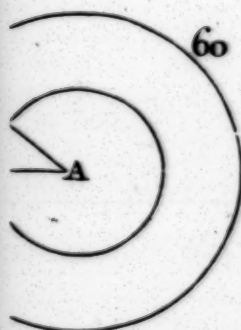
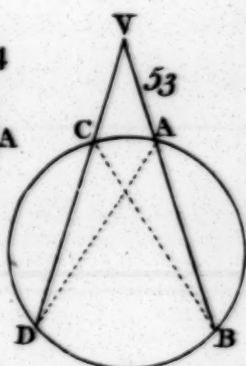
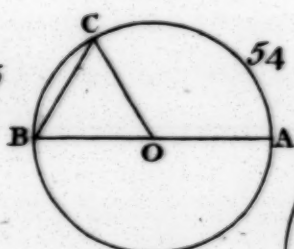
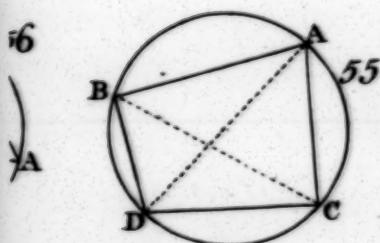
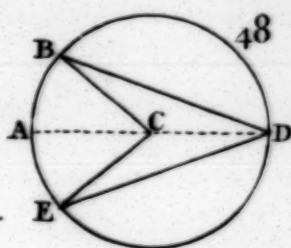
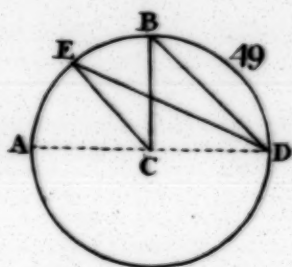
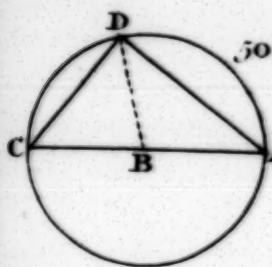
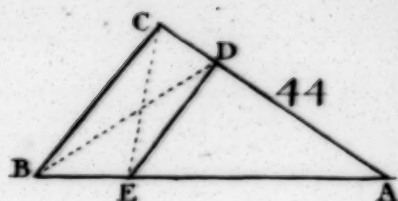
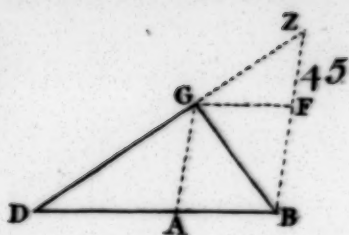
Therefore $AC : Cn :: Ak = EC : mk$, that is,

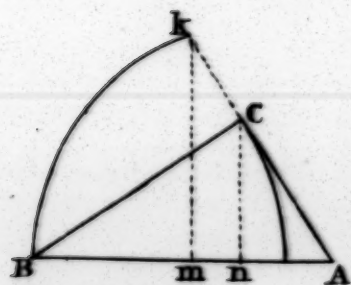
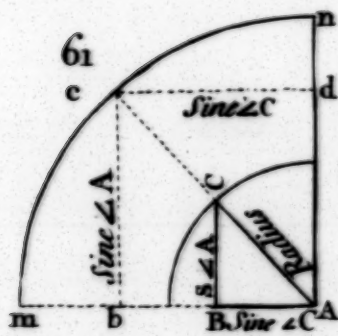
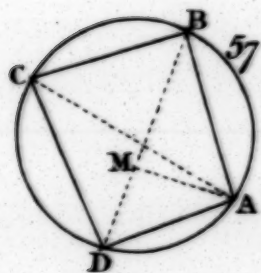
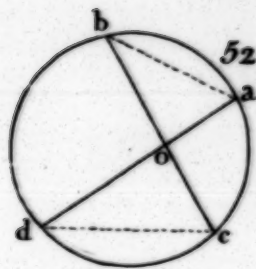
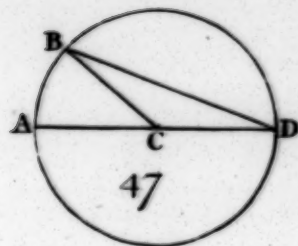
As AC : is to the sine of its opposite angle B ::

So is BC : to sine of its opposite $\angle A$

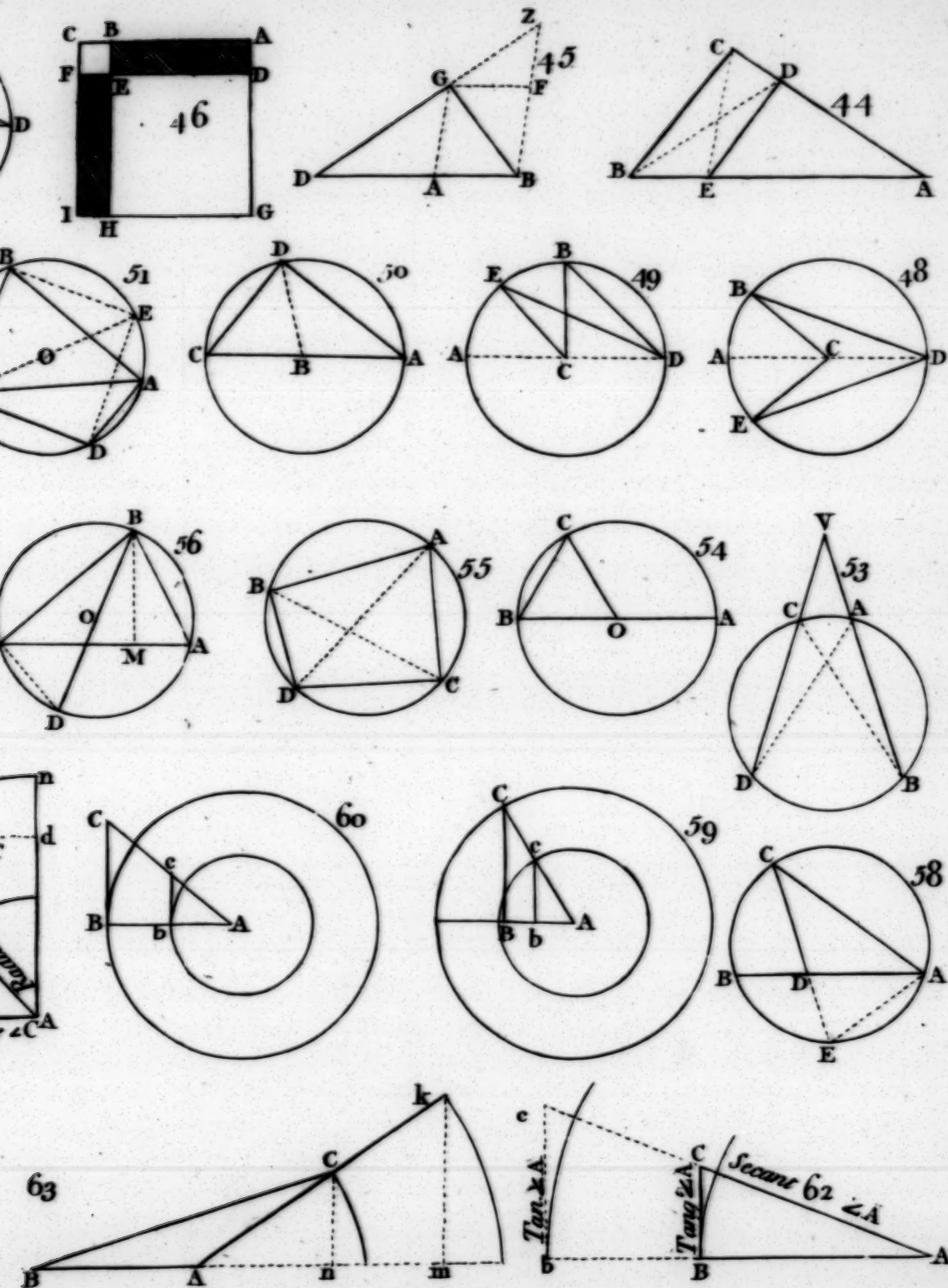
Q. E. D.

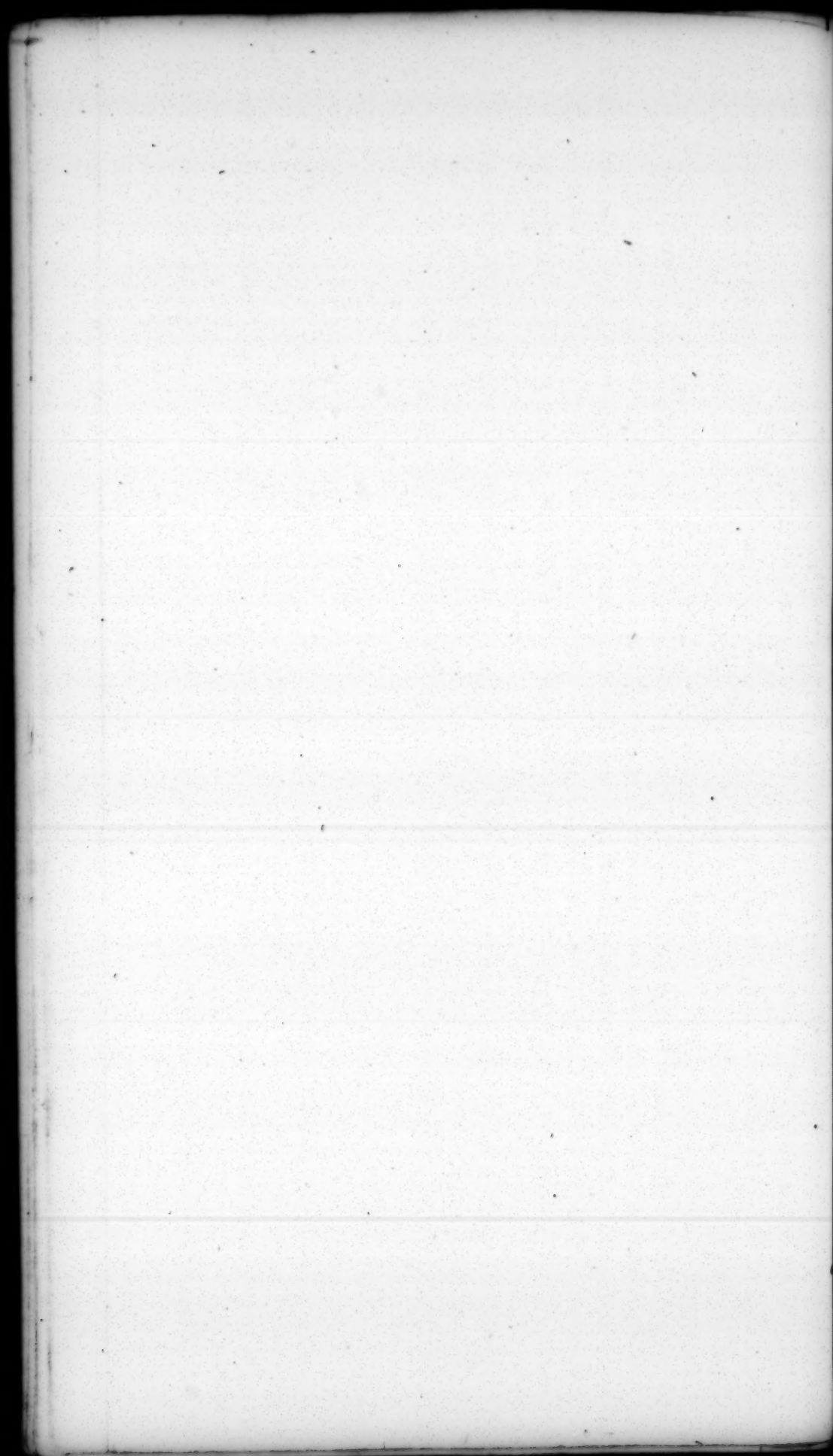
T H E O.





Geometry





THEOREM XXXVI.

Or *Axiom* 3. in Plane Trigonometry.

In any plane triangle, as the sum of any two sides is to their difference, so is the tangent of half the sum of their opposite angles, to the tangent of half their difference.

DEMONSTRATION.

Let ABC represent the triangle, on the center C with the ⁶⁴ radius CB describe a circle; produce AC to F, join FB, BD, and draw DE parallel to AB, then the sum of the sides AC, BC = AF, and difference of the sides AC, BC = AD.

But $\angle A + \angle ABD = \angle BDC = \angle DBC$, and adding $\angle ABD$ to both sides, we have $\angle A + \angle ABD + \angle ABD = \angle DBC + \angle ABD$, that is, $\angle A + 2\angle ABD = \angle ABC$, hence $2\angle ABD$ is = diff. angles A and ABC, and consequently half their difference is $= \angle ABD = \angle BDE$, whose tangent is BE.

Again since $\angle A + \angle ABD = \angle BDC$, and $\angle BDC = \angle DBC$, it will be $\angle A + \angle ABD + \angle DBC = \angle BDC + \angle DBC$, that is $\angle A + \angle ABC = 2\angle BDC$, hence $\angle BDC =$ half the sum of the angles A, and ABC, the tangent of which is BF, then since the triangles DBE, ABF are similar it will be

$$FA : DA :: FB : EB. \quad \text{Q. E. D.}$$

THEOREM XXXVII.

Being *Axiom* 4. of Plane Trigonometry

In every plane triangle, as the base is to the sum of the other two sides, so is the difference of those sides to the difference of the segments of the base, made by a perpendicular let fall from the angle opposite to the base.

DEMONSTRATION.

Upon C as a center with the radius AC describe a circle, ⁶⁵ then the sum of the sides AC, CB is = GB, and their difference, that is $CB - AC = FB$; also it is evident that EB is the difference of DB and AD the segment of the base; then from what is demonstrated in *theorem* 24. we have

$$AB : BF :: BG : EB.$$

$$AB : BG :: BF : EB.$$

Q. E. D.

To multiply FEET and INCHES.

R U L E.

1. Feet multiplied by feet produce feet.
2. Feet by inches and inches by feet cross ways, the sum of the products divided by 12 give feet, the remainder is inches.
3. Inches by inches every 12 is an inch, the rest *twelfths* of an inch.

Example 1.

Multiply 16 feet 6 inches by 9 feet 3 inches.

ft. inc.	or thus
16 6	16 6
9 3	9
144 0	inc.
8 6	3 $\frac{1}{2}$ 148 6
1 $\frac{6}{12}$	4 1 $\frac{1}{2}$
Feet 152 7 $\frac{1}{2}$	162 7 $\frac{1}{2}$

ft. ft.
For $16 \times 9 = 144$ feet

ft. inc.
 $9 \times 6 = 54$
 $16 \times 3 = 48$

12)102(8 ft. 6 inc.

$6 \times 3 = 18$) and $\frac{18}{12} = 1\frac{6}{12}$ inc.

Example. 2

ft. inc. ft inc.
Multiply 178 10 by 58 9

ft. inc.	ft inc.
178 10	58 9
1424	
890	
181 10	
7 $\frac{6}{12}$	
10506 5 $\frac{1}{2}$	

Glaziers,

P.III. To multiply FEET and INCHES. 53

Glaziers, painters, joiners, carpenters, &c. measure their work after this method, and the result given in yards, except glazier's work.

Example.

A room 18 feet 9 inches long, and 12 feet 6 inches wide is to be painted; how many yards; when it is 8 feet 6 inches high?

ft.	inc.
18	9 length
12	6 breadth
3 ¹	3
	2
62	6 compass
8	6 high
496	0
35	0
	3
9)531	3
Yards	59 0 3

And thus may boards be measured, or by the following

T A B L E.

6 Inches	-	-	-	-	$\frac{1}{2}$ Foot
4	-	-	-	-	$\frac{1}{3}$
3	-	-	-	-	$\frac{1}{4}$
2	-	-	-	-	$\frac{1}{6}$
1 $\frac{1}{2}$	-	-	-	-	$\frac{1}{8}$

Example. 1.

A board 19 feet 10 inches long, and 23 inches broad, how many square feet?

ft.	inc.		or thus
19	10		inc ft. inc.
1	11		6 19 10
			4 9 11
19	0		1 6 7 $\frac{1}{4}$
18	3		1 1 7 $\frac{3}{4}$
	9 $\frac{2}{3}$		
			38 0 $\frac{1}{4}$
Feet	36 0 $\frac{1}{2}$		

E 3

Example

54 To multiply by FEET and INCHES: P. III.

Example 2.

A board 17 feet 8 inches, by 16 inches.

ft. inc.

17 8

1 4

17 0

6 4

2 $\frac{1}{2}$

Feet 23 $6\frac{2}{3}$

or thus

inc. ft. inc.

4 $1\frac{1}{3}$ | 17 8

5 10 $\frac{8}{12}$

23 $6\frac{2}{3}$

Example 3.

A board 17 feet 8 inches, by 9 inches.

ft. inc.

17 8

9

12 9

6

Feet 13 3

or thus

ft. inc.

17 8

inc.

6 $1\frac{1}{2}$ |

3 $1\frac{1}{2}$ |

8 10

4 5

13 3

P A R T IV.

T H E

Measuring of Areas and Surfaces.

	144 square inches	1 square foot
	9 square feet - -	1 square yard
	4 ⁸ 40 yards - - -	1 acre
	1210 yards - - -	1 rood
	30.25 yards - -	1 perch
Also	100000 links - - -	1 acre
	25000 links - - -	1 rood
	625 links - - -	1 perch.

P R O P O S I T I O N. I.

To measure a Square.

R U L E.

Square the side, the product is the area, in such a name as you multiplied the side by.

Example 1.

66.

There is a square court-yard whose side AB measures 79 feet; how many square yards is the area, or content?

$$\begin{array}{r}
 79 \\
 79 \\
 \hline
 711 \\
 553 \\
 \hline
 9)6241 \text{ feet} \\
 \hline
 \text{yards } 693 \text{ } 4 \text{ feet.}
 \end{array}$$

Example 2.

There is a square clofe that measures $81\frac{1}{2}$ yards on a side, what is the area?

E 4

81.75

FIG. 56

MENSURATION.

P. IV.

$$\begin{array}{r}
 81.75 \\
 81.75 \\
 \hline
 40875 \\
 57225 \\
 8175 \\
 65400 \\
 \hline
 4840 \overline{) 6683.0625} \quad (1.3807 \text{ area} \\
 \quad \quad \quad 4 \\
 \quad \quad \quad \hline
 \quad \quad \quad 1.5228 \\
 \quad \quad \quad \quad 40 \\
 \quad \quad \quad \hline
 \text{A. R. P.} \quad \quad \quad 20.9120 \\
 \text{Answer } 1 \ 1 \ 20
 \end{array}$$

Cor. The square root of the area of any square gives the length of the side.

PROPOSITION II.

To measure a Parallelogram.

R U L E.

Multiply the length by the breadth, the product is the content.

Example 1.

67. There is a parallelogram ABCD, whose length AD is 43 feet, and breadth AB 22 feet, required the area in yards?

$$\begin{array}{r}
 43 \\
 22 \\
 \hline
 86 \\
 86 \\
 \hline
 9 \overline{) 946} \text{ area in feet.}
 \end{array}$$

yards 105 1 foot

But the finding the measure of land is best performed by Gunter's chain, the method of which I shall here briefly shew.

OF GUNTER'S CHAIN.

Gunter's chain is the most commodious of any chain for measuring land; it is 4 poles in length, or 22 yards; divided into 100 links with pieces of notched brass at every 10 links, and

and the number of notches or points on the brags denotes the tens from one end; but in the middle or 50 links, it is commonly distinguished by a round piece of brags; and it is so numbered from both ends to the middle; but those that are above 50 are the complements to 100, as 30 links from one end is 70 from the other, &c.

To cast up any survey by the chain, reduce the chains and links into links, which is no more trouble than to put down the links on the right hand of the chains.

	ch.	lin.	lin.
As	9	34	= 934
	11	59	= 1159
	13	00	= 1300
	4	08	= 408, &c.

Always remembering to put down two cyphers to the chains, if there be no links; and if the links be under ten, to put a cypher in the ten's place; then multiply according to the rule, and having cast up the contents according to the figure, set off five figures of the product, those on the left will be acres; the five that were pointed off multiply by 4, and set off five figures, those on the left are roods; the five figures that remain which were pointed off, multiply by 40, those on the left are perches.

Example 2.

There is a close inform of a parallelogram whose length measures 13 chains 55 links, and breadth 9 chains 83 links, what is the content?

$$\begin{array}{r}
 1355 \\
 983 \\
 \hline
 4065 \\
 10840 \\
 12195 \\
 \hline
 1331965 \\
 4 \\
 \hline
 127860 \\
 40 \\
 \hline
 1114400
 \end{array}$$

A. R. P.
Ans. 13 1 11

Cor.

FIG. *Cor.* If the area be divided by the $\left\{ \begin{array}{l} \text{length} \\ \text{breadth} \end{array} \right\}$ the quotient gives the $\left\{ \begin{array}{l} \text{breadth} \\ \text{length} \end{array} \right\}$ as is required.

Example 3.

A close measures 13 ch. 55 links long, and I want to set off 5 acres 1 rood 24 poles along the side of it how broad must it be?

	links
5 acres - - -	500000
1 rood - - -	25000
24 p. $\times 625$ - -	15000
	links
	1355)540000(397.78.

Or 3 chains 97 $\frac{3}{4}$ links must be the breadth.

P R O P. III.

To measure a triangle.

R U L E.

Multiply the base by half the perpendicular;

Or, Multiply the perpendicular by half the base; either product is the area.

Or, Multiply the base and perpendicular together, half the product is the area.

Example 1.

68. The base AB of a triangle measured 8 chains 36 links and perpendicular CD 5 chains 31 links, required the area?

836
531
836
2508
4180
2)4.43916
Area 2.21958

$$\begin{array}{r}
 \text{Area} \quad 2.21958 \\
 \quad \quad \quad 4 \\
 \hline
 \quad \quad .87832 \\
 \quad \quad \quad 40 \\
 \hline
 \quad 35.13280
 \end{array}$$

$$\begin{array}{r}
 \text{A. R. P.} \\
 2 \quad 0 \quad 35
 \end{array}$$

Cor. When twice the area of a triangle is divided by $\left\{ \begin{array}{l} \text{base} \\ \text{perpendicular} \end{array} \right\}$ the quotient is the $\left\{ \begin{array}{l} \text{perpendicular} \\ \text{base} \end{array} \right\}$ as is required.

Example 2.

The area of a triangle is 168, and the base 24, what is the perpendicular?

$$\begin{array}{r}
 168 \text{ area} \\
 \quad 2 \\
 \hline
 24)336(14 \text{ perpendicular} \\
 \quad 24 \\
 \hline
 \quad 96 \\
 \quad 96 \\
 \hline
 \end{array}$$

P R O P. IV.

Given the three sides of a triangle to find the area.

R U L E.

From half the sum of the three sides, subtract each side severally, multiply the half sum, and the three differences continually together, the square root of the product will be the area required.

Example.

There is a triangle whose sides are 50, 40, and 30, what is the area?

$$\begin{array}{r}
 50 + 40 + 30 = 120 \text{ the sum, half} = 60. \\
 \text{From} \quad \begin{array}{ccc} 60 & 60 & 60 \\ 50 & 40 & 30 \\ \hline 10 & 20 & 30 \end{array} \text{ d ff.}
 \end{array}$$

$$\text{Then } 60 \times 10 = 600 \times 20 = 12000 \times 30 = 360000.$$

$$\text{The square root of } 360000 = 600, \text{ area.}$$

P R O P.

PROP. V.

Given the area of an equilateral triangle to find its side.

RULE.

Divide the given area by .433, the square root of the quotient is the side required.

Example.

What is the side of an equilateral triangle whose area is 500.548.?

$$.433 \overline{) 500.548} (1156$$

The sq. root of 1156 = 34 yards, the side.

PROP. VI.

Given the side of an equilateral triangle to find the length of its perpendicular.

RULE.

Multiply the given side by .866, and the product is the perpendicular.

Example.

Suppose the side of an equilateral triangle be 34, what is the perpendicular?

$$.866 \times 34 = 29.444, \text{ perpendicular.}$$

PROP. VII.

Given the side of an equilateral triangle to find the area.

RULE.

Multiply the square of the given side by .433, the product is the area.

Example.

If the side of an equilateral triangle be 34 yards, what is the area?

$$34 \times 34 = 1156 \times .433 = 500.548, \text{ the area.}$$

PROP. VIII.

To find the area of a trapezium.

RULE.

Multiply the diagonal by half the sum of the two perpendiculars, the product is the area.

Or,

P. IV. MENSURATION.

61

Or, Multiply the sum of the perpendiculars by half the diagonal.

Or, Multiply the sum of the perpendiculars by the diagonal, and take half the product.

Example.

Let ABDC denote a trapezium, whose diagonal AD measures 18 ch. 9 links.

Perpendicular $\left\{ \begin{array}{l} CF \text{ 4 chains 21 links.} \\ EE \text{ 9 chains 37 links, required the area?} \end{array} \right.$

$$\begin{array}{r}
 421 \\
 937 \\
 \hline
 2)1358 \\
 \hline
 679 \\
 1809 \\
 \hline
 6111 \\
 54320 \\
 679 \\
 \hline
 \text{Area} \quad 12.28311 \\
 \hline
 4 \\
 \hline
 1.13244 \\
 \hline
 40 \\
 \hline
 5.29760
 \end{array}$$

A. R. P.

12 1 5

Cor. When two sides are parallel, then it is called a trapezoid, to measure which this is the

R U L E.

Multiply the sum of the two parallel sides by their distance, half the product is the area.

Example.

Let the side AB be 6 feet, the side CD be 14 feet, and the distance AE be 8 feet, required the area?

$$\begin{array}{r}
 14 \\
 6 \\
 \hline
 20 \quad \text{sum} \\
 8 \\
 \hline
 2)160 \\
 \hline
 80 \quad \text{area.}
 \end{array}$$

Of

DEFINITIONS.

AB the diameter.

AC radius.

DF chord line.

EG verfed line.

HCB sector.

AEB femicircle.

AEC quadrant.

RULES.

I. Diameter multiplied by 3.1416, gives the circumference.

II. The square of the diameter multiplied by .7854, gives the area.

III. The circumference multiplied by .31831, gives the diameter.

IV. The square of the circumference multiplied by .0795776, gives the area.

V. Multiply the square root of the area by 1.12837, gives the diameter.

VI. Multiply the square root of the area by 3.5449, gives the circumference.

Example 1.

The diameter of a circle is 50 feet, how many feet is it round?

$$\begin{array}{r} 3.1416 \\ \times 50 \\ \hline 157.0800 \end{array} \text{ circumference.}$$

Example 2.

If the diameter of a circle be 50 inches, what is the area?

$$\begin{array}{r} 50 \\ \times 50 \\ \hline 2500 \end{array}$$

Then $.7854 \times 2500 = 1963.5$ area.

Example 3.

If the circumference of a circle be 157.08 what is the diameter?

$$157.08$$

$$157.08 \times .31831 \times 50 = 0001348$$

Diameter = 50.

Example 4.

If the circumference be 157.08 what is the area?

$$157.08 \times 157.08 = 24674.1264$$

And $24674.1264 \times .079577 = 1963.492956$ the area = 1963.5 nearly.

Example 5.

If the area of a circle be 1963.4929 what is the diameter?

The square root of 1963.4929 is = 44.31.

And $1.12837 \times 44.31 = 49.998$ or 50 diameter.

Example 6.

If the area of a circle be 1963.4929, what is the circumference?

The square root of 1963.4929 = 44.31.

And $3.5449 \times 44.31 = 157.074519$.

Cor. The reason why the numbers do not correspond exactly, shews that no finite numbers can be found for determining the quadrature of the circle.

P R O B. I.

To determine whether the square or the circle has the least fencing round any given piece of land.

Example 1.

What difference is there in fencing round an acre of land laid out in a square or in a circle?

In an acre of land are 4840 yards, and the square root of $4840 = 69.57$ which is the side of the square.

Then $69.57 \times 4 = 278.28 =$ fencing round a square.

And $3.5449 \times 69.57 = 246.618693$ the circumference of the circle round it.

Then square's fencing - - = 278.28

Circle's fencing - - - = 246.618693

Yard's difference - - - - 31.66

Consequently the circle has 31.66 yards less fencing than the square.

Example 2.

A gentleman has a circular basin fronting his house that is 60 yards in diameter, and he would have a gravel walk round it of 5 yards in breadth, what will the expence for gravelling this walk come to at 2d. a yard?

First

First the diameter of the basin = 60, and $5 + 5 + 60 = 70$ the diameter of the walk and basin. Then this

R U L E 1.

From the area of the basin and walk take the area of the basin, leaves the area of the walk.

R U L E 2.

Multiply the sum of the diameters of the basin and walk, and that of the basin by their difference, and that product by .7854 gives the area of the ring round the basin or walk.

R U L E 3.

Multiply half the sum of the circumferences of each by the breadth of the walk gives the area required.

By Rule 1.

$$\begin{array}{r} 60 \times 60 = 3600 \times .7854 = 2827.44 \\ 70 \times 70 = 4900 \times .7854 = 3848.46 \\ \hline \end{array}$$

Diff. the area of the walk 1021.02.

By Rule 2.

$$\begin{array}{r} 60 + 70 = 130 \\ 70 - 60 = 10 \\ \hline \end{array}$$

$$1300 \times .7854 = 1020.92 \text{ the area.}$$

By Rule 3.

$$\begin{array}{r} 3.1416 \times 60 = 188.496 \text{ circum. of basin} \\ 3.1416 \times 70 = 219.812 \text{ circum. of both} \\ \hline \end{array}$$

$$2)408.308 \text{ sum.}$$

$$\begin{array}{r} 204.154 \\ 5 \\ \hline \end{array}$$

1020.770 area of the walk.

Now 1021 yards at 2d. a yard = £ 8 10 2.

Example 3.

I want to plant a clump of firs, and larches in a circular piece of ground that shall measure 4 acres; how long must the chord be to strike the circle?

Here

P. IV. MENSURATION. 65

Here is given the area to find the diameter, half of which is the length of the chord.

$$4840$$

$$\underline{4}$$

19360 whose square root is 139.14.

And $1.12837 \times 139.14 = 157$ yards, the diameter; therefore its half is $78\frac{1}{2}$ yards the length of the chord.

PROP. IX.

To determine the size of round water-pipes, to supply water on the same level.

RULE.

As the time in which it is now done :

To the square of the given pipe's bore ::

So is the time required :

To the square of the diameter of the pipe required, in a reciprocal proportion.

Example.

I have a water-pipe two inches in the bore, which fills my cistern in an hour and a half; but wanting this to be done in half the time, I would know what the bore of the pipe ought to be to do it?

As 90 minutes : $(2 \times 2 =) 4 :: 45$

$$\underline{4}$$

$$\begin{array}{r} 45 \overline{) 360} 8 \\ \underline{360} \end{array}$$

The square root of 8 is $= 2.82$ inches, the diameter of the pipe required.

PROP. X.

To find the area of a semicircle.

RULE.

Multiply the square of the diameter by .3927 gives the area.

Example.

If the diameter of front line of a semicircular alcove be 35 yards, what is the area?

$35 \times 35 = 1225 \times .3927 = 481.0575$, the area.

F

Cor.

FIG. *Cor.* The area of a semicircle and quadrant may be had by finding the area of the whole circle, and the half or fourth part is the area respectively.

P R O P. XI.

To find the area of a sector of a circle.

R U L E.

Multiply the semidiameter by half the arch line, the product is the area.

Example.

In a circle whose semidiameter is 25, and the length of the arch 62, required the area?

$$\begin{array}{r} 2)62 \\ \hline \end{array}$$

$$31 \times 25 = 775, \text{ area.}$$

P R O P. XII.

71.

To find the area of a circular segment DEFD, whose height or versed sine EG is given, and also its chord DF.

R U L E.

Multiply the versed sine by .626 and to the square of the product add the square of half the chord, extract the square root of the sum, then multiply twice the root by two thirds of the versed sine, gives the area.

Example.

71.

What is the area of a circular segment DEFD where chord DF is 24 feet, and versed sine EG = 4.78?

$$4.78 \times .626 = 2.99228 \times 2.99228 = 8.9537395984.$$

Also $3)4.78(1.59 \times 2 = 3.18.$

$$\begin{array}{r} 2)24 \\ \hline \end{array}$$

$$12 \times 12 = 144 + 8.9537395984 = 152.9537, \&c. \text{ whose square root is } 12.36.$$

$$\text{And } 12.36 \times 2 = 24.72 \times 3.18 = 78.6096, \text{ area.}$$

P R O P. XIII.

71.

Given the chord DF and versed sine EG, to find the diameter of that circle.

R U L E.

Divide the square of half the chord by the versed sine, to the quotient, and the versed sine gives the diameter.

Example.

Example.

Let the chord be 18 and versed sine 4, required the diameter?

$$\begin{array}{r} 2 \overline{) 8} \\ \hline \end{array}$$

$$9 \times 9 = \frac{81}{4} = 20.25 + 4 = 24.25, \text{ diameter.}$$

PROP. XIV.

Given the diameter, or radius of a circle as DC, and chord DF, 713
to find the versed sine EG.

RULE.

From the square of the radius, take the square of half the chord, the square root of the difference taken from the radius leaves the versed sine.

Example.

Let the radius DC = 20, chord DF = 38, to find the versed sine EG?

$$20 \times 20 = 400$$

$$19 \times 19 = 361$$

Diff. 39, whose square root is 6.24, and 20 - 6.24 = 13.76, versed sine.

PROP. XV.

To find the length of an arch of a circle DEF, by having the 714
chord DF.

RULE.

Make DE = EF, and draw the chord DE; then from 8 times DE take DF, divide the difference by 3, the quotient is the length of the arch DEF.

Example.

There is an arch of a circle, whose chord DF is 40 inches, and the chord DE is 25; required the length of the arch DEF?

$$\begin{array}{r} \text{Chord DE} = 25 \\ 8 \\ \hline \end{array}$$

$$200$$

$$40$$

$$\begin{array}{r} 3 \overline{) 160} \\ \hline \end{array}$$

53 $\frac{1}{3}$ the length of the arch.

F 2

PROP.

FIG.

PROP. XVI.

Given the base and height of the segment of a circle; to find the diameter.

RULE.

71. As the height of the segment GE :
 To half the chord or base DG or GF ::
 So is GF, the same half :
 To KG a fourth number,
 which added to the segment's height, gives the diameter EK.

Example.

Let the height of the segment be = 18, and base = 48;
 required its diameter?

$$GE = 18, DF = 48, DG = 24.$$

$$\text{Then } 18 : 24 :: 24 : 32$$

18 added

$$\text{Diameter} = 50.$$

PROP. XVII.

Given the diameter of a circle, to find the diameter of another, that shall be in any given proportion to it.

RULE.

Multiply or divide the given diameter by the square root of the required greater or less proportion, the product in the former, or the quotient in the latter, gives the diameter.

Example 1.

What is the diameter of that circle, whose area shall be 5 times as large as one of 18 inches diameter?

The square root of 5 is 2.236

$$\text{Then } 2.236 \times 18 = 40.248.$$

Or, $40\frac{1}{4}$ nearly inches, the diameter required.

Example 2.

I have a circular fountain that measures 50 yards diameter; but I would have one ten times less, what must the diameter be?

The square root of 10 is 3.162

$$\text{Then } 3.162 \div 50 = 0.06324$$

Answer 15.812 yards the diameter required.

PROP.

P R O P. XVIII.

FIG.

To find the area of an ellipsis.

R U L E.

Multiply the transverse diameter by the conjugate, and that product by .7854 gives the area.

Example.

72.

There is an ellipsis the transverse $mn = 50$, and conjugate axis $cd = 40$, required the area?

$$50 \times 40 = 2000$$

$$\text{And } 2000 \times .7854 = 1570.8, \text{ area}$$

P R O P. XIX.

To find the periphery of an ellipsis.

R U L E 1.

Multiply half the sum of the transverse and conjugate diameters by 3.1416, the product is the periphery nearly.

R U L E 2.

To twice the square root of the sum of the squares of the two diameters, add one third part of the conjugate, the sum will be the periphery.

Example.

72.

Let transverse $mn = 50$, conjugate $cd = 40$ yards, how many yards is it round?

$$\text{By rule 1. } 50 + 40 = 90, \text{ and } \frac{90}{2} = 45$$

$$\text{Then } 3.1416 \times 45 = 141.372, \text{ periphery.}$$

$$\text{By rule 2. } 50 \times 50 = 2500$$

$$\text{And } 40 \times 40 = 1600, \text{ root.}$$

$$\begin{array}{r} 4100 \overline{) 64.0312} \\ \underline{2} \\ 128.0624 \\ \underline{13.3333} \\ 141.3957 \end{array}$$

$\frac{40}{3} = - - - -$

Periphery = 141.3957.

F 3

P R O P.

FIG.

PROP. XX. PROB.

72. Given the two diameters of any ellipsis, to find the breadth of a walk surrounding the said ellipsis, to be every where of an equal breadth, that shall contain a given area.

RULE.

Divide the given area of the elliptical walk by 3.1416, to the quotient add one fourth of the square of half of the sum of the ellipsis's two principal diameters, from the square root of the sum take a fourth part of the sum of the ellipsis's two diameters, the remainder is the breadth of the walk.

Example 1.

72. A gentleman has a grass-plat fronting his house, and would have in it an elliptical one to front his door at a proper distance, whose transverse axis *mn* should be 50 yards, and conjugate *cd* 40 yards; now he would have a gravel walk to surround the said grass-plat, to be every where of an equal breadth, so as to take up just a rood of land; what must the breadth of the gravel walk be?

First, the area of the walk is a rood = 1210 yards,

Then $3.1416 \mid 1210 (385.154$

Ellipsis transverse = 50

conjugate = 40

90

45

And $45 \times 45 = 2025$, and $\frac{2025}{4} = 506.25$

To - - - 385.154

add - - - 506.25

891.404

whose square root is 29.85

Take $\frac{90}{4} = 22.5$

Breadth of the walk = 7.35

For

For a proof of this

$$\begin{array}{r} \text{To} \quad - \quad - \quad - \quad 50 \\ \text{add} \quad - \quad - \quad - \quad 14.7 \\ \hline \end{array}$$

72.

Transverse - - 64.7 of the walk and plat,

$$\begin{array}{r} \text{To} \quad - \quad - \quad - \quad 40 \\ \text{add} \quad - \quad - \quad - \quad 14.7 \\ \hline \end{array}$$

Conjugate - - 54.7 of the walk and plat,

Then $64.7 \times 54.7 \times .7854 = 2779.601$, the area of the whole,

From - - - 2779.601

Take - - - 1570.8, area of the plat,

Remainder is the walk 1210 nearly; the defect being for want of carrying the process to more places of decimals.

Example 2.

72:

A gentleman having in his garden an elliptical fountain, whose greater diameter is 30, and the less 24 feet, orders a free stone walk to be made round it, to be every where of an equal breadth, and to take up just the same quantity of ground as the fountain itself; what must the breadth of the walk be?

$$\begin{array}{r} 30 \\ 24 \\ \hline 54 \\ \hline \end{array}$$

27, half sum.

Then $30 \times 24 = 720 \times .7854 = 565.488$, the area of the fountain or walk.

$$3.1416)565.488(180$$

Also $27 \times 27 = 729$, and $\frac{729}{4} = 182.25$

$$\begin{array}{r} \text{To} \quad - \quad - \quad - \quad 180 \\ \text{add} \quad - \quad - \quad - \quad 182.25 \\ \hline \end{array}$$

From - - - 362.25 whose square root = 19.03286

$$\begin{array}{r} \text{Take} \quad \frac{54}{4} = \quad - \quad - \quad 13.5 \\ \hline \end{array}$$

feet
the fountain.

5.53286, breadth of the walk round

Proof.

$$5.53286 \times 2 = 11.06572$$

And 30 added is 41.06572 transverse axis of fountain and walk.

To 11.06572
add 24

35.06572 conj. axis of fountain and walk.

$$\text{Then } 41.06572 \times 35.06572 = 1440,$$

And $1440 \times .7854 = 1130.976$, area of walk and fountain.

From area, walk, and fountain - - - 1130.976

Take area of fountain - - - - - 565.488

Remains area of walk - - - - 565.488

P R O P. XXI.

To find the area of a semi-ellipsis, or the area of an elliptic arch.

R U L E.

Multiply the span or width of the arch by its height, and that product by .7854 gives the area.

Example.

There is an elliptical arch whose span is 50 feet, and height 20, required the area?

$$50 \times 20 = 1000 \times .7854 = 785.4 \text{ feet, area.}$$

P R O P. XXII.

To find the area of a quadrant of an ellipsis, viz. the area mce.

R U L E.

72. Multiply the length by the height, and that product by .7854, gives the area.

Example.

There is an elliptical quadrant whose length AE is 25 feet, and height EC 20 feet, required the area?

$$25 \times 20 = 500 \times .7854 = 392.7 \text{ feet.}$$

To measure REGULAR POLYGONS.

P R O P. XXIII.

The side of any regular polygon being given, to find its area.

R U L E.

Multiply the square of the given side

by {	1.72047	if a pentagon	} the product is the area.
	2.59807	- - - a hexagon	
	3.63391	- - - a heptagon	
	4.82842	- - - an octagon	
	6.18182	- - - a nonagon	
	7.69421	- - - a decagon	
	9.36564	- - - undecagon	
	11.19615	- - - duodecagon	

Example 1.

Let the side of a regular pentagon be 100 yards, what is the area?

$100 \times 100 = 10000 \times 1.72047 = 17204.7$ yards, the area.

P R O P. XXIV.

The area of any regular polygon being given, to find the length of its side.

R U L E.

Multiply the square root of the given area

by {	.76238	for a pentagon	} the product will be the length of the side.
	.6204	- - - a hexagon	
	.52458	- - - a heptagon	
	.45509	- - - an octagon	
	.4022	- - - a nonagon	
	.36051	- - - a decagon	
	.32676	- - - undecagon	
	.29885	- - - duodecagon	

Example.

If the area of a regular octagon be 1444 yards, what is the length of its side?

The square root of 1444 is 38.

Then $.45509 \times 38 = 17.29342$ yards.

P R O P.

PROP. XXV.

The area of any regular polygon being given, to find the radius of its circumscribing circle.

RULE.

Multiply the square root of the given area

by	{	.64852 - - -	pentagon	}	the product is the radius of the circumscribing circle.
		.62040 - - -	hexagon		
		.60451 - - -	heptagon		
		.5946 - - -	octagon		
		.58797 - - -	nonagon		
		.58331 - - -	decagon		
		.57991 - - -	undecagon		
		.57735 - - -	duodecagon		

For the radius of the inscribed circle,

Multiply the square root of the given area

by	{	.52466 - - -	pentagon	}	gives the radius of the inscribed circle.
		.53728 - - -	hexagon		
		.54465 - - -	heptagon		
		.54934 - - -	octagon		
		.55251 - - -	nonagon		
		.55642 - - -	undecagon		
		.55767 - - -	duodecagon		
		.55476 - - -	decagon		

Example 1.

If the area of a regular octagon be 1444 yards, what is the radius of the circle circumscribing that octagon?

The square root of 1444 is = 38,

And $.5946 \times 38 = 22.5948$ yards, the radius of the circumscribing circle.

Example 2.

Required the radius of the inscribed circle in that octagon, whose area is 1444 yards?

The square root of 1444 is = 38.

Then $.54934 \times 38 = 20.87492$ the radius inscribed in that octagon.

If the reader would see how the above tables are constructed, he will find the rules in *Miscellanea Curiosa Mathematica*, by the late Mr. DODSON.

PROB.

PROBLEM.

FIG.

A gentleman would have an octagonal basin fronting his house, that shall contain half an acre of land; how long must the chord be to strike the circumscribing circle, and how long must the side be?

yards
2)4840 acres

2420 half an acre, whose square root is 49.193.

Then $49.193 \times 45509 = 22.38724$, &c. yards = 22 yards 1 foot 2 inches, the side.

And $49.193 \times .5946 = 29.250157$, &c. or $29\frac{1}{4}$ yards, the length of the chord.

PROP. XXVI.

To find the surface of arches of bridges, vaulted roofs, &c.

If the arch is a circle, or any segment of a circle, its length may be found by the rules already given.

Or, Take a string and measure the length of the curve of the arch, then multiply the length of this arch, by the length of the vault, the product is the surface.

Example.

There is a vault built according to the *Emersonian* arch, (see his *MECHANICS*, 410) which measures 40 yards, and the length of the vault is $12\frac{1}{2}$ yards, what is the surface of this arch?

$$\begin{array}{r} 40 \\ 12\frac{1}{2} \\ \hline 480 \\ 20 \\ \hline \end{array}$$

Surface 500 yards.

OBSERVATION.

All irregular figures may be measured, and their contents found, as well as those that are regular, by reducing them into triangles and trapeziums, and then finding the content of each part separately, the sum of the whole will be the content.

Example.

Let ABCDEFGA represent an irregular inclosure; now 73- before the area of this can be found, reduce it into trapeziums and

FIG. and a triangle, as it will admit of; and it may be reduced different other ways, but the conclusion will come out the same.

Measure the diagonals and perpendiculars of the trapeziums; and also the base and perpendiculars of the triangle, and admit the measures be as follows, viz.

73.

1. In the trapezium BCDE.

ch. lin.

The diagonal CE = 13 17

 Perpendiculars $\begin{cases} Bm = 8 41 \\ Dn = 5 50 \end{cases}$

2. In the trapezium AEFG.

ch. lin.

The diagonal AF = 14 80

 Perpendiculars $\begin{cases} Ec = 4 84 \\ Gf = 6 36 \end{cases}$

3. In the triangle ABE.

Base BE = 16 24

Perp. Ar = 7 10

841

550

2)1391

695

BCDE

484

636

2)1120

560

1624

710

1624

11368

2)11.53040

ABE

5.76520

1317

695

6585

11853

7902

9.15315

1480

560

888

740

8.28800 AEFG.

The

The area of the trapezium BCDE = 9.15315
 trapezium AEFG = 8.28800
 triangle ABE = 5.76520

Acres
 23.20635
 4

.82540
 40

A. R. P.

Answer 23 0 33

33.01600

PROP. XXVII.

To take off-fets and parts of crooked rivers, hedges, &c. and cast up their contents.

Take a straight stationary line, at some convenient distance from the river, hedge, &c. then at every bend of it, both inwards and outwards take perpendicular lines on your stationary line by a cross staff, or by your eye if near, and measure your stationary line as you go along in taking the off-fets.

Then multiply each part of the stationary line by the sum of off-fets at each end of that part, and so proceed till you have done the whole length; then add all the results together, and take half the sum which gives the content of these irregular curves.

Example.

Let *adfgbo:vx* be a river bounding one side of a meadow, 74. to find the content of the irregular curves?

At some convenient distance I take a stationary line as AB, and measure as follows, both the distances on the stationary line, and the corresponding off fets.

⊙	A	Off-fets
	ch. lin.	right hand o
Ab =	1 84	ba = 87
Ac =	3 80	cd = 41
Ae =	6 18	ef = 95
Ag =	7 94	hg = 38
Am =	10 82	km = 98
Ar =	13 37	no = 66
As =	16 29	rs = 85
Av =	18 38	tv = 90
AB =	20 74	Bx = 94

Off-fet

$$\begin{array}{r}
 \text{Offset at A} = 0 \\
 \text{at } b = 0.87 \\
 \hline
 87 \\
 Ab = 184 \\
 \hline
 348 \\
 696 \\
 87 \\
 \hline
 .16008
 \end{array}$$

$$\begin{array}{r}
 \text{Off-set at } b = 87 \\
 \text{at } c = 41 \\
 \hline
 128 \\
 bc = 196 \\
 \hline
 768 \\
 1152 \\
 128 \\
 \hline
 .25088
 \end{array}$$

$$\begin{array}{r}
 \text{Off-set } cd = 41 \\
 ef = 95 \\
 \hline
 136 \\
 ce = 238 \\
 \hline
 1088 \\
 408 \\
 272 \\
 \hline
 .32368
 \end{array}$$

$$\begin{array}{r}
 \text{Off-set } ef = 95 \\
 bg = 38 \\
 \hline
 133 \\
 eh = 176 \\
 \hline
 798 \\
 931 \\
 133 \\
 \hline
 .23408
 \end{array}$$

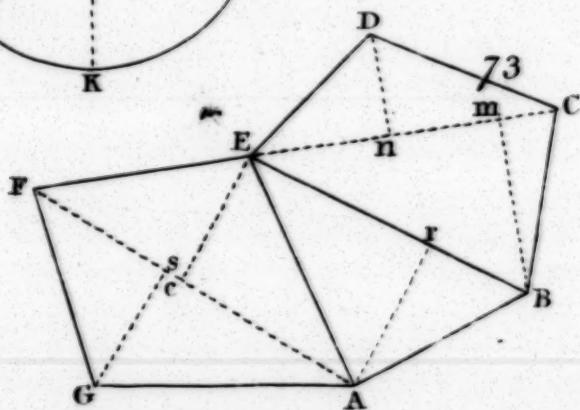
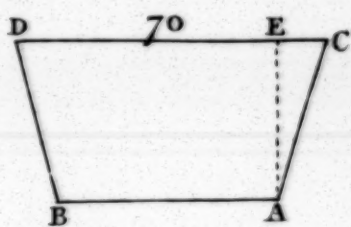
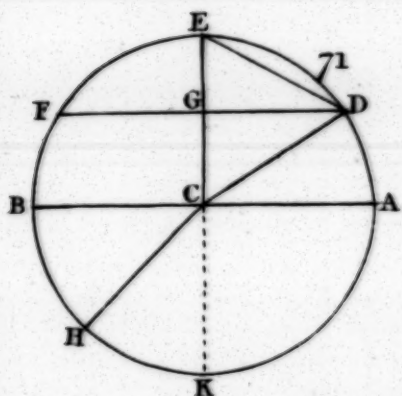
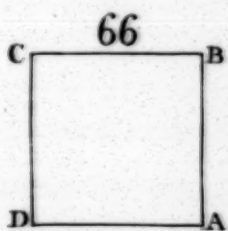
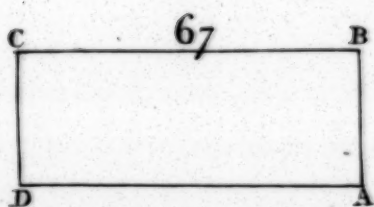
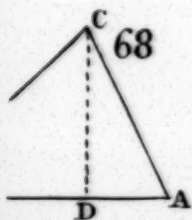
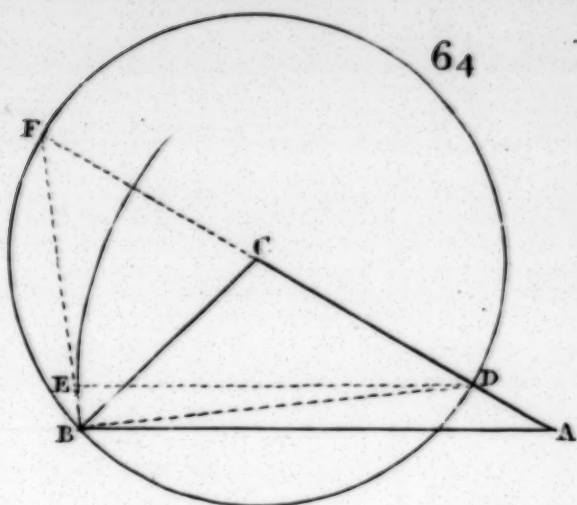
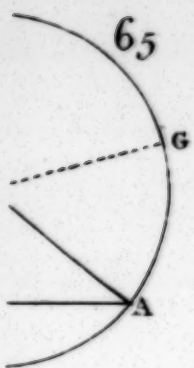
$$\begin{array}{r}
 \text{Off-set } bg = 38 \\
 km = 98 \\
 \hline
 136 \\
 bk = 288 \\
 \hline
 1088 \\
 1088 \\
 272 \\
 \hline
 .39168
 \end{array}$$

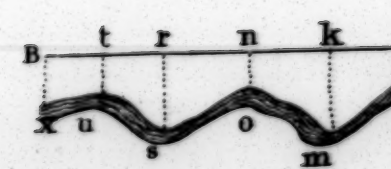
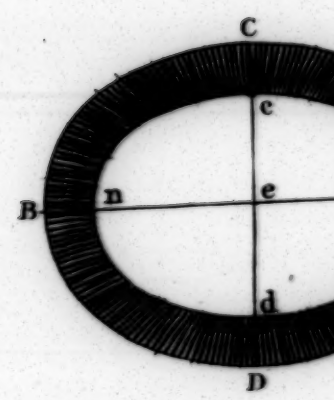
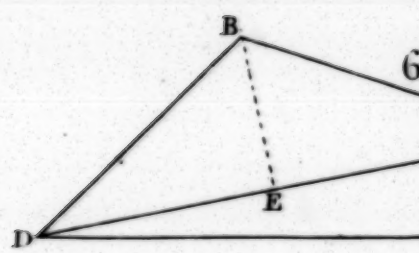
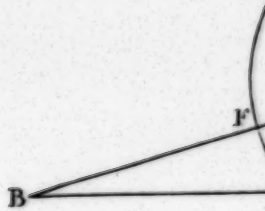
$$\begin{array}{r}
 \text{Off-set } km = 98 \\
 no = 66 \\
 \hline
 164 \\
 kn = 255 \\
 \hline
 820 \\
 820 \\
 328 \\
 \hline
 .41820
 \end{array}$$

$$\begin{array}{r}
 \text{Off-set } no = 66 \\
 rs = 85 \\
 \hline
 151 \\
 292 \\
 \hline
 302 \\
 1359 \\
 302 \\
 \hline
 .44092
 \end{array}$$

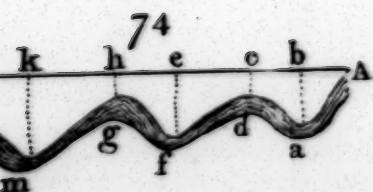
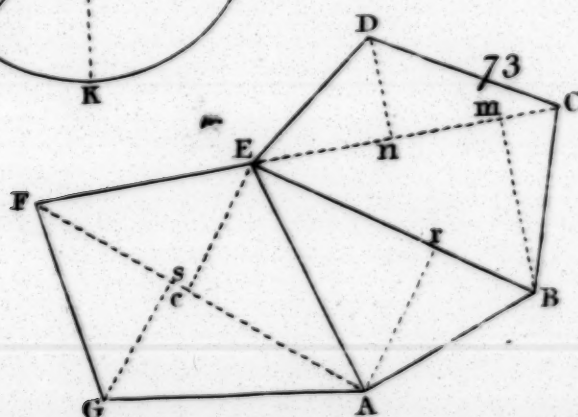
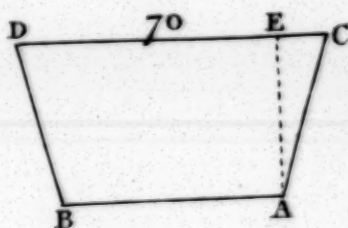
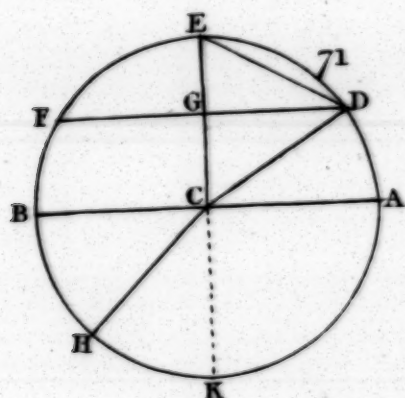
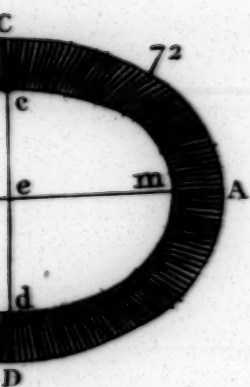
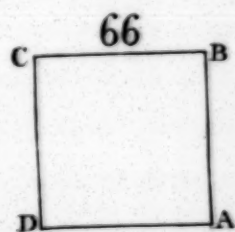
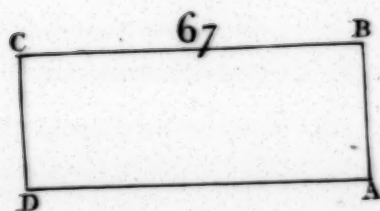
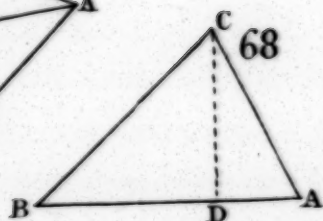
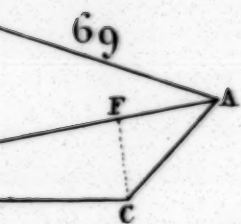
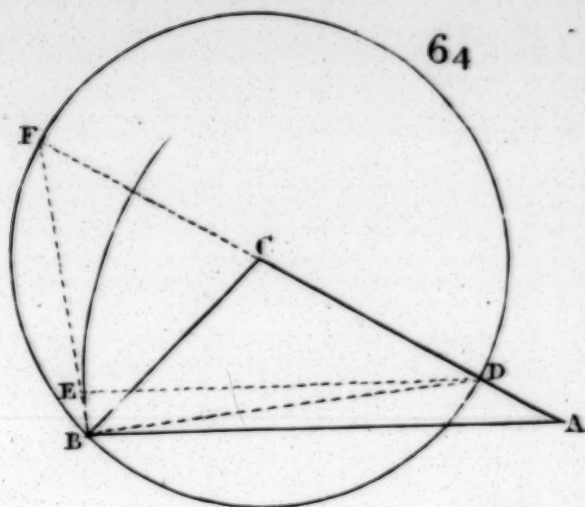
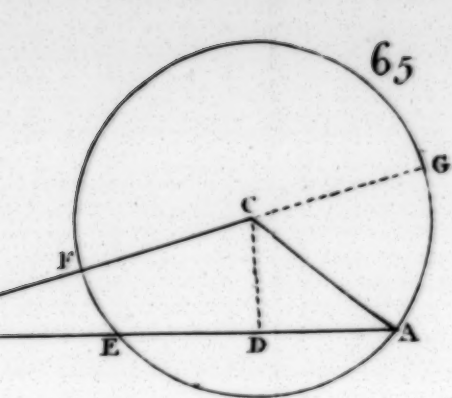
$$\begin{array}{r}
 \text{Off-set } rs = 85 \\
 tu = 90 \\
 \hline
 175 \\
 rt = 209 \\
 \hline
 1575 \\
 3500 \\
 \hline
 .36575
 \end{array}$$

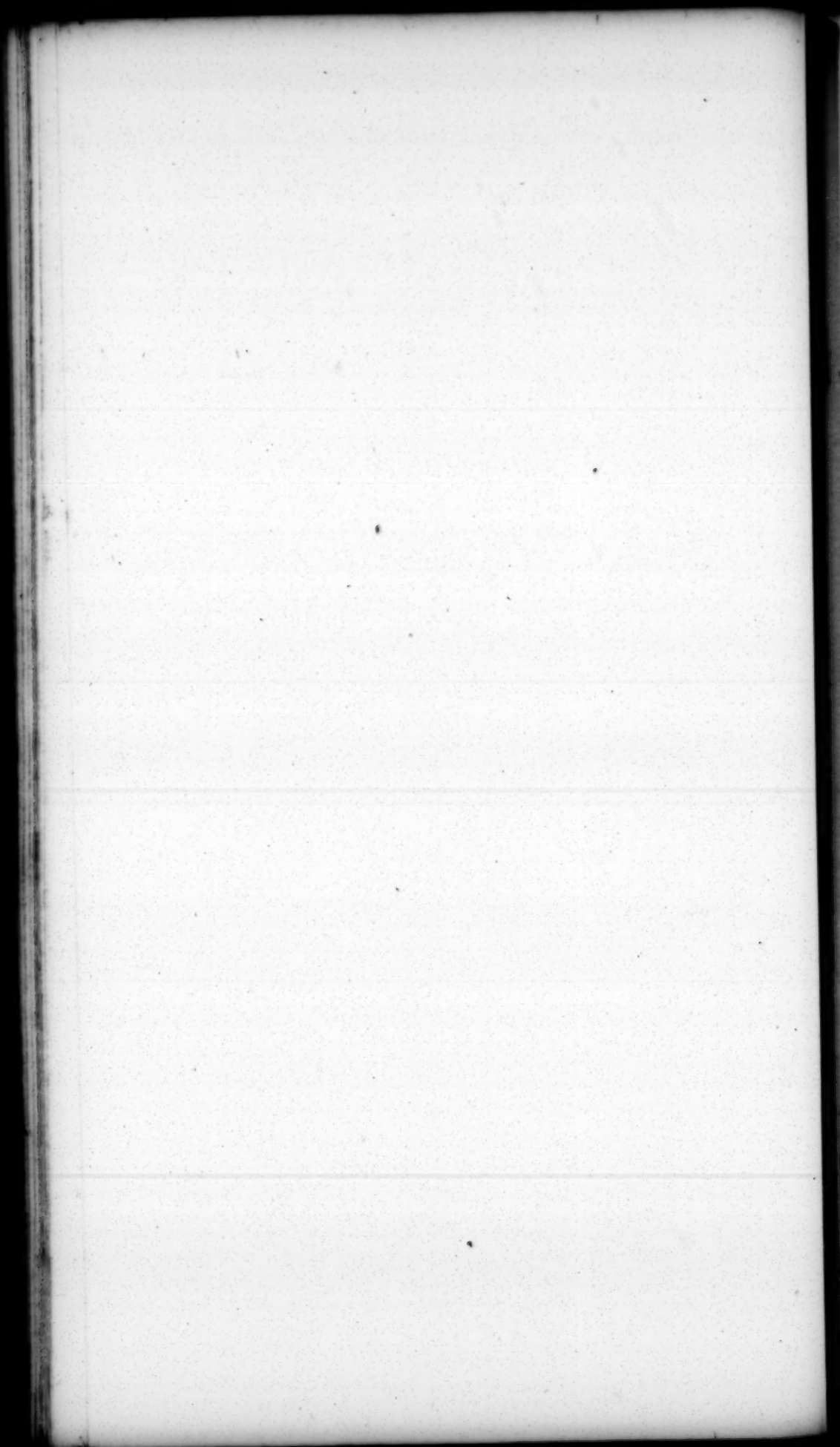
Off-set





Menfuration.





$$\begin{array}{rcl} \text{Off-set} & t u & = 90 \\ & B x & = 94 \end{array}$$

$$\begin{array}{r} 184 \\ t B = 236 \end{array}$$

$$1104$$

$$552$$

$$368$$

$$.43424$$

$$.16008$$

$$.25088$$

$$.32368$$

$$.23408$$

$$.39168$$

$$.41820$$

$$.41092$$

$$.36575$$

$$2)3.01951$$

$$\text{Area} = 1.50975$$

Or 1 acre 2 roods 1 perch.

P A R T V.
THE
MEASURING OF SOLIDS.

PROPOSITION I.

To find the Solidity of a Cube ABCD.

R U L E.

75. **M**ULTIPLY the side by itself, and that product by the same side again, and the last product is the solid content in the same sort of measure.

Example.

There is a cube whose side is 14 inches, what is the solidity?

$$\begin{array}{r}
 14 \\
 14 \\
 \hline
 56 \\
 14 \\
 \hline
 196 \\
 14 \\
 \hline
 784 \\
 196 \\
 \hline
 \end{array}$$

2744 solid inches.

Cor. If you have the solidity of any cubic vessel, and would know the dimensions of it, extract the cube root of the solidity, the root is the side.

P R O P. II.

To find the solidity of a parallelopipedon.

R U L E.

76. Multiply the length by the breadth, and that product again by the depth or thickness, this last product is the solidity.

Example.

Example 1.

There is a parallelopipedon whose length is 100 inches, breadth 20, and depth 14 inches, how many solid feet?

$$\begin{array}{r} 100 \\ 20 \\ \hline 2000 \\ 14 \\ \hline \end{array}$$

1728)28000(10 feet and 720 inches.
Inches

N. B. 282 - - - 1 ale gallon
231 - - - 1 wine gallon
1728 - - - 1 foot
2150.42 - 1 malt bushel.

Example 2.

76.

A vessel in form of a parallelopipedon, whose length AB measured $80\frac{1}{2}$ inches, the breadth CD, $20\frac{1}{4}$; and depth CA, $18\frac{3}{4}$, required the solid feet, also the area in ale, wine, and malt?

$$\begin{array}{r} 80.5 \\ 20.25 \\ \hline 4025 \\ 1610 \\ 16100 \\ \hline 1630.125 \\ 18.75 \\ \hline 8150625 \\ 11410875 \\ 13041000 \\ 1630125 \\ \hline \end{array}$$

1728)30564.84375(17.688 feet
282)30564.84375(108.386 A. G.
231)30564.84375(132.315 W. G.
2150.42)30564.84375(14.21 M B.

P R O P. III.

To find the solidity of a triangular prism.

R U L E.

Multiply the area of the base or end, by the length, the product is the solidity.

G

Example.

FIG.

Example.

77. Let ABCDEF denote a triangular prism, whose base AB is 40 inches, perpendicular cd , 20, and height or length cD , 90; how many solid feet doth it contain?

40

10

400 area of the end.

90 length.

1728)36000(20.8 feet.

PROP. IV.

To find the solidity of a cylinder.

RULE.

Multiply the area of the end by the length gives the solidity.

Example.

78. There is a cylinder whose diameter AB is 20 inches, and length cD 60, required the solidity?

 $20 \times 20 = 400$ square of the diameter,

And

.7854

400

314.1600 area of the end.

60

18849.6000, solidity.

PROP. V.

To find the solidity of a pyramid

RULE.

Multiply the area of the base or end by one third of the length or altitude, the product is the solidity.

Example 1.

79. Let ABDECc represent a pyramid standing upon a square base, whose side BD is 40 inches, and height Cc 90, required the solidity in feet?

$$\begin{array}{r}
 40 \\
 40 \\
 \hline
 1600, \text{ area of base.} \\
 \frac{1}{3} \text{ length} \quad 30 \\
 \hline
 1728)48000(27.7 \text{ feet.}
 \end{array}$$

Example 2.

There is a pyramid standing upon an octagonal base, each side measures 20 inches, and length 60, required the solidity?

$$\begin{array}{r}
 20 \\
 20 \\
 \hline
 \end{array}$$

400, the square of the side,

And $48284 \times 400 = 1931.36$, the area of the base, by *prop. xxiii.*

And $1931.36 \times 20 = 38627.2$, solid inches = 22.35, solid feet.

PROP. VI.

To find the solidity of a body bounded by four regular isosceles triangular faces, whose base and side are given.

RULE.

From the square of the side subtract half the square of the base, and the square root of the difference multiplied by one sixth of the square of the base, the product is the solidity.

Example.

A piece of timber bounded by four regular isosceles triangular faces, the base of each is 3 feet, and the length of each side is 5 feet, required the solidity?

	feet	
Side	5	Base 3
	5	3
From	25	2)9
take	4.5	
		4.5
Diff.	20.5	

The square root of 20.5 is = 4.5276

Multiplied by $\frac{1}{6}$ = . - - - 1.5

226380

45276

Solidity - - - 6.79140

G 2

The

FIG.

- The above rule is demonstrated thus: Let the base of the
 80. isosceles triangle $=m=3$ feet, side $=n=5$ feet; and take a square prism, whose side of the base is $=m$, and length $\sqrt{n^2 - \frac{1}{4}m^2}$, draw lines from the ends of two opposite sides of the upper base to the middle of the sides of the lower base, and let the two slices be cut off by those lines, which together will be half the prism. Then from the ends of the upper base, draw lines to the middle of the lower base, and let the two slices be cut off by those lines, which together, it is evident, will be a square pyramid $=\frac{1}{3}$ of the prism; hence there is $\frac{1}{2}$ of the prism cut off, and $\frac{1}{3}$ for the remaining solid.

Theorem. $\frac{m^2}{9}\sqrt{n^2 - \frac{1}{4}m^2} = 6.7914$ cubic feet the solidity required.

80. To represent this quaint solid truly. Take a piece of clear pasteboard, and draw $bd=3$ feet and $bc, dc=5$ feet, bisect the sides as m, n , and base as at r , join mn, mr, nr , these three lines cut half through and folded will represent the body truly.

PROP. VII.

To find the solidity of a frustum of a pyramid.

RULE.

Multiply the side of the greater base or end by the side of the less; take the difference of the two sides, and add one third of the square of this difference to the product, this sum multiplied by the frustum's length, gives the solidity, if the frustum stand on a square base; but if a polygon, then the last product must be multiplied by the polygon's factor by *prop. xxiii.* and it will give the solidity.

Example 1.

81. There is a frustum of a square pyramid ABDHGE, the side AB at the greater end is 20 inches EG, the side of the less end is 16 inches, and length 60; required the solidity?

20	20
16	16
320	4 diff.
5.33	4
325.33	3)16
60	
	5.33
1728)19520(11.29 feet.	

Example

Example 2.

What is the solidity of the frustum of an octagonal pyramid, the side of its greater end being 20 inches, and that of the less end 15, and length 60 inches?

20	20
15	15
—	—
300	5
8.33	5
—	—
308.33	3)25
60	8.33
—	
18500	

And $18500 \times 4.8284 = 89325.4$ cubic inches $= 51.6$ feet.

P R O P. VIII.

To find the solidity of a cone.

R U L E.

Multiply the square of the diameter by ($\frac{.7854}{3} =$) .2618 and the product by the perpendicular or height, gives the solidity.

Or, Multiply the square of the circumference by .0265, and the product by that height, gives the solidity.

82.

Example 1.

Let ABCD represent a cone, whose diameter AB is 20, and height CD 40; required the solidity?

$20 \times 20 = 400 \times .2618 = 104.72$, and $104.72 \times 40 = 4188.8$ solidity.

Example 2.

Admit the circumference of a cone's base be 100, and height 150? required the solidity?

$100 \times 100 = 10000 \times .0265 = 265$, and $265 \times 150 = 39750$, solidity.

P R O P. IX.

To find the solidity of a frustum of a cone.

R U L E.

To the product of the diameter of the two bases, add $\frac{1}{3}$ the square of their difference, the sum multiplied by the height, and the last product by .7854 gives the solidity.

G 3

Example.

FIG.

Example.

83. Let ABEF represent a frustum of a cone, whose greater diameter AB is 40, less diameter EF, 20, and height CD, 60; required the solidity?

$$40 \times 20 = 800$$

$$20 \times 20 = 400, \text{ and } \frac{400}{3} = 133.33$$

Now	800
	133.33
	933.33
	60

Mult.	-	-	56000
			7854
Prod.	-	-	43982.4

Or this,

R U L E.

Multiply the circumference of each base by one another, add the squares of the circumference of each base to the product of the circumferences, multiply the sum by the height, and the last product by .026526 gives the solidity.

Example 2.

Let the circumference of the greater base be 52 inches, lesser base 31, and height 85; required the solidity?

52	×	52	=	-	-	2704
31	×	31	=	-	-	961
52	×	31	=	-	-	1612

5277

And $5277 \times 85 = 448545 \times .026526 = 11898.10467$, solidity.

P R O P. X.

To find the solidity of a globe or sphere.

R U L E I.

84. Multiply the cube of the diameter by .5236 gives the solidity.

R U L E

R U L E 2.

Multiply the cube of the circumference by .0169, the product is the solidity.

Example 1.

If a bullet be 8 inches in diameter, what is the solidity?

$$\begin{array}{r}
 8 \\
 8 \\
 \hline
 64 \\
 8 \\
 \hline
 512 \\
 \text{Cubic inches} = 268.0632
 \end{array}
 \qquad
 \begin{array}{r}
 .5236 \\
 512 \\
 \hline
 10472 \\
 5236 \\
 \hline
 26180
 \end{array}$$

Example 2.

If the circumference of a globe be 37.7 inches, what is the solidity?

$$\begin{aligned}
 37.7 \times 37.7 \times 37.7 &= 1421.29 \times 37.7 = 53582.633, \\
 \text{And } 53582.633 \times .0169 &= 905.5464977, \text{ the solidity.}
 \end{aligned}$$

P R O P. IX.

To find the solidity of a sloping body.

R U L E.

Multiply the area of its profile at the end by the length, gives the solidity.

Example 1.

85.

Let ABCDEF be a wall broader on one side than the other; and let the length AB=50 feet, the breadth BH of the slope 6 feet, the height HK, 14 feet, and therefore the depth BC, 10 feet; to find the solidity?

$$\begin{aligned}
 BH + HC &= BC = 10, \\
 \text{Add } DK &= 4
 \end{aligned}$$

$$\begin{array}{r}
 \text{Mult. HK or DC} \\
 14 \\
 14 \\
 \hline
 56 \\
 14 \\
 \hline
 2)196
 \end{array}$$

Area of the trapezoid AKDC = 98 = area of the profile.

Mult. length, 50

Solidity of the whole, 4900 cubic feet.

G 4

P R O P.

PROP. XII.

To find the solidity of a wall sloped on both sides.

RULE.

Multiply the profile of the wall by the length, gives the solidity.

Example.

6. There is a wall ABCDK, sloped on both sides, whose length $AB=48$, BG, the breadth of the sloping body $ABGOE=6$, CH the breadth of the other sloping body $=5$, GO or HD the height of the wall $=12$, the breadth $OD=$ feet; required the solidity?

First $BG+CH+DH=BC=14$, and BCDO a trapezoid, the profile.

$$\begin{array}{r}
 \text{Then } BC=14 \\
 OD=3 \\
 \hline
 \text{Height} = \begin{array}{r} 17 \\ 12 \\ \hline \end{array} \\
 \begin{array}{r} 2)204 \\ \hline 102, \text{ area of trapezoid} \end{array} \\
 \text{Length} = \begin{array}{r} 48 \\ \hline 816 \\ 408 \\ \hline 4896 \text{ cubic feet.} \end{array}
 \end{array}$$

PROP. XIII.

To find the solidity of a sloping wall when the two sloping bodies are joined together.

RULE.

Multiply the profile of each part by the length, the several areas added together give the solidity.

Example.

87. There is a wall ABCDEFGH, whose two slopes ABCIH, and CDEFI, are joined in the point I; and let $AB=24$ feet, $BC=6$, $CD=5$, $DE=GH=FI=4$ feet, $CI=12$ feet, to find the solidity?

First

First	BC = 6
mult.	IB = 12
	2)72
Area profile	BCI = 36
Mult.	24
	144
	72
Solidity	864 of BCIHA.
	CD = 5
	CF = 12
	2)60
Profile	30
	4
Solidity of	CIDEF = 120
	GH = 4
	HI = 12
Profile	48
	24
	192
	96
Sol. of HGFI	- - - 1152
of CIDEF	- - - 120
of BCIHA	- - - 864
Cubic feet of the whole	= 2136

P R O P. XIV.

To find the solidity of a sloping body, which has two sloping bodies joined, the one on one side, and the other above it, which in this situation is called the glacis.

R U L E.

Multiply the profile of each part by the length, gives the solidity.

Example.

88.

Example.

Let length $AF = 24$ feet $= GI$.

$$\left. \begin{array}{l} AG = FI = 4 \\ BG = IM = 6 \end{array} \right\} = AB = 10 \text{ feet.}$$

$$\left. \begin{array}{l} BK = ML = 8 \\ BC = 5 \end{array} \right\} = BC = 13 \text{ feet.}$$

$$IE = GH = 8 \text{ feet.}$$

Now the body $ABCDEF$ has before it the talud or sloping body $AGHEIF$, whose base is the rectangle $FAGI$, and the profile the triangle AGH right angled at G ; and above, the glacis $EHKCDL$, whose base is the rectangle $EHLK$, and the profile, the triangle HKC right angled in K ; also $IGBKLE$, a right angled parallelopiped whose base is the rectangle $IGBM$.

OPERATION.

$$\begin{array}{l} 1. \quad AF = EH = 24 \\ \quad \quad IM = EL = 6 \end{array}$$

$$\begin{array}{r} \phantom{GH \text{ height}} \quad \quad 144 \\ GH \text{ height} \quad - \quad 8 \end{array}$$

$$\text{Parallelopiped} \quad 1152 = IGBKLE.$$

$$\begin{array}{l} 2. \quad AG = 4 \\ \quad \quad GH = 8 \end{array}$$

$$\begin{array}{r} \\ 2)32 \\ \hline 16 \\ 24 \\ \hline 64 \\ 32 \end{array}$$

$$\text{Slope} \quad 384 = AGHEF.$$

$$\begin{array}{l} HK = 6 \\ KC = 5 \end{array}$$

$$\begin{array}{r} 2)30 \\ \hline 15 \\ 24 \\ \hline 60 \end{array}$$

Glacis

$$\begin{array}{r} 30 \\ \hline 360 = EHKCDL \end{array}$$

Ta

P. V. MENSURATION.

91 FIG.

Then IGBKLE = 1152
The talud AGHEIF = 384
Glacis EHKCDL = 360

Cubic feet - - - - 1896

PROP. XV.

To find the solidity of a rampart of a fortified polygon, according to Mr. Ozanam's Method.

R U L E.

Multiply each profile by the length gives the solidity of each, their sum is the whole solidity required.

AB represents half the curtain.

BC the flank of the bastion, perpendicular to the curtain for the ease of calculation.

CD the face of the bastion.

HM the capital.

GH the line of the inward sloping body, parallelwise to EF of the rampart.

IKLM the line of the outward sloping body, parallelwise to ABCD.

So GIKLMH represents the plan of a semi-curtain and semi-bastion.

AE the breadth of the rampart above, that is to say, the breadth of the terre plain suppose 72 feet.

EG the quantity of the inward sloping body 18 feet.

AI the quantity of the outward talud or sloping body; for instance 12 feet.

The height of the rampart 18 feet, as in the profile N^o 2. which shews the rampart is about 102 feet broad below; not at all regarding, says Mr. Ozanam, whether the numbers are intirely agreeable to the maxims of a good fortification.

After he has constructed the figure from the above numbers from a scale of equal parts, he reduces it into a prism contained between the two taluds, whose base is the plane ABCDIF; he then finds this plane by reducing it into a trapezoid AEFT, and measures it from the scale; AB is produced to T, and forms the right angled triangle BCT, CTD an oblique angled triangle by the diagonal CT, this being done, the several lines in each may be measured from the scale, and their contents found by the rules already given. Thus,

EF

FIG.

EF measures about	344
AT - - - -	376
AE is supposed	72
BT measures about	160
BC - - - -	120
CD - - - -	270
TV - - - -	160

This being premised, I shall now shew the

OPERATION.

89.

1.

$$\begin{array}{r} EF = 344 \\ AT = 376 \\ \hline \end{array}$$

Mult.

$$\begin{array}{r} 720 \\ AE = 72 \\ \hline \end{array}$$

144

504

$$\begin{array}{r} 2)51840 \\ \hline \end{array}$$

Trapezoid

$$25920 \text{ AEFT.}$$

2.

$$BT = 160$$

$$BC = 120$$

$$\begin{array}{r} 2)19200 \\ \hline \end{array}$$

Right angled

9600 triangle BCT.

3.

$$CD = 270$$

$$TV = 160$$

162

27

$$\begin{array}{r} 2)43200 \\ \hline \end{array}$$

21600 oblique triangle CTD.

25920

9600

57120 surface plane ABCDEF.

18 height.

45696

5712

1028160 cubic feet the solidity of the
internal

internal prism, bounded by the two taluds or sloping bodies, FIG. and placed on the base ABCDFE.

To find the solidity of the taluds.

Reduce them to semi-prisms and pyramids, by letting fall, from the angles of their bases, perpendiculars between the two parallel sides, viz. HO, DN, CP, CQ, KR, KS; and the inward talud, whose base is the trapezoid GEFH, will be found to be divided into a prism, whose base is the rectangle GEOH, and height the same with that of the rampart, and into a pyramid, whose vertex is H, and height, the perpendicular HO, and base a rectangle, whose breadth is the line OF, and length equal to the height of the rampart; and the external talud will be found to be divided into three semi-prisms, whose common height is the same as the rampart's, and the bases, the three rectangles AK, CK, CN, and into 89. three pyramids, two of which have their vertices upwards, and therefore have the height of the rampart for the common height, and their bases the right angled triangle DNM, and the quadrilateral figure CPLQ; and the third is taken for one that has the same height as the rampart, and the square KRBS for a base, because so small a part is inconsiderable.

But in geometrical accuracy, instead of this pyramid, we ought to take the double of another, that has its point below at the point K, the perpendicular KR for its height, and a rectangle for its base, whose breadth is the line BR, and length equal to the height of the rampart.

Then from the scale the lines measure as follows, viz.

	feet
EO, or GH =	330
EG =	18
OF =	14
HO =	6
IK =	206
KQ =	109
PN =	265
AI =	12
KR =	12
QL =	8
MN =	12
DN =	12

OPERA-

OPERATION.

89.

1.

$$\begin{array}{r} GO = 330 \\ EG = 18 \end{array}$$

264

33

5940 area of the rectangle GO.
9 half rampart's height.

53460 solidity of the semi-prism,
whose base is the rectangle GO.

2.

$$OF = 14$$

18 rampart's height.

112

14

252

6 one third of HO.

1512 solidity of the pyramid whose
vertex is H, and height HO. But $1512 + 53460 = 54972$,
solid internal talud.

3.

$$IK = 206$$

$$KQ = 109$$

$$PN = 265$$

580

12 = AI.

6960

9 half rampart's height.

62640 solidity of the three prisms,
whose three rectangles AK, CK, CN represent the bases.

4.

$$KR = 12$$

$$QL = 8$$

$$\text{Half } MN = 6$$

26

12 DN common breadth.

312 sum of the bases of the three
pyramids, contained in the external talud.

Then

Then 312
 6 the $\frac{1}{3}$ rampart's height.

 1872 solidity of the three pyramids.
 62640 solidity of the three semi-prisms.

 64512 solidity of the external talud.
 Then to 1028160 found before
 add 64512 external talud.
 54972 internal talud.

 1147644 cubic feet,
 or 42505 cubic yards.

P R O P. XVI.

To find the solidity or quantity of earth dug up in making a ditch with sloping sides.

Example.

90.

There is a fosse or ditch as ABCDEF, whose breadth AB at the bottom is 24 feet, breadth DC at the top 40 feet, perpendicular depth 10 feet, and length 100 feet. Required the cubic yards dug up in making it?

	feet.
Breadth top	40
Bottom	24
Depth	64
	10
	640
	100
	2)64000
	32000 cubic feet.

Then 27)32000(1185 cubic yards.

Cf

Of the STRENGTH of BEAMS of TIMBER.

Mr. *Emerson*, at page 117 of his *Principles of Mechanics*, treats of the strength of beams of timber in all positions; and their stress, by any weights acting upon them, or by any forces applied to them; where that excellent author says, *The lateral strength of any piece of timber, in any place, whose section is a rectangle, is directly as the breadth and square of the depth.*

Whence to find the strength of any square piece of timber this is the

R U L E.

Multiply the square of the thickness in inches, into its breadth in inches, and that again into the weight that a piece of the same sort of an inch square and a foot long would bear, and divide the product by the length of the timber in feet, gives the answer.

Now to apply this rule to practice, Mr. *Emerson* has been pleased to give us some experiments he made, with pieces of wood a foot long and an inch square, supported horizontally at both ends, what they will bear in the middle before they break, as follows

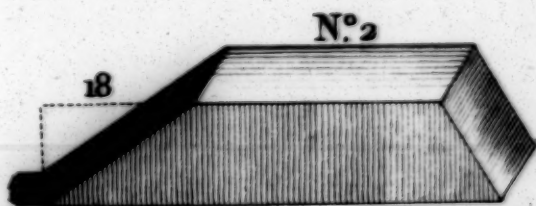
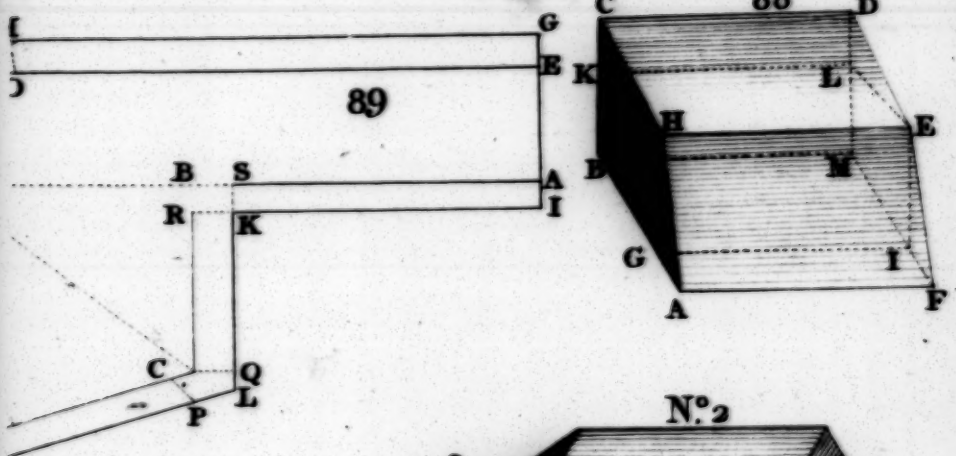
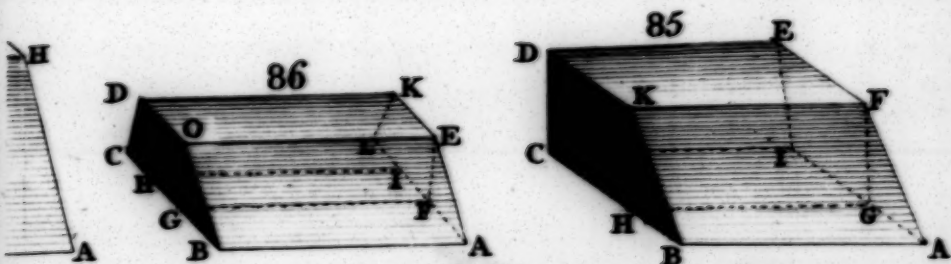
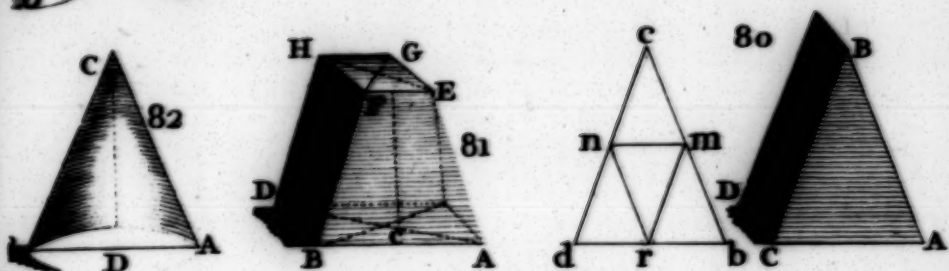
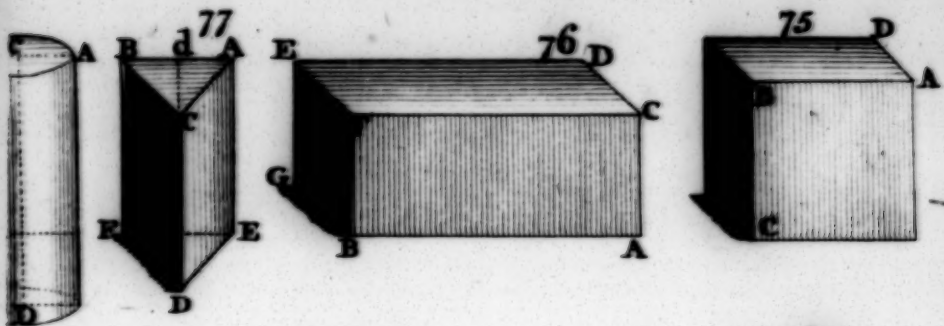
Oak	—	320 pounds.
Elm	—	310
Beech	—	290
Fir	—	280

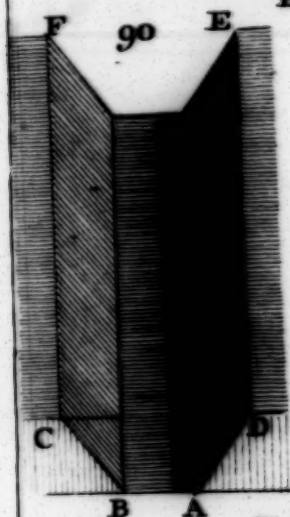
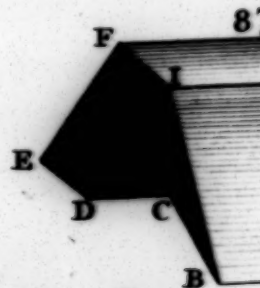
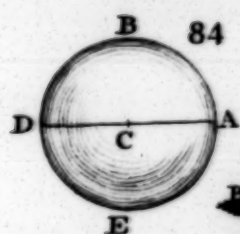
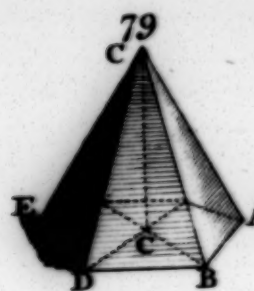
Example.

What weight will a joist of oak sustain that is 12 feet long, 8 inches deep, and 6 inches wide?

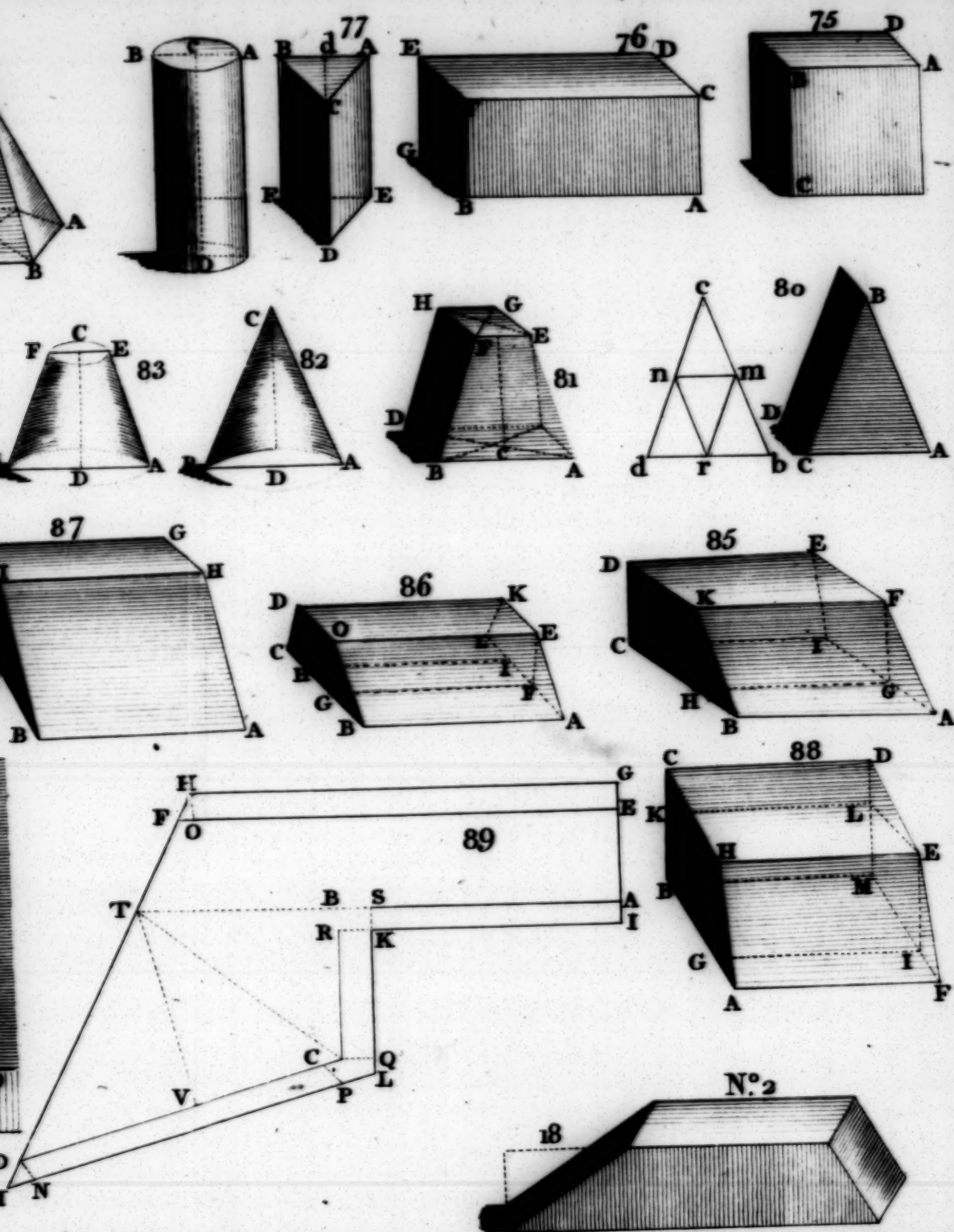
$$\begin{array}{r}
 8 \\
 8 \\
 \hline
 64 \text{ square of the depth.} \\
 6 \\
 \hline
 384 \\
 320 \\
 \hline
 7680 \\
 1152 \\
 \hline
 12 \overline{) 122880} \\
 \hline
 10240 \text{ lb. or } 91 \text{ C. } 1 \text{ Qr. } 20 \text{ lb. the} \\
 \text{weight the joist will bear, being} \\
 \text{hung in the middle.}
 \end{array}$$

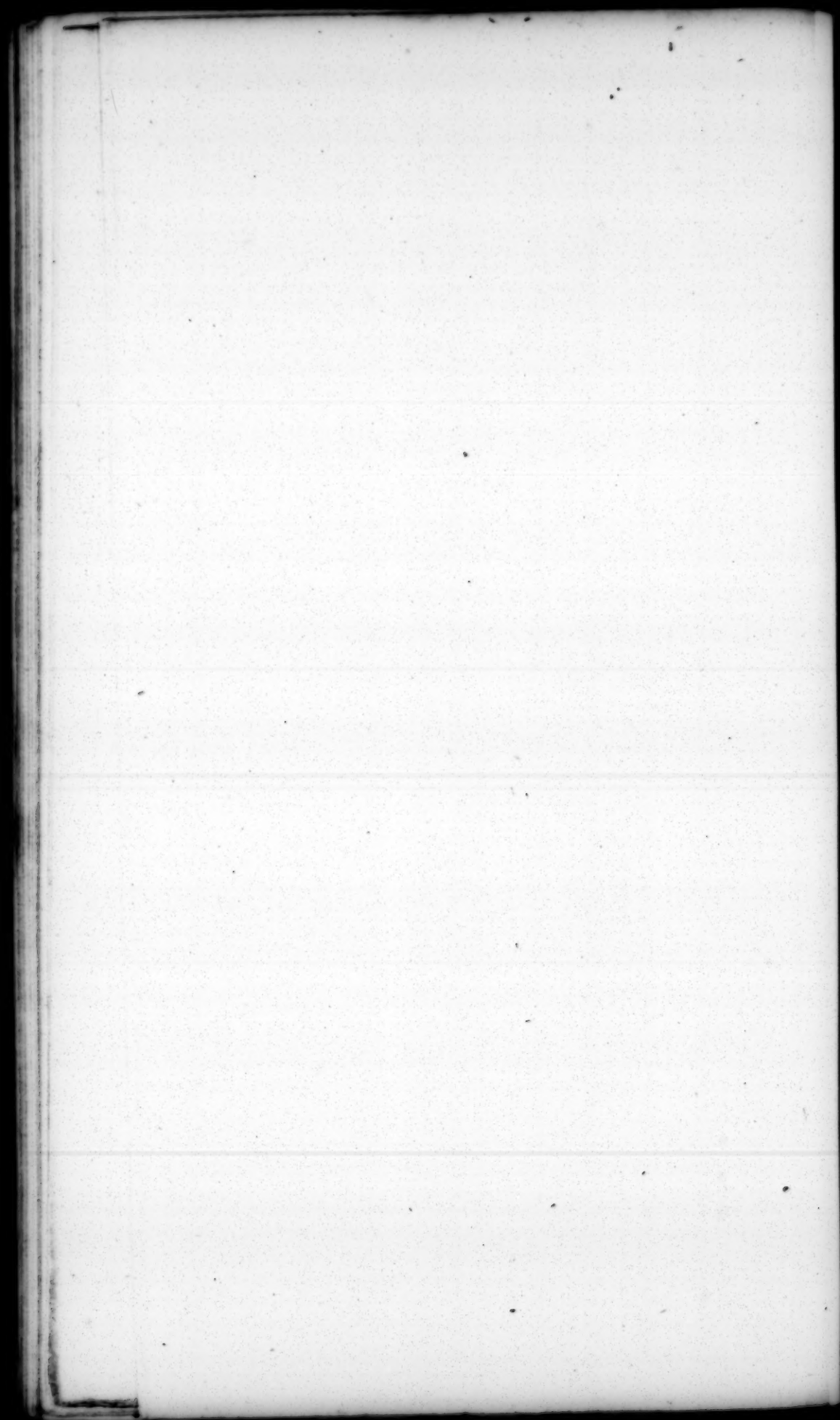
But





Mensuration.





But if the joist be laid flat, then the side 6 inches is the depth, and 8 inches the breadth, and this is the strength.

$$\begin{array}{r}
 6 \\
 6 \\
 \hline
 36 \\
 8 \\
 \hline
 288 \\
 320 \\
 \hline
 576 \\
 864 \\
 \hline
 12 \overline{)92160}
 \end{array}$$

7680lb. or 68C. 2Qrs. 8lb. the joist
 — will bear, when hung in the
 middle.

Since the printing the above, Mr. *Emerson* has published a second edition of his *Mechanics* much enlarged in-4to. and in 1769, he published another *Book of Mechanics* in-8vo. as an introduction to that in 4to. where, at page 94, he has given some more experiments of the strength of timber, viz.

A piece of wood a foot long and an inch square will bear, as follows

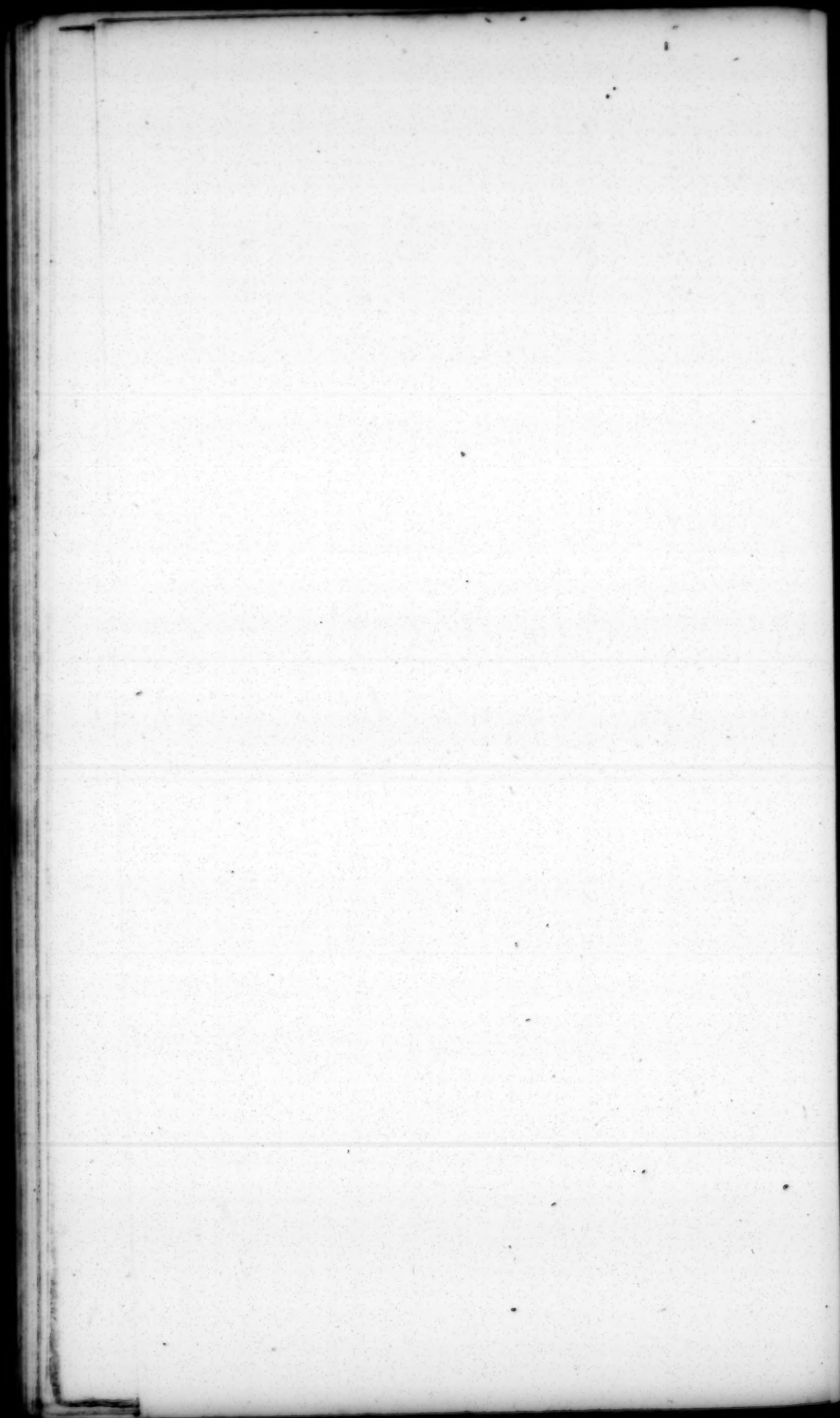
Oak from 320 to 1100 pounds

Elm from 310 to 930

Fir from 280 to 770, according to the goodness; and has shewn how to adjust several pieces of timber in regard to one another, that the *strength* may be always proportioned to the *stress*, they are to endure; and demonstrates from indisputable mathematical principles, that the *breadth multiplied by the square of the depth or thickness, must be as the length multiplied by the weight to be born*, for then, says he, the strength will be as the stress.

From those propositions above quoted and their corollaries may be found the proper dimensions of fir and oak scantlings used in buildings, and those tables given by carpenters and architects examined whether the dimensions they give be true or not, very necessary to be understood by all carpenters.

Now from the experiments the above mentioned author has given us, I find oak is in proportion to fir as 320 to 280, that is as 8 is to 7.



But if the joist be laid flat, then the side 6 inches is the depth, and 8 inches the breadth, and this is the strength.

$$\begin{array}{r}
 6 \\
 6 \\
 \hline
 36 \\
 8 \\
 \hline
 288 \\
 320 \\
 \hline
 576 \\
 864 \\
 \hline
 12 \overline{)92160}
 \end{array}$$

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H

The

The dimensions of oak and fir scantlings, which builders say were appointed by act of parliament after the great fire of London, are for fir girders as under.

T A B L E.

Length. feet.	Breadth. inches.	Depth. inches.
10	8	10
12	$8\frac{1}{2}$	10
14	9	$10\frac{1}{2}$
16	$9\frac{1}{2}$	$10\frac{1}{2}$
18	10	11
20	11	12
22	$11\frac{1}{2}$	13
24	12	14

I shall now set down the first scantling of each table, which are given in Books of Carpentry.

The dimensions of the first scantling, and consequently the shortest, are as under.

	Length in feet.	Breadth inches.	Depth. inches.
<i>Tie Beams.</i>			
Of fir - - -	12	6	8
Of oak - - -	12	5	8.2
<i>Fir bridging joists.</i>			
In small buildings - - -	6	$2\frac{1}{2}$	5
In large buildings - - -	6	3	5.4
Of fir common joists - -	6	2	8
Fir trimming joists - -	5	3	7
Fir girders - - -	10	8	10
Oak girders - - -	10	7	10
Fir small rafters - -	8	$2\frac{1}{2}$	$3\frac{1}{2}$
Oak small rafters - -	8	3	$4\frac{1}{2}$
Fir principal rafters - -	18	4	$5\frac{1}{2}$
Oak principal rafters - -	18	3	$6\frac{1}{4}$

Now

Now given the length and either the $\left\{ \begin{array}{l} \text{breadth} \\ \text{depth} \end{array} \right\}$ the $\left\{ \begin{array}{l} \text{depth} \\ \text{breadth} \end{array} \right\}$ may be found by the following rules.

Let l = length of any beam, joist, girder, &c.

b = breadth

d = depth

w = weight it will bear,

Then $w = \frac{bdd}{l}$ the strength of it.

Example 1.

Let us take the first scantling in the table for fir girders, which we will suppose the dimensions to be right because the shortest, viz. 10 feet long, 8 inches broad, and 10 deep.

Here $l = 10 \text{ feet} = 120 \text{ inches}$

$b = 8$ } inches,

$d = 10$ }

Then $\frac{bdd}{l} = \frac{800}{120} = \frac{20}{3}$ the strength.

Whence $d = \sqrt{\frac{20l}{3b}} = 10$,

And $b = \frac{20l}{3dd} = 8$, as in the table.

This being supposed to be right, that is sufficiently strong, all the others in the table may be examined by the above rules, and which I now come to shew.

Example 2.

Let the length be 12 feet, the depth is 10 inches, required the breadth?

Here $l = 12 \text{ feet} = 144 \text{ inches}$,

And $d = 10 \text{ inches}$, then $dd = 100$.

Whence $b = \frac{20l}{3dd} = 9.6$ inches, breadth.

But in the table it is $8\frac{1}{2}$ which is too little by 1.1 inch.

Example 3.

Let the length be 14 feet, depth 10.5 inches.

Then $b = \frac{20l}{3dd} = 10.1$, breadth.

But in the table it is 9, therefore 1.1 inch too little. Also by the same method those of 16 and 18 feet long, will be found too little.

Example 4.

Let the length be 22 feet long; depth 13 inches.

$$\text{Then } b = \frac{20l}{3dd} = 10.4 \text{ breadth.}$$

But in the table it is $11\frac{1}{2}$, therefore 1.1 too great.

Example 5.

Let length be 24 feet, and 14 inches deep, required the breadth?

$$\text{Then } b = \frac{20l}{3dd} = 9.8 \text{ inches nearly.}$$

But in the table it is 12, which is 2.2 inches too great; which fully prove they are not computed from mathematical principles, but are mere guess work, and consequently erroneous.

But for the sake of those that do not understand algebra, I shall here set down rules in words at length for finding the dimensions of timber in building, whereby any tables given by others may be examined, by having two dimensions given to find the the third, whether a breadth or a depth.

PROPOSITION I.

Given the length and breadth of a fir scantling, or girder, to find its proper depth or thickness.

RULE.

Multiply the length in inches by 20, and divide the product by three times the breadth, the square root of the quotient gives the depth.

Example.

There is a fir girder whose length is 14 feet, and breadth 9 inches, what depth must it be, that the strength may be as the stress?

ft. inc.

$$14 \times 12 \times 20 = 168 \times 20 = 3360$$

$$9 \times 3 = 27. \quad \text{Then } 27 \overline{) 3360} \begin{matrix} \text{root} \\ 124.44 \end{matrix} (11.1, \text{ depth.}$$

PROP.

P R O P. II.

Given the length and depth of a fir girder, to find its proper breadth.

R U L E.

Multiply the length in inches by 20, and divide the product by three times the square of the depth, gives the breadth.

Example.

Let the length be 14 feet, and depth 11.1 inches, required its proper breadth?

ft.
 $14 \times 12 \times 20 = 168 \times 20 = 3360$. And $11.1 \times 11.1 = 123.21$.
 And $123.21 \times 3 = 369.63$ 3360 (9 inches, breadth.

P R O P. III.

Given the length and breadth of a fir common joist, to find the depth.

Here I take the first dimensions in the table for fir common joists, which I have given before, viz. length 6 feet, breadth 2 inches, and depth 8 inches, which I suppose to be of sufficient strength.

Here $l = 6 \text{ feet} = 72 \text{ inches}$

$b = 2 \text{ inches}$

$d = 8 \text{ inches}$

Then $\frac{bdd}{l} = \frac{128}{72} = \frac{16}{9}$.

R U L E.

Multiply the length in inches by 16, and divide the product by nine times the breadth, the square root of the quotient is the proper depth.

Example.

Wanted a fir common joist to be 8 feet long and $2\frac{1}{2}$ broad, what must be the depth?

ft. inc.

$8 \times 12 \times 16 = 96 \times 16 = 1536$

$2.5 \times 9 = 22.5$ 1536.00 (68.26 (8.2, depth.

P R O P. IV.

Given the length and depth of a fir common joist, to find the breadth.

H 3

R U L E.

R U L E.

Divide 16 times the length in inches, by 9 times the square of the depth, gives the breadth.

Example.

Given the length of a fir common joist = 12 feet, and depth = 8 inches, required the breadth.

ft. inc.

$$12 \times 12 = 144 \text{ which } \times 16 = 2304$$

$$8 \times 8 = 64 \text{ which } \times 9 = 576 \quad 2304 \div 576 = 4 \text{ inc. breadth.}$$

P R O P. V.

Given the length and breadth of a fir common joist, to find its proper breadth.

Here I take the first dimension in the table for fir trimming joists, which I have given before, viz.

Length 5 feet, breadth 3 inches, depth 7 inches.

Here $l = 5 \text{ feet} = 60 \text{ inches}$

$b = 3$
 $d = 7$ } inches

$$\text{Then } \frac{bdd}{l} = \frac{147}{60} = \frac{49}{20}.$$

R U L E.

Multiply the length in inches by 49, and divide the product by 20 times the breadth, the square root of the quotient is the depth.

Example.

Let the length of a fir trimming joist be 7 feet, breadth $3\frac{1}{2}$ inches, required its proper depth?

ft. inc.

$$7 \times 12 = 84 \text{ and } 84 \times 49 = 4116$$

$$3.5 \times 20 = 70 \quad 4116 \div 70 = 58.80 \quad (7.6 \text{ inches, the depth.})$$

P R O P. VI.

Given the length and depth of a fir trimming joist, to find the proper breadth.

R U L E.

Multiply the length in inches by 49, and divide the product by 20 times the square of the depth, gives the breadth.

Example.

Example.

Let the length be 5 feet, and depth 7 inches, required the breadth?

ft. inc.

$$5 \times 12 = 60 \text{ which } \times 49 = 2940$$

$$7 \times 7 \times 20 = 49 \times 20 = 980 \quad 2940 \div 980 = 3 \text{ inches, the breadth.}$$

P R O P. VII.

Given the length and breadth of a fir bridging joist, in small buildings, to find the depth.

The first in the table is 6 feet long, $2\frac{1}{2}$ inches broad, and 5 inches deep, which I suppose of sufficient strength.

Then $l = 6 \text{ feet} = 72 \text{ inches.}$

$$b = 2\frac{1}{2} \text{ inches}$$

$$d = 5$$

$$\text{And } \frac{bdd}{l} = \frac{62.5}{72}$$

R U L E.

Multiply the length in inches by 62.5, and multiply the breadth by 72, divide the former by the latter, and the square root of the quotient is the answer.

Example.

Let the length be 9 feet, the breadth 3 inches, required the depth?

$$9 \times 12 \times 62.5 = 108 \times 62.5 = 6750$$

$$72 \times 3 = 216 \quad 6750 \div 216 = 31.25 \quad \sqrt{31.25} = 5.6 \text{ inches, the depth.}$$

P R O P. VIII.

Given the length and breadth of a fir bridging joist, in small buildings to find the breadth.

R U L E.

Multiply the length in inches by 62.5, and divide the product by 72 times the square of the depth, the quotient is the breadth.

Example.

Let the length be 10 feet, depth 6 inches, to find the breadth.

ft.

$$10 \times 12 \times 62.5 = 120 \times 62.5 = 7500$$

$$6 \times 6 \times 72 = 36 \times 72 = 2592 \quad 7500 \div 2592 = 2.89 \text{ inches nearly.}$$

P R O P. IX.

Given the length and breadth of a fir bridging joist in large buildings, to find the depth.

Here the first in the table is 6 feet long, 3 inches broad, and $5\frac{1}{8}$ inches deep.

$$\text{Whence } \frac{bdd}{l} = 1.215.$$

R U L E.

Multiply the length in inches by 1.215, and divide the product by the breadth, the square root of the quotient is the depth.

Example.

Wanted a fir bridging joist to be 9 feet long, and 3 inches broad, what depth must it be, that the strength shall be as the firsts?

$$9 \times 12 \times 1.215 = 108 \times 1.215 = 131.22,$$

$$\text{And } 3)131.22(43.74(6.6 \text{ inches, depth.}$$

P R O P. X.

Given the length and depth of a fir bridging joist in large buildings, to find the breadth.

R U L E.

Multiply the length in inches by 1.215, and divide the product by the square root of the depth, gives the breadth.

Example.

What breadth must that fir bridging joist be that is 9 feet long, and 6.6 inches deep?

$$9 \times 12 = 108 \text{ and } 108 \times 1.215 = 131.22$$

$$6.6 \times 6.6 = 43.56)131.22(3 \text{ inches, the breadth.}$$

P R O P. XI.

Given the length and breadth of a fir tie beam, to find the depth.

Here the length, breadth, and depth of a fir tie beam, in the table, is 12 feet, 6 and 8 inches respectively: this being supposed to be of sufficient strength, we have

$$\frac{bdd}{l} = \frac{384}{144} = \frac{8}{3}.$$

R U L E.

R U L E.

Multiply the length in inches by 8, and divide the product by 3 times the breadth, the square root of the quotient is the depth.

Example.

Given the length of a fir tie beam = 16 feet, breadth = 7 inches, what must the depth be?

$$16 \times 12 = 192 \text{ and } 192 \times 8 = 1536$$

$$7 \times 3 = 21 \mid 1536 (73.14 (85 \text{ inches, depth.}$$

P R O P. XII.

Given the length and breadth of a fir tie beam, to find the breadth.

R U L E.

Divide 8 times the length in inches by 3 times, the square of the depth, gives the breadth.

Example.

Let length be 36 feet, depth 12 inches, to find the breadth?

$$36 \times 12 = 432 \text{ and } 432 \times 8 = 3456$$

$$12 \times 12 = 144 \text{ which } \times 3 = 432 \mid 3456 (8 \text{ inches, breadth.}$$

P R O P. XIII.

Given the length and breadth of a fir small rafter, to find the depth.

The first in the table is 8 feet long, $2\frac{1}{2}$ inches broad, and $3\frac{1}{2}$ inches deep.

$$\text{Then } \frac{b d d}{l} = \frac{30.625}{96} = .32.$$

R U L E.

Multiply the length in inches by .32, and divide the product by the breadth, the square root of the quotient is the depth.

Example.

Wanted a fir small rafter to be 10 feet long, its breadth $2\frac{1}{2}$ inches, required its depth?

$$10 \times 12 = 120 \text{ and } 102 \times .32 = 38.4$$

$2.5 \mid 38.4 (15.36 (3.9 \text{ or } 4 \text{ inches nearly, the depth; which is half an inch less than Mr. Price makes it.}$

P R O P.

P R O P. XIV.

Given the length and depth of a fir small rafter, to find its breadth.

R U L E.

Multiply the length in inches by .32, and divide the product by the square of the depth, gives the breadth.

Example.

Let the length = 10 feet, depth 3.9 inches, required the breadth?

$$10 \times 12 \times .32 = 120 \times .32 = 38.40$$

$$3.9 \times 3.9 = 15.21 \quad 38.40 \div 15.21 = 2.5 \text{ inches, breadth.}$$

P R O P. XV.

Given the length and breadth of a fir principal rafter, to find the depth.

The first in the table is that of 18 feet long, 4 inches broad, and $5\frac{1}{2}$ deep.

$$\text{Then } \frac{b d d}{l} = \frac{121}{216} = .5602.$$

R U L E.

Multiply the length in inches by .5602, and divide the product by the breadth, the square root of the quotient is the depth.

Example.

Required the depth of a fir principal rafter, whose length is 24 feet, breadth 5 inches?

$$268 \text{ length in inches} \times .5602 = 161.3376$$

$$\text{And } 5 \overline{) 161.3376} \quad 32.26 \text{ (5.7 inches, depth.)}$$

P R O P. XVI.

Given the length and depth of a fir principal rafter, to find the breadth.

R U L E.

Multiply the length in inches by .5602, and divide the product by the square of the depth, gives the breadth.

Example.

Let the length of a fir principal rafter be 30 feet, and depth 6.6 inches, required its breadth.

30 x

$$30 \times 12 \times .5602 = 360 \times .5602 = 201.672$$

$$6.6 \times 6.6 = 43.56 \quad 201.672 \div 4.6 \text{ inches, the breadth.}$$

By the same method may those of oak be found, as the fir; for taking one of any sort, especially the first in the table, and supposing it of sufficient strength, may a table be deduced from it.

For instance, we have given rules for finding the depth and breadth of fir girders, from this equation $\frac{bdd}{l} = \frac{20}{3}$.

Now let us suppose an oaken girder, whose dimensions are 10 feet long, 7 inches broad, and 10 inches deep, as in the table.

Then $l = 10 \text{ feet} = 120 \text{ inches.}$

$$\left. \begin{array}{l} b = 7 \\ d = 10 \end{array} \right\} \text{ inches.}$$

Whence $\frac{bdd}{l} = \frac{700}{120} = \frac{35}{6}$.

R U L E.

Then if the length in inches be multiplied by 35, and divided by 6 times the breadth, the square root of the quotient is the depth.

And, If the length in inches be multiplied by 35, and divided by 6 times the square of the depth, the quotient gives the breadth. But since the equation for a fir girder is $\frac{bdd}{l} = \frac{20}{3}$, the same may be applied to oak, by considering what has been said, viz. that oak is to fir as 8 to 7.

Now, if we reduce $\frac{20}{3}$ in proportion, as 8 to 7, we shall have $\frac{35}{6}$ for the strength of oaken scantlings.

For $8 : 7 :: \frac{20}{3} : \frac{140}{24} = \frac{35}{6}$, the very same numbers as above.

For Oaken Tie Beams.

The first, in the table, is 12 feet long, 5 inches broad, and 8.2 deep.

Then

Then $\frac{bdd}{l} = \frac{336.2}{144} = \frac{7}{3}$ the strength of oak.

But the fir is for the same dimensions $\frac{bdd}{l} = \frac{8}{3}$.

Now if we reduce the $\frac{8}{3}$ in proportion as 8 to 7, we shall have $\frac{7}{3}$ for the strength as above.

Thus have I shewn how the proper dimensions of fir scantlings may be found, that the strength may be as the firs; and by the same method may those of oak scantlings be found, by making an equality from the dimensions of the first in a table, as was done for the fir. Or the fir equality may be reduced to that of oak, in the proportion of 8 to 7, as has been already mentioned.

Now as I always took the shortest scantlings for my standard ones, in order to get an equation for the strength, so I am persuaded they are the likeliest to have been used, and found to be of a sufficient strength, therefore if tables be made from the first of each, all the others made from thence by the foregoing rules, are presumed to be strong enough, for any buildings, nay, perhaps stronger than they need be. So that these rules are founded upon true mathematical principles, and shew those tables given by carpenters and architects are not right, but greatly erroneous; and this fully demonstrates that practice by itself is not sufficient to determine the proper size of scantlings. Some are given a great deal too big, and some a great deal too little. However these rules are true, and if the tables be corrected by them, much wood would be saved.

P A R T VI.

T H E

C O M P U T A T I O N

O F

B A L L S A N D S H E L L S.

P R O P O S I T I O N I.

Given the Diameter of an Iron Bullet, to find its Weight.

R U L E.

AS 100 is to the cube of the bullet's diameter in inches,
so is 14 to the weight in pounds.

Example 1.

If the diameter of an iron bullet be 5 inches, what is the weight?

$$\begin{array}{r}
 5 \\
 5 \\
 \hline
 25 \\
 5 \\
 \hline
 125 \text{ cube of } 5. \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 \text{As } 100 : 125 :: 14 \\
 \hline
 14 \\
 \hline
 500 \\
 125 \\
 \hline
 1.00) 17.50 \\
 \hline
 17\frac{1}{2} \text{ lb. the weight} \\
 \hline
 \end{array}$$

Example 2.

If the diameter of an iron bullet be $3\frac{1}{2}$ inches, what is the weight?

The

The cube of 3.5 inches = 42.875
 As 100 : 42.875 :: 14

$$\begin{array}{r} 14 \\ \hline 171500 \\ 42875 \\ \hline \end{array}$$

1,00)6,00.250 the weight 6lb.

But if the bullet be lead, to find its weight, when the diameter is known, this is the

R U L E.

Cube the diameter, and multiply this cube by .5236, and and this last product by .41, gives the weight.

Example 1.

What is the weight of a leaden bullet, whose diameter is 5 inches?

$$\begin{array}{r} \text{Multiply} \quad .5236 \\ \text{by cube of } 5 = 125 \\ \hline 26180 \\ 10472 \\ 5236 \\ \hline 65.4500, \text{ solidity.} \\ .41 \\ \hline 6545 \\ 26180 \\ \hline \text{lb. } 26.834500, \text{ the weight.} \\ \hline \end{array}$$

If a leaden bullet be 3 inches diameter, what is the weight?

Multiply

Multiply .5236
by cube of 3 = 27

36652
10472

14.1372, solidity.
-41

141372
565488

lb. 5.796252, the weight.

P R O P. II.

The weight of an iron bullet being given, to find its diameter

R U L E.

As 14 is to the given weight, so is 100 to the cube of its diameter, the cube thereof is the diameter sought.

Example 1.

If an iron bullet weigh 6 pounds, what is the diameter?

lb.

As 14 : 6 :: 100
6

14)600(42.85, the cube of the diameter.
56

40
28

Its log. is 1.63144

$\frac{1}{3}$ of it is .54381

120 its number, is 3.5 the diameter.

112

80

70

10

Example

Example 2.

lb.
As 14 : 17.5 :: 100
100

14)1750.0(125, the cube of the diameter;

Whose cube root is 5 inches, the diameter sought.

PROP. III.

The weight of a leaden bullet being given, to find its diameter.

RULE.

Divide the given weight by .41 (the specific gravity of lead in pounds) and that quotient again by .5236, the last quotient gives the cube of the diameter, whose cube root is the diameter sought.

Example.

If a leaden bullet weigh 26.8345 pounds, what is the diameter?

.41)26 8345(65 45
246
223
205
184
164
205
205

.5236)65.45(125, the cube of the diameter.
5236

13090
10472

Its log. 2.09691

A third part .69897

Its number 5, the diameter sought.

26180
26180

Or the weight of a leaden bullet may be found thus.

The

The proportion that lead, brads, and ir on bear to one another:

Cast iron is to $\left\{ \begin{array}{l} \text{cast brads as 1 to 1.15} \\ \text{cast lead as 1 to 1.578} \end{array} \right.$
 Cast lead is to $\left\{ \begin{array}{l} \text{cast brads as 1 to .728} \\ \text{cast iron as 1 to .6336} \end{array} \right.$
 Cast brads is to $\left\{ \begin{array}{l} \text{cast iron as 1 to .8692} \\ \text{cast lead as 1 to 1.371.} \end{array} \right.$

Example 1.

An iron bullet or shot 4 inches diameter, weighs 9 pounds, what does a ball of lead weigh, whose diameter is 4 inches?

R U L E.

Multiply the weight in pounds of the given bullet by the proper multiplier, the product gives the weight of the other in pounds, which is required.

Thus 1.578 multiplier for lead
 9 lb. iron

lb. 14.202 lead

Answer 14.2 lb. the lead ball.

Example.

What is the weight both of an iron and leaden ball, whose diameter is 2 inches?

The cube of 2 is 8

Then $100 : 8 :: 14$
 8

1.12 lb. the iron ball,

And

1.578
 1.12

3156
 1578
 1578

1 76736

1.76 lb. the leaden ball.

Example 2.

What is the weight of a shot both iron and lead, whose diameter is $2\frac{1}{4}$ inches?

I

2.25

2.25 inches cubed is = 11.39,

Then 100 : 11.39 :: 14

$$\begin{array}{r}
 14 \\
 \hline
 4556 \\
 1139 \\
 \hline
 100 \overline{)159.46} \\
 \hline
 1.59 \text{ lb. the iron ball,}
 \end{array}$$

Then

$$\begin{array}{r}
 1.578 \\
 1.59 \\
 \hline
 14202 \\
 7890 \\
 1578 \\
 \hline
 2.50902
 \end{array}$$

Answer 25 lb. the lead shot.

By proceeding in this manner is made the following table.

T A B L E

T A B L E I.

A Table shewing the Weight of any Ball of Iron or Lead, from two to seven Inches Diameter.

Diam. Inches.	Iron lb.	Lead lb.
2	1.12	1.76
2 $\frac{1}{4}$	1.59	2.5
2 $\frac{1}{2}$	2.18	3.44
2 $\frac{3}{4}$	2.91	4.59
3	3.78	5.96
3 $\frac{1}{4}$	4.8	7.57
3 $\frac{1}{2}$	6.	9.46
3 $\frac{3}{4}$	7.38	11.64
4	9.	14.2
4 $\frac{1}{4}$	10.75	16.96
4 $\frac{1}{2}$	12.76	20.13
4 $\frac{3}{4}$	15.	23.67
5	17.5	27.61
5 $\frac{1}{4}$	20.25	31.95
5 $\frac{1}{2}$	23.29	36.75
5 $\frac{3}{4}$	26.61	42.
6	30.24	47.7
6 $\frac{1}{4}$	34.17	53.92
6 $\frac{1}{2}$	38.44	60.65
6 $\frac{3}{4}$	43.05	67.93
7	48.02	75.77

When guns are fortified alike, that is, when they are in the same proportion in weight and thickness, their requisite charges of powder may be found by the following

P R O P. IV.

Given the requisite charge of powder for one gun, to find the requisite charge of powder for another gun.

R U L E.

As the cube of the known gun's diameter at the bore, is to its given charge of powder, so is the cube of the diameter of that gun, whose charge of powder is required, to its charge required.

I 2

Example.

Example.

The diameter at the bore of a nine pounder is 4.3 inches and its charge is $4\frac{1}{2}$ pound of powder what will a twenty-four pounder require, whose diameter at the bore is 5.9 inches?

4.3	5.9
43	59
129	531
172	295
18.49	3481
43	59
5547	31329
7396	17405
79.507	205.379

lb.

As 79.507 : 4.5 :: 205.379

4.5

1026895
821516

79.507)924.2055(11.6 lb. required.

Whence the following table is easy to be understood by inspection.

T A B L E II.
A T A B L E of G U N N E R Y.

Guns. Pound- ers.	Diameter of the Bore.	Diameter of the Bullet.	Powder for Proof.		Powder for Service.	
			Brafs. Land Service.	Brafs and Iron. Sea Service.	Brafs. Land Service.	Brafs and Iron. Sea Service.
	Inches.	Inches.	lb. oz.	lb. oz.	lb. oz.	lb. oz.
$\frac{1}{2}$	1.7	1.5	0 : 8	0 : 8	0 : 4	0 : 4
3	2.9	2.7	3 : 0	3 : 0	1 : 8	1 : 8
6	3.6	3.4	6 : 0	6 : 0	3 : 0	3 : 0
9	4.2	4.	9 : 0	9 : 0	4 : 8	4 : 8
12	4.6	4.4	12 : 0	12 : 0	6 : 0	6 : 0
18	5.3	5.	18 : 0	15 : 0	9 : 0	9 : 0
24	5.9	5.5	21 : 0	18 : 0	12 : 0	11 : 0
32	6.4	6.1	26 : 0	21 : 8	16 : 0	14 : 0
42	7.	6.6	31 : 8	25 : 0	21 : 0	17 : 0

To

To find the proper adjustment of powder for any piece of ordnance, whether true bored or not, so as to render the bullet capable, of doing the greatest execution, is a very difficult problem in gunnery, and though several rules have been given, engineers have found them often fail: and this I take to lie in the different strength of powder, or in the piece of ordnance, or in both; therefore experience will be your best and surest guide. Engineers usually allowed two thirds of the weight of the ball, for its proper charge; but experience shewed that this was more than necessary, and then they allowed about one half its weight, which continues pretty nearly the same to this day. In October 1739, the French engineers made several experiments at La Fere, to adjust the charge to the piece, by which it appeared from the result of the experiments, says Mr. Bigot de Morogues,

That a 24 pounder ought to be charged with no more than 9 pounds of powder.

16 Pounders with 6 pounds.

12 Pounders with 5 pounds.

8 Pounders with 3 pounds.

And that larger charges than these do not (says he) the least augment the flight of the bullet.

In the French Memoirs, of Royal Academy of Sciences, are several experiments relating to gun-powder and artillery, made by the Chevalier d'Arcy, and M. le Roy.

These gentlemen made several gutters of sand, and laid trains of gunpowder in them. One was 576 feet long, 8 lines ($\frac{2}{3}$ inch) broad, and 4 lines ($\frac{1}{3}$ inch) deep. Another train was 384 feet long, 4 lines deep, and 4 lines broad. The fire ran from one end to the other in the first, in one minute $15\frac{1}{2}$ seconds, and in the shorter in $70\frac{1}{2}$ seconds; had this been 576 feet long it would have taken up $105\frac{3}{4}$ seconds.

They made some other experiments for the better settling this proportion: they made two trains, one of $136\frac{1}{2}$ feet long, with the same dimensions as the above, and the other $136\frac{1}{2}$ feet long, and 4 lines broad, and as many deep, as above related. The fire took up 18 seconds in running through the train that was 8 lines broad, and $25\frac{1}{2}$ seconds in that of 4 lines broad, which gives the same proportion of 5 to 7, very different to what most authors say of this subject.

In order to find the relative velocities of powder inclosed and open, they made a train like in all respects to the last mentioned, but closed at the top, and the flame passed quite

through in $7\frac{1}{4}$ seconds, instead of $25\frac{1}{2}$ seconds, being almost four times the velocity.

It would be natural to suppose from this experiment, that in powder inclosed in fire arms, the inflammation would be so quick as to be considered as instantaneous; yet it is far otherwise, and proved by several undoubted experiments, that powder takes up time in inflaming, and that such time is sufficient to produce effects truly sensible.

Having thus settled the laws of the inflammation of powder, the next points to be determined were, 1. The most advantageous charge for a given cannon. 2. The most advantageous cannon for a given charge; and 3. At what part of the charge the fire should be applied, that the inflammation may be the quickest possible.

But the result of the above 40 trials, (they made after Mr. Robins's Method, as the most certain for measuring the efforts of different charges of powder) it appeared that the charge capable of producing the greatest effect, is that which takes up between the third and the half of the length of the cavity of a given barrel; which agrees nearly with what Mr. Robins had determined from theory. But what is remarkable, if the charge be farther increased, the force and velocity of the bullet will be diminished.

In order to find the length of the cannon proper to produce the greatest effect with a given charge, the above gentlemen made use of ordinary musket barrels of different lengths, the first of 4 feet, the second of 5 feet, and the third of 6 feet, and it was found that the longest always gave the greatest force of the bullet, though they could not precisely determine what length was the best to stop at.

As for the best place for making the touch hole, or applying the fire to the charge, they caused a cannon to be suspended by a long rod, like a pendulum, and drilled in it several touch-holes, at unequal distances from the breech; from this method of mounting the piece, there were two ways of discovering which was the most advantageous touch-hole, first by the greatest excursions of the paller against which the bullet was fired; and secondly by the recoil of the cannon.

Upon trial it seemed to appear, that the touch-hole which gave the quickest inflammation to the powder, was at somewhat above half the length of the charge from the breech, but the violence of the blows having broken several pallets in pieces, this article was referred to some further examination.

PROP. V.

To find the cubic inches in any bomb, and how much powder will fill it.

RULE.

Multiply the cube of the diameter of the hollow part of the bomb by .5236 gives the cubic inches, and that product again by .032, the last product will give the quantity of powder required in pounds.

Example.

A bomb for a 13 inch mortar its diameter is $12\frac{1}{2}$ inches, its concave axis being about $8\frac{1}{2}$ inches, it is required to know how much powder will be sufficient to fill this shell?

$$\begin{array}{r}
 8.25 \\
 8.25 \\
 \hline
 4125 \\
 1650 \\
 6600 \\
 \hline
 68.0625 \\
 8.25 \\
 \hline
 3403125 \\
 1361250 \\
 \hline
 5445000 \\
 561.515625 \text{ the cube of the hollow part's diameter.} \\
 \hline
 .5236 \\
 \hline
 3369093750 \\
 1684546875 \\
 \hline
 1123031250 \\
 2807578125 \\
 \hline
 294.0095812500 \\
 \hline
 \text{And } 294 \\
 .032 \\
 \hline
 588 \\
 882 \\
 \hline
 0.408 \text{ lb. of powder to fill the shell.}
 \end{array}$$

I shall take the liberty to insert here a table founded on experiment made in the year 1743-4. by the ingenious Mathematician Mr. W. Mountaine, F. R. S. master of the academy in Gainsford-street, Southwark, from page 116 of his Practical Sea Gunner's Companion, which the reader may compare with the above.

T A B L E III.

Of the quantity of powder for filling shells, and weight of composition requisite to fill each fuze, length of the fuze, and number of seconds each fuze burns, when cut to its length for fixing in the Shell.

Experimented in the year 1743 4.

Nature of shells.	Quantity of powder to fill each shell.	Length and diameter of fuze-holes		Weight of fuzes when		Weight of comp. with No of ladles. to fill each fuze.	Length of solid comp when cut to fix in shells.	Time of fuzes burning.	Time of one inch of comp. burning.	Weight of shells empty.
		Length	Diam.	empty.	filled with comp.					
Inches.	lb. oz.	In. Pts.	In. Pts.	oz. dr.	oz. dr.	Comp. oz. dr.	In. Pts.	Seconds.	Seconds.	lb. oz.
12 $\frac{1}{2}$	9: 4 $\frac{1}{2}$	8: 4	0: 6	9: 4	11: 10	2: 6	8. 2	37	4 $\frac{1}{2}$ ferè	190: 01
9 $\frac{1}{2}$	4: 14 $\frac{1}{2}$	7: 3	0: 5	5: 12	7: 3	1: 7	7. 2	31	4	96: 6 $\frac{1}{2}$
7 $\frac{1}{2}$	: 3 $\frac{1}{2}$	6: 6	0: 44	3: 12	4: 11	0: 15	6. 2	30	4 $\frac{1}{2}$	46: 12
5 $\frac{1}{2}$	1: 1 $\frac{1}{2}$	3: 7	0: 20	1: 4	1: 8	0: 4	3. 6	16	4 $\frac{1}{2}$ ferè	14: 4 $\frac{1}{2}$
4 $\frac{1}{2}$	0: 8	3: 4	0: 20	1: 1	1: 4	0: 3	3. 1	15	4 $\frac{1}{2}$	7: 12 $\frac{1}{2}$
Cochon	0: 4 $\frac{1}{2}$	2: 5	0: 20	0: 7	0: 9	0: 2	2. 2	10 $\frac{1}{2}$	4 $\frac{1}{2}$	4: 6
Hann Granad. Sea.	0: 2									1: 11. 1
Ditto Land	0: 1 $\frac{1}{2}$									1: 3. 14
Musket.										

TABLE

T A B L E IV.
Of MORTARS, and the DIMENSIONS of SHELLS.

Nature of mortars in inches.	Diameter of bombs in inches.	Diameter of the hollow part in inches.	Thickness of the shell at the fuze-hole in inches.	Diameter of the fuze-hole in inches.	Thickness of the metal at the bottom	
13	12 $\frac{1}{2}$	8.25	1.7	1.74	2.68	
10	9 $\frac{1}{2}$	6.48	1.47	1.6	1.78	
8	7 $\frac{1}{2}$	4.81	0.93	1.24	2.03	
5 $\frac{3}{4}$	5 $\frac{1}{2}$	3.98	0.62	0.6	0.9	
Coehorn	4 $\frac{1}{2}$	2.98	0.53	0.6	0.9	
	3 $\frac{1}{2}$	2.25	0.49	0.46	0.62	
	3 $\frac{1}{4}$	2.04	0.45	0.46	0.63	
	2 $\frac{3}{4}$	1.71	0.42	0.46	0.67	
	2 $\frac{1}{2}$	1.59	0.34	0.46	0.41	
Musket shell	2 $\frac{1}{2}$					

Diameter is equal to a 12 pounder
- - - - - 6 pounder
- - - - - 5 $\frac{1}{4}$ pounder
- - - - - 3 pounder
- - - - - 2 pounder

These sorts of muskets have a chamber fixt upon the muzzle filled to contain such a shell.

TABLE V.

The following table is in Mr. Stone's Mathematical Dictionary, and is Mr. Anderson's, shewing the requisite weight of powder for all mortars, from 6 to 20 inches diameter.

Diameter in inches	Weight. Pounds. Ounces	Diameter in inches.	Weight Pounds. Ounces.
6	0 : 13	13½	9 : 10
6½	1 : 1	14	10 : 11½
7	1 : 5	14½	11 : 14
7½	1 : 10	15	13 : 3
8	2 : 0	15½	14 : 9
8½	2 : 6	16	16 : 0
9	2 : 14	16	17 : 9
9½	3 : 6	17	19 : 3
10	3 : 14½	17½	20 : 15
10½	4 : 8	18	22 : 12½
11	5 : 3	18½	24 : 11
11½	5 : 15	19	26 : 13
12	6 : 12	19½	28 : 14
12½	7 : 10	20	31 : 4
13	8 : 9		:

For proof, the chamber of the mortar is mostly filled full of powder; but for service, the quantity is diminished, or increased, as the random is less or greater.

PROPOSITION I.

To compute the number of cannon balls in any pile, whether square, oblong, or triangular.

From *prop.* 1. page 17 of my Translation of Mr. Stirling's Summation of Series; if x = number of shot in the side, then for the number of shot in any finished square pile, we have this

$$\text{theorem } \frac{x \times x + 1 \times 2x + 1}{6} = \text{number of shot.}$$

2.

If l = length of the bottom, and b = breadth, then the number of shot in an oblong finished pile is found by this

$$\text{theorem. } \frac{b \times b + 1 \times 3l + 1 - b}{6} = \text{number of shot.}$$

If

3.

If x = number of balls in the lower tier, then for the number of shot when piled in a triangular manner this is the

theorem. $\frac{x \times x + 1}{2} \times \frac{x + 2}{3} = \text{number of shot.}$

PROP. IV.

To find the number of shot in any square, oblong, finished or unfinished pile.

This *prop.* is a general one; for let b and c denote the upper sides of an unfinished pile, and put $m = b + c$, the sum of the upper sides, and x = number of ranges;

Then bcx = sum of the first top column.

And $\frac{mx \times x - 1}{2}$ = sum of the second column.

And $\frac{x \times x - 1 \times 2x - 1}{6}$ = sum of the third column.

Whence to find the whole sum, we have this *theorem*, $bcx + \frac{mx \times x - 1}{2} + \frac{x \times x - 1 \times 2x - 1}{6}$ = the number of shot contained in the pile.

But as the above theorems require some knowledge in algebra, I shall here illustrate the same by examples, and rules in words at length.

Example 1.

To find the number of shot in a square pile.

RULE.

To the number of shot in the side add one, and multiply the sum by the number of shot in the side, and reserve the product; to twice the number of shot in the side add one, multiply this sum by the reserved product, a sixth part of the last product is the number of shot in the pile.

Example 2.

There is a square finished pile, whose side is 30 shot; how many balls in that pile?

To

$$\begin{array}{r} \text{To} \quad 30 \\ \text{add} \quad 1 \\ \hline \end{array}$$

$$\begin{array}{r} \text{Sum} \quad 31 \\ \times \text{ by} \quad 30 \\ \hline \end{array}$$

$$\begin{array}{r} \text{Product} \quad 930 \text{ reserved} \\ \quad 61 \\ \hline \end{array}$$

$$\begin{array}{r} 93 \\ 558 \\ \hline \end{array}$$

$$\begin{array}{r} 6)56730 \\ \hline \end{array}$$

9455 = number of shot.

II.

To find the number of shot in an oblong finished pile.

R U L E.

To three times the number of shot in the length of the base add one, and from the sum take the number of shot in the breadth, and note the remainder; to the number of shot in the breadth add one, and multiply that sum by the number of shot in the breadth, and this product by the remainder, a sixth of the last product is the number of shot required.

Example.

There is an oblong pile, whose length at the base is 38 shot, and breadth 12; required the number of shot in that pile?

$$\begin{array}{r} \text{To } 38 \times 3 = 114 \\ \text{add} \quad 1 \\ \hline \end{array}$$

$$\begin{array}{r} \text{From} \quad 115 \\ \text{take} \quad 12 \\ \hline \end{array}$$

$$\begin{array}{r} \text{Rem.} \quad 103 \\ \quad 156 \\ \hline \end{array}$$

$$\begin{array}{r} 618 \\ 515 \\ 103 \\ \hline \end{array}$$

$$\begin{array}{r} 6)16068 \\ \hline \end{array}$$

2678 = N^o shot.

$$\begin{array}{r} \text{To} \quad 12 \\ \text{add} \quad 1 \\ \hline \end{array}$$

$$\begin{array}{r} 13 \\ 12 \\ \hline \end{array}$$

$$156$$

To

III.

To find the number of cannon balls, when they are piled in a triangular manner.

R U L E.

To the number of balls in the lower tier add one, multiply the sum by the number of shot in the lower tier, and reserve half the product; to the number of balls in the lower tier add two, and divide the sum by three, and the quotient multiplied by the reserved half-product gives the number of shot required.

Example.

Admit there is a triangular pile, the length of whose lower tier is 18 shot; required the number of shot in that pile?

To 18
add 1

mult. 19
18

152
19

2)342

half 171 reserved.
6 $\frac{2}{3}$

1026

57

57

1140 = number of shot.

To 18
add 2

3)20
6 $\frac{2}{3}$

IV.

To find the number of shot contained in any pile, whether square or oblong, finished or not finished.

R U L E.

1. Multiply the upper sides together, and reserve the product; take one from the corner row, and multiply the sum of the upper sides by the remainder, and reserve half the product.
2. Multiply twice the corner row less one, by the corner row less one, and reserve one sixth part of the product; the sum of these three last results multiplied by the corner row, the product is the number required.

Example.

Example 1.

Let it be required to find the number of balls, in a square finished pile, whose side is 30 shot?

Product upper sides $1 \times 1 = 1$

$$29 \times 2 = 58, \text{ and } \frac{58}{2} = 29$$

$$59 \times 29 = 1711 \text{ and } \frac{1711}{6} = \frac{285\frac{1}{6}}{315\frac{1}{6}}$$

$$\begin{array}{r} \text{Sum} \quad \quad 315\frac{1}{6} \\ \text{Mult.} \quad 30 \\ \hline \end{array}$$

$$\text{N}^\circ \text{ shot} = 9455$$

Example 2.

Let it be required to find the number of shot in a square unfinished pile, whose upper sides are 6, and the corner row 8?

$$6 \times 6 - - - = 36$$

$$12 \times 7 = 84, \text{ and } \frac{84}{2} = 42$$

$$15 \times 7 = 105, \text{ and } \frac{105}{6} = 17\frac{1}{2}$$

$$\begin{array}{r} \text{Mult.} \quad \quad 95\frac{1}{2} \\ \quad \quad 8 \\ \hline \end{array}$$

$$\text{N}^\circ \text{ shot} = 764$$

Example 3.

There is a finished oblong pile, length of whose upper range is 43 shot, and the side row is 16; how many shot in that pile?

$$43 \times 1 - - - = 43$$

$$44 \times 15 = 660, \text{ and } \frac{660}{2} = 330$$

$$31 \times 15 = 465, \text{ and } \frac{465}{6} = 77\frac{1}{2}$$

$$\begin{array}{r} \text{Mult.} \quad \quad 450\frac{1}{2} \\ \quad \quad 16 \\ \hline \end{array}$$

$$\text{N}^\circ \text{ shot} - - = 7208$$

Example

Example 4.

There is an unfinished oblong pile of shot, whose upper sides are 30 and 6 shot, and the corner row 8; how many shot?

$$30 \times 6 = - - - 180$$

$$36 \times 7 = 252, \text{ and } \frac{252}{2} = 126$$

$$15 \times 7 = 105, \text{ and } \frac{105}{6} = 17\frac{1}{2}$$

$$\begin{array}{r} 323\frac{1}{2} \\ \hline 8 \end{array}$$

$$\begin{array}{r} \text{N}^{\circ} \text{ shot} \quad - - - 2588 \\ \hline \end{array}$$

Example 1.

Let it be required to find the number of balls, in a square finished pile, whose side is 30 shot?

Product upper sides $1 \times 1 = 1$

$$29 \times 2 = 58, \text{ and } \frac{58}{2} = 29$$

$$59 \times 29 = 1711 \text{ and } \frac{1711}{6} = \frac{285\frac{1}{2}}{315\frac{1}{6}}$$

Sum	315½
Mult.	30
	9455

$$\text{N}^\circ \text{ shot} = 9455$$

Example 2.

Let it be required to find the number of shot in a square unfinished pile, whose upper sides are 6, and the corner row 8?

$$6 \times 6 - - - = 36$$

$$12 \times 7 = 84, \text{ and } \frac{84}{2} = 42$$

$$15 \times 7 = 105, \text{ and } \frac{105}{6} = 17\frac{1}{2}$$

Mult.	95½
	8

$$\text{N}^\circ \text{ shot} = 764$$

Example 3.

There is a finished oblong pile, length of whose upper range is 43 shot, and the side row is 16; how many shot in that pile?

$$43 \times 1 - - - = 43$$

$$44 \times 15 = 660, \text{ and } \frac{660}{2} = 330$$

$$31 \times 15 = 465, \text{ and } \frac{465}{6} = 77\frac{1}{2}$$

Mult.	450½
	16

$$\text{N}^\circ \text{ shot} - - = 7208$$

Example

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There is an unfinished oblong pile of shot, whose upper sides are 30 and 6 shot, and the corner row 8; how many shot?

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$$\begin{array}{r} 323\frac{1}{2} \\ 8 \end{array}$$

$$\text{N}^{\circ}. \text{shot} - - \underline{\underline{2588}}$$

P A R T VII.

O F

LOGARITHMICAL ARITHMETIC.

LOGARITHMICAL Arithmetic shews how these parts of vulgar and decimal arithmetic, that consist of *multiplication* and *division*, may be performed by *addition* and *subtraction* only.

Logarithms are artificial numbers which are disposed into tables, and placed against their natural numbers, proceeding from 1 to 10000, and are such that the *sum* of any two logarithms corresponds to the *multiplication* of their natural numbers, that is, the *sum* of any two logarithms stands against that number which is the *product* of those numbers answering to the two given logarithms.

Thus,

Let the number 84 its logarithm 1.92427
be multiplied by 5 — — — 0.69897

The product is 420 standing agst. its log. 2.62324, the sum.

The whole number (as 2 in this example) prefixed to the decimal part of any logarithm, is called the *index* of that logarithm, and is always less by unity than the places of whole numbers, in the number answering to that logarithm.

Thus 5742 the logarithm is 3.75906

574.2 — — — 2.75906

57.42 — — — 1.75906

5.742 — — — 0.75906

.5742 — — — 1.75906

PROPOSITION I.

To find the logarithm of any whole number.

R U L E.

Find the number in the left hand column, in the tables, and right against it, is its logarithm required.

Examples.

Examples.

4273,	its	logarithm	by	the	rule	is	3.6307329
769	—	—	—	—	—	—	2.8859263
41	—	—	—	—	—	—	1.6127839
8	—	—	—	—	—	—	0.9030900

PROP. II.

To find the logarithm of any mixed number.

RULE.

Take the decimal part of the logarithm answering the given mixed number, as if a whole number, to which prefix an index which must be always the less by unity than the places of whole numbers in the given mixed number.

Examples.

371.7,	its	logarithm	is	2.5701926
41.17	—	—	—	1.6145809
3.72	—	—	—	0.5705429

PROP. III.

To find the logarithm of any whole or mixed number, that exceeds the limits of the tables.

RULE.

Take the difference between the two logarithms in the tables, which are the next greater, and next less than your given number; multiply this difference by the figures in the given number, which are more than the tabular figures, cut off as many figures from the right hand of the product as you multiply by, the rest being added to the less logarithm, is the answer.

Example 1.

Required the logarithm of 471285?

OPERATION.

471200	logarithm	—	5.6732053
471300	—	—	5.6732974
			921
			.85
			4605
			7368
			782.85

$$471285 \log. = 5.6732053 + 782.85 \times \frac{5.6732974 - 5.6732053}{100} = 5.6732835, \text{ answer.}$$

K

Example

825.814 logarithm = $\overline{2} 9168822$, answer.

Any logarithm being given to find the number answering thereto.

Find the given logarithm in the tables, and right against it is the number required.

Required the number answering the log. 2 6732053?

Examine the tables, and you will find the number to be 471.2.

But when the number answering the given logarithm exceeds the limits of the tables, then find the difference between the given logarithm and the nearest less in the tables, to which add cyphers, and divide the result by the difference between the next greater and next less tabular logarithm, the quotient (which will consist of as many figures as you brought down cyphers) being annexed to the right hand of the number answering to the next less logarithm, is the answer.

Required the number answering the log. 3.7284153 ?

5350.75, answer.

Example

Example 2.

Required the number answering to the log. 6.4213713?

OPERATION.

Next the greater logarithm — 6.4214394

Next less logarithm — — 6.4212748

Difference — — — — 1646

Given logarithm — 6.4213713

2638000.0 — — 6.4212748

586.2

2638586.2, answer.

1646).965.0000(586.2

8230

14200

13168

Etc.

MULTIPLICATION *by logarithms.*

CASE I.

To find the product of two given whole or mixed numbers.

RULE.

Find the logarithm of each given number, and their sum will be the logarithm of the product, whose corresponding number had from the tables, is the answer.

Example 1.

Multiply 84 logarithm 1.92427

by 25 — — 1.39794

Product 2100 — — 3.32221

Example 2.

Multiply 41.5 logarithm 1.61804

by 7.24 — — 0.85973

Product 300.4 2.47777

CASE II.

When both or either of the factors are less than unity.

K 2

RULE.

R U L E.

Remove the decimal point towards the right hand till the first figure becomes a whole number, with which take out the logarithms, and find the product as by *case* I. then remove the decimal point as many places towards the left hand as you before removed it to the right, and it is the true product.

Example 1.

removed.

Multiply	54	—	5.4	logarithm	0.73239
by	.25	—	2.5	—	0.32794
Product	1.35		13.5	—	1.13033

Example 2.

removed.

Multiply	.74	—	7.4	logarithm	0.86923
by	5.25	—	5.25	—	0.72016
Product	3.885		38.85		1.58939

Example 3.

removed.

Multiply	.742	—	7.42	logarithm	0.87040
by	.421	—	4.21	—	0.62428
Product	.312382		31.2382		1.49468

DIVISION *by logarithms.*

C A S E I.

To divide a whole or mixed number by a less whole or mixed number.

R U L E.

From the logarithm of the dividend subtract the logarithm of the divisor, and the remainder is the logarithm of the quotient.

Example 1.

Divide	—	624	logarithm	2.79518
By	—	26	—	1.41497
Quotient	24	—	—	1.38021

Example

Example 2.

$$\begin{array}{rcl} \text{Divide} & - & 1221 \text{ logarithm} = 3.08671 \\ \text{by} & - & 81.4 - - = 1.91062 \\ \hline \text{Quotient} & = & 15 - - = 1.17609 \end{array}$$

Example 3.

$$\begin{array}{rcl} \text{Divide} & - & 34.86 \text{ logarithm} = 1.54232 \\ \text{by} & - & 8.3 - - = 0.91907 \\ \hline \text{Quotient} & = & 4.2 - - = 0.62325 \end{array}$$

CASE II.

When both or either of the factors are less than unity.

RULE.

Remove the decimal points till the factors contain whole numbers, and the dividend the greatest, with which find the product as before.

Then if the dividend be more places removed than the divisor, by the excess remove the quotient to the left hand. But if the divisor be more places removed, by the excess remove the quotient to the right, and it is done.

Example 1.
removed.

$$\begin{array}{rcl} \text{Divide} & 72.42 - & 72.42 \text{ logarithm} = 1.85985 \\ \text{by} & - & .543 - 5.43 - - = 0.73480 \\ \hline \text{Quotient} & 1.33370 & 1.33370 - - = 1.12505 \end{array}$$

Example 2.
removed.

$$\begin{array}{rcl} \text{Divide} & 875 - & 87.5 \text{ logarithm} = 1.94200 \\ \text{by} & - & 51.43 - 51.43 - - = 1.71121 \\ \hline \text{Quotient} & .17013 & 1.7013 - - = 0.23079 \end{array}$$

Example 3.
removed.

$$\begin{array}{rcl} \text{Divide} & .7824 - & 7.824 \text{ logarithm} = 0.89342 \\ \text{by} & - & .4321 - 4.321 - - = 0.63558 \\ \hline \text{Quo.} & 1.81069 & 1.8106 - - = 0.25784 \end{array}$$

PROP. XV.

The square, cube, &c. any given number by logarithms.

R U L E.

Multiply the logarithm of the given number, by the index of the power sought.

Note, 2 is the index of the square, or second power.

3 is the index of the cube or third power.

&c.

&c.

&c.

Example 1.

What is the square of 51?

51 logarithm = 1.70757

Index — — 2

Answer 2601 = 341514

Example 2.

What is the cube of 12?

12 logarithm = 1.07918

Index — — 3

Cube is 1728 = 3.23754

PROP. VI.

To extract the roots in logarithms.

R U L E.

This is only the reverse of the former rule, namely, divide the logarithm of the given number by the proposed index, the number answering to the quotient is the required root.

Example 1.

What is the square root of 144?

144 logarithm = 2.15836

Answer 12 = 1.07918

Example 2.

What is the cube root of 1728?

1728 logarithm = 3.23754

Answer 12 = 1.07918

To work the rule of three direct logarithmically.

Add the logarithm of the second and third terms together, from which sum take the logarithm of the first term, and the remainder is the logarithm of the fourth term sought, where radius is 10. As is evident to any one that pursues the solutions to the following cases in trigonometry.

P A R T

P A R T VIII.

O F

PLANE TRIGONOMETRY.

TRIGONOMETRY is a rule by which we learn how to compute the length of the sides and quantity of the angles of all manner of triangles, from proper *data*.

In order to which there is calculated and disposed into tables, the length of the sines, tangents, and secants of each degree and minute of the quadrant of a circle whose radius is 10000000000, and these are called *natural* sines, tangents and secants.

But with regard for accommodating the aforesaid tables to practice, the logarithms of the natural sines, tangents, and secants are disposed into tables, which are called *artificial* sines, tangents, and secants. And it is not only requisite that the circumference of a circle be divided into degrees and minutes, but likewise certain lines both within and without the circle are to be expressed by parts of the radius.

D E F I N I T I O N S.

1. CB = radius of the circle AGIBE.
2. IB = chord, a right line joining the extremities of the arch IB together.
3. IF = chord of the arches IBF, and IAF.
4. ID = sine of the arches IB and IA.
5. CD = co-sine of the arch IB.
6. BT = tangent of the arches IB and IA.
7. CT = secant of the arches IB and IA.
8. DB = versed line of the arch IB, and
9. DA = versed sine of the arch IA.
10. The complement of an arch is what it wants of a quadrant, or 90 degrees; then GI is the complement of BI, for BG is a quadrant.
11. GH = co-tangent of the arch IB.
12. CH = co-secant of the arch IB.
13. GK = co-versed sine of the arch IB.

K 4

CO.

COROLLARY.

Hence the co-fine, co-tangent, co-secant, co-versed fine of an arch, is equal to the fine, tangent, secant, versed fine of its complement respectively.

SCHOLIUM.

As co-fine	: fine	:: radius	: tangent.
radius	: fine	:: secant	: tangent.
radius	: co-fine	:: secant	: radius.
radius	: fine	:: co-secant	: radius.
tangent	: radius	:: radius	: co-tangent.
co-tangent	: co-fine	:: co-secant	: radius.
fine	: co-fine	:: tangent	: radius.
fine	: co-fine	:: radius	: co-tangent.
co-fine	: radius	:: radius	: secant.
fine	: radius	:: radius	: co-secant.
secant	: tangent	:: radius	: fine.
secant	: radius	:: radius	: co-fine.

Hence 1. Radius square = fine square + co-fine square = secant square - tangent square = tangent multiplied by the co-tangent.

$$2. \text{Sine} = \sqrt{\text{radius square} - \text{co-fine square.}}$$

$$3. \text{Co-fine} = \sqrt{\text{radius square} - \text{fine square.}}$$

$$4. \text{Tangent} = \frac{\text{fine}}{\text{co-fine}} = \frac{\text{secant}}{\text{co-secant}}$$

$$5. \text{Secant} = \frac{\text{tangent}}{\text{fine}} = \frac{\text{co-secant}}{\text{co-tangent}} = \frac{\text{rad. sq.}}{\text{co-fine}} = \sqrt{\text{rad. sq.} + \text{tangent square.}}$$

$$6. \text{Versed fine} = \text{radius} - \text{co-fine.}$$

$$7. \text{Co-tangent} = \frac{\text{rad. square}}{\text{tangent}}$$

$$8. \text{Co-secant} = \frac{\text{rad. square}}{\text{fine}}$$

The foregoing properties of fines, tangents, &c. are the natural fines, natural tangents, &c. and they must be actually multiplied, and divided, and here radius is always 1.

Example.

What is the co-secant of $15^{\circ} 01'$?

The natural side of $15^{\circ} 01'$ is $\approx .2591$, and radius ≈ 1 , and consequently its square 1.

Then $.2591 \times 1.0000000 (3.85951$, the co-secant required; and so of the rest.

But

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But what here follows, belongs to the logarithmic lines, logarithmic tangents, &c. and the radius here is always 10 : and then addition and subtraction performs the work. Thus,

1. Log. radius + log. sine — log. co-sine = log. tangent.
2. Twice log. radius — log. co-sine = log. secant.
3. Twice log. radius — log. tangent = log. tangent of the complement of that arch to a quadrant.

Example 1.

What is the log. tangent of $30^{\circ} 10'$?

The log. radius	10.0000000
+ log. sine of $38^{\circ} 10'$	9 7909541
Sum	19.7909541
Take log. cosine, $38^{\circ} 10'$	9 8955422
Log. tangent of $38^{\circ} 10'$	9.8954119

Example 2.

What is the log. secant of $38^{\circ} 10'$?

2 log. radius	20.
— log. cosine $38^{\circ} 10'$	9 8955422
Log. secant of $38^{\circ} 10'$	10.1044578

Example 3.

What is the log. co-tangent of $38^{\circ} 10'$?

2 log. radius	20.
— log. tangent	9 8954119
Log. co-tangent of $38^{\circ} 10'$	10.1045881

PROP. I. PROB.

To find the sine, co-sine, tangent, &c. to 90 degrees, of the intermediate parts of a minute.

RULE.

Take the difference between the lines, &c. of the given degrees and minutes, and of the minute next greater. Then, as 1 : is to that difference :: so is the given intermediate part of a minute in decimals : to a fourth number. Therefore multiply that difference by such decimal part, and add the product to the sine, tangent, and secant ; or, subtract it from the

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the co-sine, co-tangent and co-secant of the given degrees and minutes, the sum or remainder, will be the sine, tangent, &c. required.

Example 1.

What is the natural sine of $1^{\circ} 48' 28'' 12'''$?

The nat. sine of $1^{\circ} 49'$	317015
nat. sine of $1^{\circ} 48'$	314108

Difference	2907
------------	------

Now as $1 : 2907 :: (28'' 12''' =) 0'.47 : 1366$

The natural sine of $1^{\circ} 48'$	314108
-------------------------------------	--------

The nat. sine of $1^{\circ} 48' 28'' 12'''$	315474
---	--------

Example 2.

What is the log. sine of $1^{\circ} 48' 28'' 12'''$?

The log. sine of $1^{\circ} 49'$	8.5010798
log. sine of $1^{\circ} 48'$	8.4970784

Difference,	40014
-------------	-------

Now $28'' 12''' = 0'.47$

Then $40014 \times .47 =$ 18805

Add log. sine of $1^{\circ} 48' =$ 8.4970784

Log. sine $1^{\circ} 48' 28'' 12''' =$	8.4989589
--	-----------

Example 3.

To find the log. co-sine of $88^{\circ} 11' 31'' 48'''$?

Log. co sine of $88^{\circ} 11'$	8.5010798
----------------------------------	-----------

Log. co-sine of $88^{\circ} 12'$	8.4970784
----------------------------------	-----------

Difference	40014
------------	-------

But $31'' 48''' = 0'.53$

Then $40014 \times 0'.53$ 21207

From co-sine $88^{\circ} 11'$ 8.5010798

Take 21207

Log. co-sine $88^{\circ} 11' 31'' 48''' =$	8.4979591
--	-----------

Example

*Example 4.*Required the log. tangent of $2^{\circ} 23' 33'' 36'''$?Log. tangent of $2^{\circ} 24'$ 8.6223427Log. tangent of $2^{\circ} 23'$ 8.6193127

Difference 30300

But $33'' 36''' = 0' 56''$ To log. tangent of $2^{\circ} 23'$ 8.6193127add $30300 \times .56 =$ 16968Log. tangent $2^{\circ} 23' 33'' 36''' =$ 8.6210095

P R O P. II.

Given the natural or logarithmic sine, co-sine, tangent, &c. to find the arc.

R U L E.

Take the next less of the same kind found in the tables, and subtract it from that given, observing the degrees and minutes answering to it; then annex 2 or 3 cyphers to the remainder; divide it by the difference between it and the next greater, the quotient will be the decimal part of a minute, to be added or subtracted from the degrees and minutes before found.

Example 1.

Given this log. sine 9.8393859 to find the arc?

First. the given log. sine is= 9.8393859

The next less is $43^{\circ} 41' =$ 9.8392719

Difference 1140

The diff. between the $43^{\circ} 41' =$ 9.8392719And the next greater $43^{\circ} 42' =$ 9.8394041

Is 1322

Then $1322/1140 = 0'.862$ Answer $43^{\circ} 41'.862 = 43^{\circ} 41' 51'' 43'''$.*Example 2.*

Given the log. tangent 9.6766687 to find the arc?

First the given log. tangent is 9.6766687

Next less is log. tangent $25^{\circ} 24' =$ 9.6765426

Difference 1261

The

The difference between it and the next greater, viz. 25° is 3260, then $3260)1261.000(0'.387$
 And $25^\circ 24'.387 = 25^\circ 24' 23'' 13'''$, answer.

Example 3.

Given log. co-sine. 9.9994206 to find the arc?

First log. co-sine given 2.9994206

Log. co-sine of $2^\circ 58'$ next less 9.9994176

Difference 30

Diff. between $2^\circ 58'$ and $2^\circ 59' = 65$

Then $65)30.0000(0'.4615$

From $2^\circ 58'$

Take 0 0.4615

2 57.5385

Or, $2^\circ 57' 32'' 19'''$, answer.

PROP. III. PROB.

Given the radius and arch of a circle in degrees and minutes, to find the length of that arch.

RULE.

Multiply the radius of that arch by .017453, and this product by the number of degrees and decimal parts, gives the length of the arch, in the same measure the radius is of.

Example.

What is the length of an arch of $106^\circ 28'$ whose radius is 24 yards?

.01745

24 radius

6980

3490

41880

106.46 = $106^\circ 28'$

251280

167520

251280

418800

445854480 yards, answer.

This

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This rule is useful for finding the measure or length of the circular part before a bastion, of the ditch before a ravelin, of the retired flank, &c.

PROP. IV. PROB.

Given the sum of two angles, and the product of their sines, to find the angles separately.

RULE.

Divide the product of the sines, by .5, and to the quotient add the co-sine of the sum of the angles, and the sum will be the co-sine of the difference of the angles; which difference added to, or subtracted from the sum, the sum or difference is double the greater, or double the less angle respectively.

Example 1.

Suppose the product of the sines of two angles be .294114, and their sum $81^{\circ} 30'$, what are the two angles?

$$\begin{array}{r}
 .5) .294114(\\
 \hline
 .58822 \\
 .14780 \\
 \hline
 \text{Add cosine } 81^{\circ} 30' = .14780 \\
 \hline
 \text{Sum } \quad \quad \quad .73602 \text{ co-sine of the difference} \\
 \text{of the angles} = 42^{\circ} 36'. \\
 \begin{array}{r}
 \text{To } 81^{\circ} 30' \\
 \text{Add } 42 \quad 36 \\
 \hline
 2) 124 \quad 06 \\
 \hline
 62 \quad 3 \\
 \hline
 \text{Answer } \left\{ \begin{array}{l} \text{greater angle} = 62^{\circ} 3' \\ \text{less angle} = 19 \quad 27. \end{array} \right.
 \end{array}
 \end{array}$$

Example 2.

Suppose the product of the sines of two angles be .44131, and their sum $= 84^{\circ} 32'$, what are the two angles?

$$\begin{array}{r}
 .5) .44131(\\
 \hline
 .88262 \\
 \text{Add co-sine } .09526 \text{ of } 84^{\circ} 32' \\
 \hline
 \text{Co-sine} = .97788 \text{ of the difference of the angles} = 12^{\circ} 4'. \\
 \text{To}
 \end{array}$$

FIG.

To $84^{\circ} 32'$
Add 12 4

2)96 36

48 18

From $84^{\circ} 32'$
Take 12 4

2)72 28

36 14

Answer { greater angle = $48^{\circ} 18'$
less angle = 36 14

AXIOMS.

92. I. In right angled triangles, any side being made the radius, semidiameter or sweep of a circle, the other two will be lines, tangents or secants, which words being writ upon, or expounded by them, it will be

93. As the word on the given side,
Is to the given side;
So is the word on the side required,
To the side required.

94. N. B. The word on any side of a triangle, is to be understood the sine, tangent or secant of the degrees and minutes which are contained in the angle expressed by that word.

II. The sides of any plane triangle, whether right or oblique angled, are in proportion to one another, as the sines of their opposite angles.

- III. In any plane triangle,
As the sum of any two sides
Is to their difference.
So is the tangent of half the sum of their opposite angles
To the tangent of half their difference.

- IV. In any plane triangle
As the base is to the sum of the two other sides, so is the difference of those sides to the difference of the segments of the base made by letting fall a perpendicular from the angle opposite the base to the base.

- V. Half the sum of any two numbers being added to half their difference, gives the greater number. But,
Half the difference taken from the half sum, gives the less number.

- VI. Having the three sides of an oblique triangle given, to find an angle.

Set

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FIG.

Set down the side opposite to the angle required, and under it the other two sides; from their half sum subtract each side in the order they stand, then find the complement arithmetical of the logarithms of the half sum, and first remainder, and also the logarithms of the other two remainders, then will the half sum of these four logarithms be the tangent of half the angle sought.

The *arith. com.* of a logarithm is found by beginning at the index, and writing down what each figure wants of 9, all but the last, which must be taken from 10.

When there are two figures in the index, reject the first, and work as before.

VII. Any two sides of a plane triangle taken together, are greater than the third that remains.

VIII. The greatest side of every triangle is opposite to the greatest angle, and the greatest angle opposite the greatest side.

IX. The sum of the three angles of every plane triangle is equal to 180° ; therefore

X. If two angles of a plane triangle are known, the third is also known, being found by taking their sum from 180° degrees.

XI. If any angle be obtuse or greater than 90 degrees, each of the other two will be acute or less than 90 degrees.

XII. If one angle be right or 90 degrees, the sum of the other two will be 90 degrees.

XIII. If one of the acute angles, in a right angled plane triangle, be given, the other is also known, being found by taking the given angle from 90° , and this defect is called its complement.

CASE I.

The hypotenuse and angle at the base being given, to find the perpendicular.

It will be proper to observe,

1. When a side is required, begin with an angle opposite to a given side.

2. When an angle is required, begin the proportion with a side opposite to a given angle.

Example.

In the triangle ABC, right angled at B.

95.

Are given $\begin{cases} AC = 34 \\ \angle A = 8^\circ 4' \end{cases}$
Required AB and BC.

Geometrical

Geometrical solution.

Draw any right line AB, make $\angle A = 28^\circ 4'$ and draw the hypotenuse $AC = 34$ from any scale of equal parts, fitted to your line of chords, from the point C, let fall the perpendicular BC, and it is done.

Measure $\left\{ \begin{array}{l} AB = 30 \\ BC = 16 \end{array} \right\}$ on the same scale of equal parts.

Numerical solution.

1. By the natural sines,

Hypotenuse AC radius.

From $90^\circ 0'$

Take $\angle A = 28 \quad 4$

$\angle C = 61 \quad 56$ by *ax. XIII.*

The natural sine of $28^\circ 4' = .47049$

natural sine of $61 \quad 56 = .88240$

Radius = 1. Hypotenuse $AC = 34$,

rad. AC nat. s. $\angle A$.

As $1 : 34 :: .47049$

34

188196

141147

15.99666 or 16, very nearly, the perpendicular

BC.

rad. AC nat. s. $\angle C$

As $1 : 34 :: .8824$

34

35296

26472

30.0016 = AB.

2. By logarithms.

As radius, sine $9^\circ (\angle B)$

10.000000

To hypot. AC, 34 its log.

15.31479

So is sine $\angle A = 28^\circ 4'$

9.672558

To $BC = 16$ its log.

1.204037

To find AB.

Fig.

As rad. fine 90°	10.000000
Is to $AC=34$ log.	1.531479
So is fine $\angle C=61^\circ 56'$	9.945060
To $AB=30$ log.	1.377145

II. Base AB radius.

By the natural tangents, &c.

sec. $\angle A$	AC	tang. $\angle A$
As 1.13327 : 34 :: .5332		
	34	
	21328	
	15996	
	1.13327)18.1288(16=BC	

sec. $\angle A$	AC	rad.
As 1.13327 : 34 :: 1 :		

$$1.13327)34.00000(30=AB$$

By logarithms.

As secant $\angle A=28^\circ 4'$ co. ar.	9.945666
To $AC=34$	1.531479
So is tangent $\angle A=28^\circ 4'$	9.726892
To $BC=16$	1.204037
As secant $\angle A=28^\circ 04'$	10.054334
To $AC=34$	1.531479
So is radius, fine 90°	10.000000
To $AB=30$	1.477145

III.

Perpendicular BC radius,

984

By natural secants and tangents.

nat. sec. $\angle C$	AC	rad. BC
As 2.1254 : 34 :: 1 : 16		
sec. $\angle C$	AC	tang. $\angle C$ AB
2.1254 : 34 :: 1.87545 : 30		

L

By

By the logarithms.

As secant $\angle C = 61^\circ 56'$	10.327442
Is to $AC = 34$	1.531479
So is the radius, sine 90°	10.000000
To $BC = 16$	1.204037
As secant $\angle C = 61^\circ 56'$ co. ar.	9.672558
To $AC = 34$	1.531479
So is tangent $\angle C = 61^\circ 56'$	10.271107
To $AB = 30$	1.477144

Instrumentally.

The extent from 90° to $28^\circ 4'$ on the sines, will reach from 34 to $16 = BC$ on the numbers. The extent from 90° to $61^\circ 56'$ on the sines reaches from 34 to $30 = AB$ on the numbers.

CASE II.

Given the angle at the base and the perpendicular, to find the hypotenuse and base.

Example.

99. In the triangle ABC right angled at B
are given $\left\{ \begin{array}{l} BC = 12 \\ \angle A = 36^\circ 52' \end{array} \right\}$ required AC and AB .

Geometrical solution.

At any point B draw BC perpendicular to AB , which make $= 12$ from any scale of equal parts; then from the line of chords, make the $\angle C = 53^\circ 8'$ (for $90^\circ - 36^\circ 52' = 53^\circ 8'$) draw the hypotenuse CA and it is done.

Measure $\left\{ \begin{array}{l} AC = 20 \\ AB = 16 \end{array} \right.$

I shall not any more give the operation by the natural sines, natural tangents, &c. but leave it to the exercise of the learner; which is very proper he should understand.

96.

Numerical solution.

I. Hypotenuse AC radius.

$$\begin{array}{r} \text{From} \quad 90^\circ \\ \text{Take } \angle A = \quad 36^\circ 52' \\ \hline \angle C = \quad 53^\circ 8' \end{array}$$

Fig.

As sine $\angle A = 36^\circ 52'$	9.778119
To BC = 12	1.079181
So is radius 90°	10.000000
To AC = 20	1.301062
As sine $\angle A = 36^\circ 52'$ co. ar.	0.221881
To BC = 12	1.079181
So is sine $\angle C = 53^\circ 8'$	9.903108
To AB = 16	1.204170

II. Base AB radius.

97.

As tangent $\angle A = 36^\circ 52'$ co. ar.	0.124990
To BC = 12	1.079181
So is secant $\angle A = 36^\circ 52'$	10.096892
To AC = 20	1.301063
As tangent $\angle A = 36^\circ 52'$	9.875010
To BC = 12	1.079181
So is radius, tangent 45°	10.000000
To AB = 16	1.204171

III. Perpendicular BC radius.

98.

As radius, tangent 45°	10.000000
To BC = 12	1.079181
So is secant $\angle C = 53^\circ 8'$	10.221881
To AC = 20	1.301062
As radius, tangent 45°	10.000000
To BC = 12	1.079181
So is tangent $\angle C = 53^\circ 8'$	1.0124990
To AB = 16	1.204171

Instrumentally.

The extent from $36^\circ 52'$ to 90° on the sines, will reach from 12 to 20 = AC on the number, the extent from $36^\circ 52'$ to $53^\circ 8'$ on the sines reacheth from 12 to 16 = AB on the numbers.

L₂

CASE

CASE III.

The two acute angles and base being given, to find the hypotenuse and perpendicular.

Example.

100. Let the base $AB=36$, and the angle $A=36^{\circ} 52'$; required the hypotenuse AC , and perpendicular BC .

Geometrical solution.

Draw the base AB , which makes $=36$ from a scale of equal parts, raise the perpendicular BC , make the angle $A=36^{\circ} 52'$, drawing the hypotenuse AC to cut the perpendicular BC in C , and it is done.

$$\text{Measure } \begin{cases} AC=45 \\ BC=27 \end{cases}$$

Numerical solution.

96.

I. Hypotenuse AC radius.

$$\text{As fine } \angle C 53^{\circ} 8' \quad \text{co. ar. } 9.903108$$

$$\text{To radius, fine } 90^{\circ} \quad 10.000000$$

$$\text{So is } AB=36 \text{ log. } 1.556302$$

$$\text{To hypoth. } AC=45 \quad 1.653194$$

$$\text{As fine } \angle C=53^{\circ} 8' \quad \text{co. ar. } 0.096892$$

$$\text{Is to } AB=36 \quad 1.556302$$

$$\text{So is fine } \angle A=36^{\circ} 52' \quad 9.778119$$

$$\text{To perpendicular } BC=27 \quad 1.431313$$

II. Base AB radius.

97.

$$\text{As radius, fine } 90^{\circ} \quad 10.000000$$

$$\text{To } AB=36 \text{ log. } 1.556302$$

$$\text{So is secant } \angle A=36^{\circ} 52' \quad 10.096892$$

$$\text{To hypoth. } AC=45 \quad 1.653194$$

$$\text{As radius } 90^{\circ} \quad 10.000000$$

$$\text{Is to } AB=36 \quad 1.556302$$

$$\text{So is tangent } \angle A=36^{\circ} 52' \quad 9.875010$$

$$\text{To perpend. } BC=27 \quad 1.431312$$

III.

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Fig.
98.

III. Perpendicular BC radius.

As tangent $\angle C = 53^\circ 8'$ co. ar.	9.875010
To AB = 36	1.556302
So is secant $\angle C = 53^\circ 8'$	10.221881
To hypoth. AC = 45	1.653193
As tangent $\angle C = 53^\circ 8'$	10.124990
Is to AB = 36	1.556302
So is the radius, 90°	10.000000
To perpendicular BC = 27	1.431312

Instrumentally.

The extent from $53^\circ 8'$ to 90° on the fines, will reach from 36 to AC = 45 on the numbers.

The extent from $53^\circ 8'$ to $36^\circ 52'$ on the fines, will reach from 36 to BC = 27 on the numbers.

CASE IV.

The hypotenuse and perpendicular being given, to find the base and angles.

Example.

Let the hypotenuse AC = 40, perpendicular BC = 24; required the base AB, and angles.

Geometrical solution.

Draw AB, upon which raise the perpendicular BC = 24, then with the extent of 40 and one foot in C, cut BA in A, join AC, and it is done.

Measure	{	AB = 32	
	{	$\angle A =$	$36^\circ 52'$
			$90^\circ 00'$
Remains $\angle C =$			$53^\circ 08'$

Numerical solution.

I. Hypotenuse AC radius.

As AC = 40	1.602060
To radius 90°	10.000000
So is BC = 24	1.380211
To sine $\angle A = 36^\circ 52'$	9.778151
	$90^\circ 00'$
$\angle C =$	$53^\circ 08'$

L 3

A9

FIG. 150

TRIGONOMETRY.

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98.

As radius 90° 10.000000

Is to $AC=40$ 1.602060

So is sine $\angle C=53^\circ 08'$ 9.903108

To $AB=32$ 1.505168

II. Perpendicular BC radius.

As $BC=24$ 1.380211

Is to radius 90° 10.000000

So is $AC=40$ 1.602060

To secant $\angle C=53^\circ 8'$ 10.221849
 $90^\circ 0'$

$\angle A=36^\circ 52'$

As radius, 90° 10.000000

To $BC=24$ 1.380211

So is tangent $\angle C=53^\circ 8'$ 10.124990

To $AB=32$ 1.505201

Instrumentally.

The extent from 40 to 24 on the numbers, will reach from 90 to $\angle A=36^\circ 52'$ on the lines.

The extent from 90 to $53^\circ 8'$ on the lines, will reach from 40 to $AB=32$ on the numbers.

CASE V.

Given the hypotenuse and base, to find the perpendicular and the angles.

Example.

99. Let the hypotenuse $AC=73$, base $AB=55$, required the perpendicular and angles.

Geometrical solution.

Draw the base $AB=55$, at B erect the perpendicular BC , then with the extent of 73 and one foot in A , cut BC in C ; draw the hypotenuse AC , and it is done.

Measure $\left\{ \begin{array}{l} BC=48 \\ \angle A=41^\circ 07' \end{array} \right.$
 $\angle C=48^\circ 53'$

Num-

Numerical solution.

I. Hypothenufe AC radius.

As AC=73	1.863323
To radius, 90°	10.000000
So is AB=55	1.740363
To sine $\angle C=48^\circ 53'$	9.877040
90 00	

$$\angle A = 41^\circ 07'$$

As radius, sine 90°	10.000000
To AC=73	1.863323
So is sine $\angle A=41^\circ 07'$	2.817958
To BC=48	1.681281

II. Base AB radius.

As AB=55	1.740263
To radius, 90°	10.000000
So is AC=73	1.863323
To secant $\angle A=41^\circ 07'$	10.122960
90 00	

$$\angle C = 48^\circ 53'$$

As radius, 90°	10.000000
To AB=55	1.740363
So is tangent $\angle A=41^\circ 07'$	9.940946
To BC=48	1.681312

Instrumental solution.

The extent from 73 to 55 on the numbers, will reach from 90° to $\angle C=48^\circ 53'$ on the fines.

The extent from 90° to $41^\circ 07'$ on the fines, will reach from 73 to 48=BC on the numbers.

C A S E VI.

Given the base and perpendicular to find the hypothenufe and angles.

L 4

Example.

FIG.

Example.

99. Let the base $AB=45$, perpendicular $BC=28$, required the hypotenuse AC and angles.

Geometrical solution.

From any scale of equal parts, make $AB=45$, raise the perpendicular BC , which make equal to 28 from the same scale, join the points A and C , and it is done.

$$\begin{array}{rcl} \text{Measure } \left\{ \begin{array}{l} AC=53 \\ \angle A= \end{array} \right. & & \begin{array}{r} 31^{\circ} 53' \\ 90 \quad 00 \\ \hline \end{array} \\ & & \angle C = \begin{array}{r} 58 \quad 07 \end{array} \end{array}$$

Numerical solution.

97.

I. Base AB radius.

$$\begin{array}{rcl} \text{As } AB=45 & & 1.653212 \\ \hline \text{To radius, tangent } 45^{\circ} & & 10.000000 \\ \text{So is } BC=28 & & 1.447158 \\ \hline \text{To tangent } \angle A=31^{\circ} 53' & & 9.793946 \\ \text{As radius, tangent } 45^{\circ} & & 10.000000 \\ \hline \text{To } AB=45 & & 1.653212 \\ \text{So is secant } \angle A=31^{\circ} 53' & & 10.071028 \\ \hline \text{To } AC=53 & & 1.724240 \end{array}$$

98.

II. Perpendicular BC radius.

$$\begin{array}{rcl} \text{As } BC=28 & & 1.447158 \\ \hline \text{To radius, tangent } 45^{\circ} & & 10.000000 \\ \text{So is } AB=45 & & 1.653212 \\ \hline \text{To tangent } \angle C=58^{\circ} 7' & & 10.206054 \\ \text{As radius, tangent } 45^{\circ} & & 10.000000 \\ \hline \text{To } BC=28 & & 1.447158 \\ \text{So is secant } \angle C=58^{\circ} 7' & & 10.277209 \\ \hline \text{To } AC=53 & & 1.724367 \end{array}$$

Isiru.

Instrumentally.

Fig.

The extent from 45 to 28 on the numbers, will reach from 45° to $31^{\circ} 53' = \angle A$ on the tangents.

The extent from $31^{\circ} 53'$ to 90° , on the sines, reaches from 28 to $AC = 53$ on the numbers.

Observation.

By making the hypotenuse radius, each side becomes sines of their opposite angles, and therefore the sides are in proportion to the sines of their opposite angles, and the contrary by *axiom* 2, and this must be allowed the most elegant when it can be used; which I have done in every case except the last.

OBLIQUE PLANE TRIANGLES.

CASE I.

Given one side and the angles to find the other two sides.

Example.

101.

In the oblique triangle ACB

are given $\left\{ \begin{array}{l} \angle A = 28^{\circ} 4' \\ \angle B = 53^{\circ} 8' \\ AB = 42 \end{array} \right\}$ required AC and BC

Geometrical construction.

Make $AB = 42$, draw AC making the angle $A = 28^{\circ} 4'$, also making the $\angle B = 53^{\circ} 8'$, draw BC, and where it meets AC, the triangle is constructed.

Calculation.

From	$180^{\circ} 00'$
Take $\left\{ \begin{array}{l} \angle A = 28^{\circ} : 4' \\ \angle B = 53 : 8 \end{array} \right\}$	
Sum =	<u>$81 \ 12$</u>
Remains $\angle C =$	$98 \ 48$ its supplement is $81^{\circ} 12'$

which use.

As

FIG.

As fine $\angle C = 90^\circ 48'$ 9 99485

101.

Is to $AB = 42$ 1.62325So is fine $\angle A = 28^\circ 4'$ 9 67255To $BC = 20$ 1.30095As fine $\angle A = 28^\circ 4'$ 9.67255Is to $BC = 20$ 1.30095So is fine $\angle B = 53^\circ 8'$ 9.90310To $AC = 34$ 1.53150

Or you may find the sides thus.

As fine $\angle C = 91^\circ 48'$ co. ar. 0.00514Is to $AB = 42$ 1.62325So is the fine $\angle A = 28^\circ 4'$ 9 67255To $BC = 20$ 11.30094As fine $\angle C = 98^\circ 48'$ co. ar. 0.00514Is to $AB = 42$ 1.62325So is fine $\angle B = 53^\circ 8'$ 9.90310To $AC = 34$ 1.53149

C A S E II.

Two sides and one angle opposite to them being given, to find the rest.

N. B. When the given angle is opposite to the greater given side, this case has one certain answer: but when the given angle is opposite to the less given side, this case will then have two true answers, as will evidently appear by the following examples.

101.

Example 1.

In the oblique triangle ACB

are given $\left\{ \begin{array}{l} AC = 85 \\ BC = 50 \\ \angle B = 53^\circ 8' \end{array} \right\}$ required AB and $\angle C$ and A .

Geometrical construction.

Draw AB , and also $BC = 50$ making $\angle B = 53^\circ 8'$, then with the extent of 85, and one foot in C , cut BA in A , and it is done.

Measured

$$\begin{array}{rcl}
 \text{Measured } \left\{ \begin{array}{l} AB=105 \\ \angle A=28^{\circ} 4' \end{array} \right. & & \\
 \text{As } AC=85 & & 1.92941 \\
 \text{To fine } \angle B=53^{\circ} 8' & & 9.90310 \\
 \text{So is } BC=50 & & 1.69897 \\
 \hline
 \text{To fine } \angle A=28^{\circ} 4' & & 9.67266 \\
 \hline
 \text{From } 180^{\circ} 00' & & \\
 \text{Take } \left\{ \begin{array}{l} \angle A=28^{\circ} 4' \\ \angle B=53^{\circ} 8' \end{array} \right. & & \\
 \hline
 \text{Sum } = 81 \quad 12 & & \\
 \hline
 \text{Remains } \angle C=98 \quad 48 \text{ by ar. 10.} & &
 \end{array}$$

$$\begin{array}{rcl}
 \text{As fine } \angle B=53^{\circ} 8' & & 9 \quad 90310 \\
 \hline
 \text{Is to } AC=85 & & 1.92941 \\
 \text{So is fine } \angle C=98^{\circ} 48' & & 9.99485 \\
 \hline
 \text{To } AB=105 & & 2.02116 \\
 \hline
 \end{array}$$

Or thus.

$$\begin{array}{rcl}
 \text{As } AC=85 & \text{co. ar. } 8.07058 & \\
 \text{To fine } \angle B=53^{\circ} 8' & & 9.90310 \\
 \text{So is } BC=50 & & 1.69897 \\
 \hline
 \text{To fine } \angle A=28 \quad 4' & & 9.67265 \\
 \angle B=53 \quad 8 & & \hline
 \hline
 \text{Take } 81 \quad 12 & & \\
 \text{From } 180 \quad 00 & & \\
 \hline
 \angle C=98 \quad 48 & & \\
 \hline
 \text{As fine } \angle B=53^{\circ} 8' & \text{co. ar. } 0.09689 & \\
 \text{Is to } AC=85 & & 1.92942 \\
 \text{So is fine } \angle C=98^{\circ} 48' & & 9.99485 \\
 \hline
 \text{To } AB=105 & & 2.02119
 \end{array}$$

Example

Example 2.

Let $\left\{ \begin{array}{l} BC=50 \\ AC=85 \\ \angle A=28^{\circ} 4' \end{array} \right\}$ required AB and $\angle B$ and C?

Geometrically.

Draw AB, and AC=85, making the $\angle A=28^{\circ} 4'$, then with the extent of 50, and one foot in C, cut AB, in the two points B, b, join BC, bC, and either of the triangles will be that required.

102. Measure $\left\{ \begin{array}{l} AB=105 \\ \angle ACB=98^{\circ} 48' \end{array} \right\}$ $\left\{ \begin{array}{l} Ab=45 \\ \angle ACb=25^{\circ} 4' \end{array} \right\}$.

Calculation.

As BC=50 co. ar. 8.30103
Is to the fine $\angle A=28^{\circ} 4'$ 9.67255
So is AC=85 1.92942

To fine $\angle B=\angle CbB=53^{\circ} 8'$ 9.90300

From 180° 00'

Take $\left\{ \begin{array}{l} \angle B=53^{\circ} 8' \\ \angle A=28^{\circ} 4' \end{array} \right\}$

Sum 81 12

Remains $\angle ACB$ 98 48

From $\angle CbB=53^{\circ} 8'$

Take $\angle A=28^{\circ} 4'$

Remains $\angle ACb=25^{\circ} 4'$

And $\angle AbC=126^{\circ} 52'$ by ar. 10.

As fine $\angle A=28^{\circ} 4'$ co. ar. 0.32744

Is to BC=50 1.69897

So is fine $\angle ACB=98^{\circ} 48'$ 9.99485

To AB=105 2.02126

As fine $\angle A=28^{\circ} 4'$ co. ar. 0.32744

Is to BC=50 1.69897

So is fine $\angle ACb=25^{\circ} 4'$ 9.62703

To Ab=45 1.65344

CASE

CASE III.

Two sides and their contained angle being given, to find the rest.

Example.

Let $\left\{ \begin{array}{l} AB=64 \\ AC=48 \\ \angle A=36^{\circ} 14' \end{array} \right\}$ required BC and $\angle C$ and B.

Geometrically.

101.

Make $AB=64$, and $\angle A=36^{\circ} 14'$, draw AC which make $=48$, join BC , then will the triangle ACB be that required, BC measures 38, and $\angle B=48^{\circ} 18'$.

Calculation by axiom 3.

$AB=64$	From $180^{\circ} 00'$
$AC=48$	take $\angle A=36 14$
Sum sides $=112$	Sum $=143 46 = \angle C$ and B.
Half sum $=$	71 53

As sum sides 112	co. ar. 7.95078
Is to their diff. 16	1.20412
So is tang. half $\angle C$ & B	$=71^{\circ} 53' 10.48522$

To tangent half their diff. $=23^{\circ} 35' 9.64012$

To half sum $=71^{\circ} 53'$	From half sum $=71^{\circ} 53'$
Add half diff. $=23 35$	Take half diff. $=23 35$
Greater $\angle C=95 28$	Less $\angle B=48 18$

As sine $\angle B=48^{\circ} 18'$	cor. ar. 0.12689
Is to $AC=48$	1.68124
So is sine $\angle A=36^{\circ} 14'$	9.77164

To $BC=38$	1.57977
------------	---------

CASE IV.

The three sides being given, to find the angles.

Example.

103.

Example.

Let $\begin{cases} AB=52 \\ BC=32 \\ AC=40 \end{cases}$ required $\angle A, C,$ and $B?$

Geometrically.

Make $AB=52$, with the extent of 40, and one foot in A, strike an arch also, with the extent of 32, and one foot in B, cut the former arch in C, join AC and BC, and it is done. Make the perpendicular CD.

Measure $\begin{cases} \angle A=37^{\circ} 57' \\ \angle B=50^{\circ} 15' \end{cases}$

Calculation by axiom 4.

AC=40	AC=40
BC=32	BC=32
Sum=72	diff=8

Base. sum. diff.

AS $52 : 72 :: 8 : 11.07 = \text{diff. segments of the base.}$

$5.53 = \text{half diff. seg.}$

To half base = 26

Add half diff. seg. = 5.53

Sum = greatest seg. = $31.53 = AD.$

From half base = 26

Take half diff. segm = 5.53

Remains less seg. = $DB 20.47$

Given $\begin{cases} AC \text{ the hyp.} = 40 \\ AD \text{ the base} = 31.53 \end{cases}$ to find the $\angle s A$ and $ACD.$

As $AC=40$ 1.60206

Is to radius, sine 90° 10.00000

So is $AD = 31.53$ 1.49872

To find $\angle ACD = 52^{\circ} 01'$ 9.89666

From $90^{\circ} 00'$

Remains $\angle A = 37^{\circ} 59'$

As

P. VIII. TRIGONOMETRY.

159 FIG.

$$\text{As } BC=32 \quad 1.50515$$

$$\text{Is to radius, sine } 90^\circ \quad 10.00000$$

$$\text{So is } BD=20.47 \quad 1.31111$$

$$\begin{array}{r} \text{To sine } \angle BCD=39^\circ 46' \quad 9.80596 \\ \text{From } 90 \quad 00 \end{array}$$

$$\text{Remains } \angle B=50 \quad 14$$

$$\text{Now } \angle C=\angle ACD+\angle BCD=91^\circ 47'.$$

$$\text{For } ACD=52^\circ 01'$$

$$\text{And } BCD=39 \quad 46$$

$$91 \quad 47$$

Otherwise by *axiom 6*.

$$BC=32$$

$$AB=52$$

$$AC=40$$

$$124$$

$$\text{Half sum}=62$$

$$\text{co. ar. } 8.20760$$

$$\text{Remainders } \left\{ \begin{array}{l} 30 \\ 10 \text{ log.} \\ 22 \end{array} \right.$$

$$\text{co. ar. } 8.52288$$

$$1.00000$$

$$1.34243$$

$$19.07291$$

$$\text{Half is tangent of } 18^\circ 58\frac{1}{2}' \quad 9.53645$$

$\angle A=37 \quad 57$ as above, hence the rest will be found by *axiom 2*.

Or thus.

103.

$$\text{A theorem } \frac{AB^2 + BC^2 - AC^2}{2 \times AB \times BC} = \text{co-sine } \angle B.$$

$$\text{And theorem } \frac{AB^2 + AC^2 - BC^2}{2 \times AC \times AB} = \text{co-sine of the } \angle A.$$

To find the angle B.

In numbers.

$$\text{Here } AB=52, \text{ its square}=2704$$

$$BC=32, \text{ its square}=1024$$

$$AC=40, \text{ its square}=1600$$

And

FIG.

And $AB \times BC = 52 \times 32 = 1664 \times 2 = 3328$

2704

1024

3728

1600

3328)2128.00000(.63942 the natural co-sine of
 $50^\circ 15' = \angle B$.

To find the angle A.

 $52 \times 40 = 2080 \times 2 = 4160$

Then

2704

1600

4304

1024

4160)3280.00000(.78846 the natural co-sine
 of $37^\circ 58' = \angle A$.

Whence their sum taken from 180° , leaves the angle $C = 91^\circ 47'$.

I shall now proceed to shew the various uses of a right angled triangle, and the oblique, with regard to the taking of heights and distances, &c.

P R O B L E M I.

To find the height of a tower, castle, church, &c. when you can come to the foot of it.

104. With any instrument to take angles, as suppose a quadrant, you stand some distance from the foot of the tower, as at A, and there taking your quadrant, and directing the sights of it to C, and find the string cuts the limb at $48^\circ 12'$, then find by measuring in a straight line how far you stood from the foot of the tower, and note the measure, suppose here 45 yards, then may the height of the tower be found.

BC denotes the tower or perpendicular of the right angled triangle ABC.

AB the distance measured 45 yards, or base of the triangle.

AC the visual sight from the quadrant to the tower's top, or hypotenuse.

Now

Now there is given $\angle A = 48^\circ : 12'$
 $\angle C = 41 : 48$ } to find BC.
 $AB = 55$ yards

FIG.

By Case III.

As sine $\angle C = 41^\circ 48'$	9.82382
Is to $AB = 45$	1.65321
So is $\angle A = 48^\circ 12'$	9.87243
To $BC = 50.33$	1.70182

The height of the instrument above ground must always be added, which here suppose was two yards above ground, then 52.33 yards is the tower's height.

P R O B. II.

To find the breadth of a river.

104.

An engineer comes to the side of a river, as at B, where he is to lay a bridge over it to pass the army, and wants to know the breadth of it, in order to know how many pontons he must use upon that occasion. Standing at B, he sees some object, suppose at C, on the other side of the river, and then making a right angle with his instrument, he observes another object by the river side at A, then measuring to it finds it 49 yards, and the $\angle A = 53^\circ 20'$, hence the breadth of the river may be found thus.

Given $\angle A = 53^\circ 20'$
 $\angle C = 36 40$ } to find BC.
 $AB = 49$ yards

As radius	10.00000
Is to $AB, 49$	1.69019
So is tangent $\angle A, 53^\circ 20'$	10.12815
To $EC = 65.82$	1.81834

But when the case happens you cannot come to the foot of a tower, &c. by reason of water, enemies, or other obstacle, it is then called an inaccessible height, and may be found by the following

M

P R O B.

FIG.

PROB. III.

To measure an inaccessible height.

105. At any convenient distance from the tower, as at A, find the angle of altitude, suppose $28^{\circ} 34'$, then measure in a straight line towards the tower, as to D, and suppose it measures from A to D 30 yards, then at D take another angle of altitude, which let be $50^{\circ} 9'$.

Here the right lines AC, DC, and the distance line AD form an oblique triangle ADC, wherein are given the distance $AD=30$, and all the angles to find DC, and consequently BC.

$$\begin{aligned} \text{First } AD &= 30 \\ \angle CAB &= 28^{\circ} 34' \\ \angle BDC &= 50^{\circ} 9' \\ \angle DCB &= 39^{\circ} 51' \text{ by ax. 13.} \end{aligned}$$

But $\angle ADC = 129^{\circ} 51' = \angle DCB + \angle B = 39^{\circ} 51' + 90^{\circ}$ by theorem 8. or $180^{\circ} - 50^{\circ} 9' = 129^{\circ} 51'$ by theorem 1.

$$\begin{aligned} \text{Then } \angle A &= 28^{\circ} 44' \\ \angle ADC &= 129^{\circ} 51' \end{aligned}$$

$$\begin{array}{r} \text{Take } 158 \quad 25 \\ \text{from } 180 \quad 00 \\ \hline \end{array}$$

$$\text{Remains } \angle ACD = 21 \quad 35 \text{ by ax. 10.}$$

Or thus.

$$\begin{array}{r} \text{From } \angle CDB = 50^{\circ} 9' \\ \text{Take } \angle A = 28 \quad 34 \\ \hline \end{array}$$

$$\text{Remains } \angle ACD = 21 \quad 35$$

$$\text{As sine } \angle ACD = 21^{\circ} 35' \quad 9.56567$$

$$\text{Is to } AD = 30 \quad 1.47712$$

$$\text{So is sine } \angle A = 28^{\circ} 34' \quad 9.67959$$

$$\text{To right line } DC = 39 \quad 1.59104$$

$$\text{As radius} \quad 10.00000$$

$$\text{Is to hyp. } DC = 39 \quad 1.59104$$

$$\text{So is sine } \angle D = 50^{\circ} 9' \quad 9.88520$$

$$\text{To } BC = 30 \text{ ferè, altitude} \quad 1.47624$$

N.B.

N. B. You may make use of this problem for finding the length of unapproachable lines, as those of places besieged; but never make the angle DCA less than two degrees.

PROB. IV.

106.

To find the height of a castle standing upon a hill, and your distance from it.

First find the $\angle CAD$ suppose $=40^\circ$, and $\angle GAB=22^\circ$, then measure in a straight line towards the castle, as from A to D, let be 40 yards; then at D find $\angle CDB$, let $=63^\circ 20'$, and $\angle GDB=49^\circ$, and you may from these data find the height, and distance DB.

Solution.

$$\angle CAD=40^\circ$$

$$\angle GAB=22^\circ$$

$$\angle CDB=63^\circ 20'$$

$$\angle GDB=49^\circ$$

$$\angle DCB=26^\circ 40' \text{ by ax. 13.}$$

$$\angle B=90^\circ$$

$$\angle ADC=116^\circ 40'=\angle DCB+\angle B, \text{ by theorem 8.}$$

$$\angle ACD=23^\circ 20' \text{ by ax. 10.}$$

And AD = 40 yards.

As sine $\angle ACD=23^\circ 20'$	9.59778
-----------------------------------	---------

Is to AD=40	1.60206
-------------	---------

So is sine $\angle CAD=40^\circ$	9.80806
----------------------------------	---------

To sight line CD=64.91	1.81234
------------------------	---------

As radius	10.00000
-----------	----------

Is to CD=64.91	1.81234
----------------	---------

So is sine $\angle CDB=63^\circ 20'$	9.95115
--------------------------------------	---------

To BC=58 the hill and castle	1.76349
------------------------------	---------

As radius	10.00000
-----------	----------

Is to CD=64.91	1.81234
----------------	---------

So is sine $\angle DCB=26^\circ 40'$	9.65205
--------------------------------------	---------

To DB=29.13	1.46439
-------------	---------

M 2

As

FIG.

As radius

10.00000

To DB=29.13

1.46439

So is the tangent $\angle GDB=49^\circ$

10.06083

To BG=33.51 the alt. hill

1.52522

From altitude of hill and castle BC=58 yards.

Take BG the perpend. hill

33.51

Remains CG the Castle's height=24.49

P R O B. V.

107.

Case I.

Let A, B, C be three churches whose distances are, viz. AC=106, BC=65.5, AB=53.25, and suppose at some station, as at E, I can see all those objects, and would know their respective distances from me, I take an angle AEB=13° 30', also $\angle CEB=29^\circ 50'$, from these *data* their distance from me may be found as follows,

Through A, C, and E describe a circle, and draw AE, CE, and BE, continue BE to D, and join AD, DC, then there are given

$$AC=106$$

$$BC=65.5$$

$$AB=53.25$$

$$\angle AEB=13^\circ 30' = \angle AED.$$

$$\angle CEB=29^\circ 50' = \angle CED.$$

$$\angle AEC=43^\circ 20' = \angle AEB + \angle CEB.$$

$$\angle ADC=136^\circ 40' = 180^\circ - \angle AEC, \text{ by } \textit{theo. 8.}$$

$$\left. \begin{array}{l} \angle ACD=13^\circ 30' = \angle AED \\ \angle CAD=29^\circ 50' = \angle CED \end{array} \right\} \text{ by } \textit{cor. theo. 20.}$$

Make the perpendicular Bb.

1. In the triangle ABC, all the sides are given to find the angles. By *case IV.*

$$BC + AB = 118.75 \text{ the sum of the sides.}$$

$$BC - AB = 12.25 \text{ the difference of the sides.}$$

$$\text{As base } AC=106$$

$$2.02530$$

$$\text{Is to the sum sides}=118.75$$

$$2.07445$$

$$\text{So is the diff. } 12.25$$

$$1.08813$$

$$\text{To the diff. seg. base}=13.71$$

$$1.13728$$

Now

Now half diff. 6.855

Half base 53

Greater seg. $bC = 59.855$ Less seg. $Ab = 46.145$ As $AB = 53.25$ 1.72632Is to radius 90°

10.00000

So is $Ab = 46.14$ 1.66407To find $\angle ABb = 60^\circ 3'$ 9.93775Then $\angle BA b = 29^\circ 57'$ As $BC = 65.5$ 1.81624

Is to radius

10.00000

So is $bC = 59.85$ 1.77706To find $\angle bBC = 66^\circ 2'$ 9.96082Then $\angle BCB = 23^\circ 58'$ And $\angle bBC + \angle ABb = 126^\circ 5' = \angle ABC.$ $29^\circ 57' = \angle CAB.$ $23^\circ 58' = \angle ACB.$

2. In the triangle ADC

are given $\left\{ \begin{array}{l} AC = 106 \\ \angle DAC = 26^\circ 50' \\ \angle ACD = 13^\circ 30' \\ \angle ADC = 136^\circ 40' \end{array} \right\}$ to find AD, DC.

As sine $\angle D = 136^\circ 40'$ 9.83647Is to $AC = 106$

2.02530

So is sine $\angle C = 13^\circ 30'$ 9.36818To $AD = 36.06$ 1.55701As sine $\angle D = 136^\circ 40'$ 9.83647Is to $AC = 106$

2.02530

So is sine $\angle A = 29^\circ 50'$ 9.69677To $DC = 76.85$ 1.88560And $\angle DCB = 37^\circ 28' = \angle ACD + \angle ACB.$

M 3

3. In

3. In the triangle BCD, are given two sides and the contained angle to find the angles, by *case III*.

$$\left. \begin{array}{l} DC = 76.85 \\ BC = 65.5 \\ \angle DCB = 37^\circ 28' \end{array} \right\} \text{to find the angles D and B.}$$

$$DC + BC = 142.35 \text{ sum sides.}$$

$$DC - BC = 11.35 \text{ diff. sides.}$$

$$180^\circ - \angle 37^\circ 28' = 142^\circ 32' \text{ sum angles D and B.}$$

$$\underline{\underline{71 \ 16 = \text{half sum.}}}$$

As sum sides 142.35	2.15320
Is to the diff. 11.35	1.05499
So is tang. half sum = $71^\circ 16'$	<u>10.46963</u>
To tang. half diff. = $13^\circ 14'$	9.37142
Half sum = $71 \ 16$	<u> </u>

$$\angle CBD = 84 \ 30 = \text{greater angle}$$

$$\angle BDC = 58 \ 02 = \text{less angle.}$$

From 180°

$$\underline{\underline{84 \ 30}}$$

$$\angle CBE = 95 \ 30$$

$$\angle CEB = 29 \ 50$$

$$\underline{\underline{125 \ 20}}$$

From 180°

$$\underline{\underline{125 \ 20}}$$

$$\angle ECB = 54 \ 40$$

4. In the triangle EBC

$$\text{are given } \left\{ \begin{array}{l} BC = 65.5 \\ \angle CBE = 95^\circ 30' \\ \angle CEB = 29^\circ 50' \\ \angle ECB = 54 \ 40 \text{ by ax. 10.} \end{array} \right\} \text{to find BE, EC}$$

$$\text{As sine } \angle E = 29^\circ 50' \quad 9.69677$$

$$\text{Is to } BC = 65.5 \quad 1.81624$$

$$\text{So is sine } \angle C = 54^\circ 40' \quad 9.91158$$

$$\text{To BE} = 107.4 \quad \underline{\underline{2.03105}}$$

As

As sine $\angle E = 29^\circ 50'$	6.69677
Is to $BC = 65.5$	1.81624
So is sine $\angle B = 95^\circ 30'$	9.99799
To $EC = 131$	<u>2.11746</u>

FIG.

5 In the triangle ABE

are given	$\left\{ \begin{array}{l} AB = 53.25 \\ BE = 107.4 \\ \angle E = 13^\circ 30' \\ \angle A = 28^\circ 5' \\ \angle B = 138^\circ 25' \text{ by ax. 10.} \end{array} \right\}$	to find AE.
-----------	--	-------------

As sine $\angle E = 13^\circ 30'$	9.36818
Is to $AB = 53.25$	1.72631
So is sine $\angle B = 138^\circ 25'$	9.82197
To $AE = 151.4$	<u>2.18010</u>

PROB. V. Case II.

108

Suppose the three objects A, B, C were seen from E, and stood as *per* figure, whose distances are $AC = 12$ furlongs, $AB = 9$ furlongs, and $BC = 6$ furlongs, the angle $AEB = 33^\circ 45'$, and $CEB = 22^\circ 30'$, it is required to determine how far E is from A, B, C?

First make a triangle ABC, with the three given sides, draw CD as to make an angle with $AC = \angle AEB = 33^\circ 45'$, and at A draw AD so as to make an angle with $AC = \angle CEB = 22^\circ 30'$ through the three points A, C, E describe a circle, produce BD to the station place E, and join AE, CE, then EA, EB, EC are the lines, required by the question.

1. In the triangle ABC, all the sides are given to find the angles. By ax. 6.

To find $\angle A$.

6	
12	
9	
<u>Sum = 27</u>	
Half sum = 13.5 co. ar.	8.86967
Remainders $\left\{ \begin{array}{l} 7.5 \text{ co. ar.} \\ 1.5 \text{ log.} \\ 4.5 \text{ log.} \end{array} \right.$	9.12494
	0.17609
	0.65331
	<u>18.82391</u>
Tang. $14^\circ 28\frac{1}{2}'$	9.41195
2	
$\angle A = 28 \quad 57$	

M 4

By

FIG. By the same method you will find the $\angle C = 46^\circ 34'$ and $\angle D = 104^\circ 29'$, whence there are

$$\text{given } \left\{ \begin{array}{l} \angle BAC = 27^\circ 57' \\ \angle ABC = 104^\circ 29' \\ \angle ACB = 46^\circ 34' \\ \angle DAC = 22^\circ 30' = \angle CED, \text{ by cor. th. 20.} \\ \angle DAB = 6^\circ 27' = \angle BAC - \angle DAC. \\ \angle ADC = 123^\circ 45' \text{ by ax. 10.} \\ \angle DCB = 12^\circ 49' = \angle ACB - \angle ACD. \\ \angle DCA = 33^\circ 45' = \angle AED, \text{ by cor. th. 20.} \\ \angle AEC = 56^\circ 15' = \angle AEB + \angle BEC. \end{array} \right.$$

2. In the triangle ADC, are given all the angles and side AC to find AD.

As sine $\angle D = 123^\circ 45'$	9.91984
Is to AC = 12	1.07918
So is sine $\angle C = 33^\circ 45'$	9.74473
To AD = 8.018	<u>0.90407</u>

108.

3. In the triangle ADB

are given $\left\{ \begin{array}{l} AB = 9 \\ AD = 8.018 \\ \angle DAB = 6^\circ 27' \end{array} \right\}$ to find the angles D and B.

As sum sides 17.018	1.23090
Is to their diff. .982	1.99211
So is tang. of half $\angle D \& B = 86^\circ 46'$	<u>11.24801</u>
To tang. half diff. = $45^\circ 37'$	10.00922
From 86 46	<u> </u>

$$\angle AEB = 41^\circ 09'$$

$$\angle ADE = 47^\circ 36' = \angle ABE + \angle DAB = \angle ACE.$$

4. In the triangle ABE

are given $\left\{ \begin{array}{l} \angle AED = 33^\circ 45' \\ \angle ABE = 41^\circ 09' \\ \angle BAE = 105^\circ 6' \\ AB = 9 \end{array} \right\}$ to find AE and EB.

As

As sine $\angle AEB = 33^\circ 45'$	9.74473
Is to $AB = 9$	0.95424
So is sine $\angle AEB = 41^\circ 9'$	9.81824
To $AE = 10.6$	1.02775
As sine $\angle AEB = 33^\circ 45'$	9.74473
Is to $AB = 9$	0.95424
So is $\angle BAE = 105^\circ 6'$	9.98474
To $EB = 15.64$	1.19425

Fig.

5. In the triangle EAC

are given $\begin{cases} \angle CAE = 76^\circ 9' = \angle BAE - \angle BAC. \\ \angle AEC = 56^\circ 15' \\ AC = 12 \end{cases}$ to find EC.

As sine $\angle E = 56^\circ 15'$	9.91984
Is to $AC = 12$	1.07918
So is sine $\angle A = 76^\circ 9'$	9.98718
To $EC = 14.01$	1.14652

180:

Hence $AE = 10.6$
 $EB = 15.64$
 $EC = 14.01$

P R O P. V. Case III.

109.

Let us again suppose the place of our station was at D, somewhere within the triangle of the three objects, as in the annexed figure, whose distances from each other are as in the problem, where being at D, I observed,

given $\begin{cases} \text{The } \angle DAC = 22^\circ 30' \\ \angle ACD = 33^\circ 45' \\ \angle ADC = 123^\circ 45' \text{ by ax. 10.} \\ \text{And } AC = 12 \text{ to find AD, DC.} \\ AB = 9. BC = 6. \end{cases}$

As sine $\angle ADC = 123^\circ 45'$	9.91984
Is to $AC = 12$	1.07918
So is sine $\angle DAC = 22^\circ 30'$	9.58283
To $DC = 5.523$	0.74217

As

FIG.	As sine $\angle ADC = 123^\circ 45'$	9 919846
109.	Is to $AC = 12$	1.079181
	So is sine $\angle ACD = 33^\circ 45'$	9.744739
	To $AD = 8.018$	0.904074

Given two sides and the contained angle in the triangle ADB, to find DB.

As sum sides 17.018	1.230908
To their diff. = .982	1.992111
So is tang. half sum $\angle s = 86^\circ 46'$	11.248010
To tang. half diff. = $45^\circ 36'$	10.009213
Hence $\angle ABD = 41^\circ 10'$, and $\angle BAD = 6^\circ 28'$.	
As sine $\angle ABD = 41^\circ 10'$	9.818391
To $AD = 8.018$	0.904074
So is sine $\angle BAD = 6^\circ 28'$	9.051635
To $BD = 1.372$	0.137318

Geometrical solution.

Make $AC = 12$, $AB = 9$, $BC = 6$; and $\angle A = 22^\circ 30'$, draw AD ; make $\angle C = 33^\circ 45'$, and where CD and AD meet, as in D , that is the place of the station, join DB , and it is done.

AD measures 8, DC 5.5, and DB 1.4 nearly.

I have been more particular in the solution of this problem, since the several varieties of it arise from the different positions of the station E , than any of the foregoing, because this problem is of very great use for drawing maps of countries, or laying down rocks, shoals or sands in sea charts, as it is done at one station.

P R O B. VI.

To find the distance of any visible object from you.

Example.

110.

Suppose C a castle, and you want find a proper place to plant a battery to hit it.

Pitch upon any convenient distance as at B , there put a mark, and place your theodolite as level as you can, laying your

your index with the sights upon it, upon the north and south diameter; then turn the theodolite about upon the staff till can see the castle at C through the sights, the index being still kept on the diameter, north and south, there screw your instrument fast, where the degrees take their beginning towards C; then from the foot of your staff at B, measure any proper distance as to A, suppose 60 poles, and at A set up another mark; then come again to B, and turn your index about till you can see the mark at A through the sights, and see what degrees and minutes the index cuts upon the limb, suppose $100^{\circ} 40'$, and this is the measure of the angle B.

Now remove your instrument to A, and there place it as you did at B, lay your index on the north and south diameter as before, turn your instrument about till you can see through the sights the mark at B, there screw your instrument fast where the degrees take their beginning towards the mark at B, and move the index, till through the sights you can see the castle at C, and observe how many degrees and minutes are cut by the index upon the limb, suppose $58^{\circ} 20'$, and that will be the measure of the angle A, and you have enough given to determine the distances of BC and AC.

For there are given the side $AB=60$ poles, and all the angles, whence the distances BC and AC, may be found by *case I.* of oblique angled triangles.

Geometrical solution.

Draw the stationary line $BA=60$, and at B make an angle $=100^{\circ} 40'$, and draw BC; then at A make another angle $=58^{\circ} 20'$, and draw AC, and where AC and BC meet as in C, the lines AC and BC are the required distances from the two stations B and A to C; BC measures 143, and AC about 165 poles.

Another way.

At B the place of your station make a right angle, and from thence go in a perpendicular line towards D, suppose 54 poles, and at D take another angle, suppose $69^{\circ} 14'$; then lines drawn from B and D, will meet in C and the stationary line BD being laid down from a scale of equal parts and the angle D protracted from the lines of chords, CB measures 143 nearly, and DC 152 poles.

P R O B.

FIG.

P R O B. VII.

To take a long distance on a level plane, without the help of any instrument, unless a staff to set off a right angle.

Example.

Observing a battery of cannon erected at a distance from me, I would know the distance, in order to erect another to silence it.

111. At any station as A, move in a right line between A and C, which may easily be done by ranging a person, or setting up a pole to be in a right line with you at A, and the object at C; then measure any distance as to D, suppose 29 poles, then at D, set up your staff, and go from a perpendicular line from D to E, (making a right angle at D) any distance, suppose 43 poles, and set up another mark at E, then come back again to A, and go in a perpendicular line from A to B, as you did at D to E, till you see the mark set up at E and the point C, range in a right line with you at B; there make a mark, and measure from A to B, suppose 54 poles; to find the distance AC.

Say, as $AB-DE : AD :: AB : AC$,

That is, as $11 : 29 :: 54 : 142\frac{1}{2}$ poles, nearly the same as before = distance AC.

To find the distance from D to C.

Say, as $AB-DE : AD :: DE : DC$,

That is, as $11 : 29 :: 43 : 113\frac{1}{2}$ poles, the distance from D to C.

For from $AC = 142\frac{1}{2}$

Take $AD = 29$

$113\frac{1}{2}$, proof.

112.

P R O B. VIII.

To find the distance of two objects, as two batteries A and B, from two stations R and S, whose distance is also given.

Suppose at some convenient distances, as at R and S, I pitch upon for the two places of my station, whose distance I measure suppose 155 paces, and at R and S, I take the following angles.

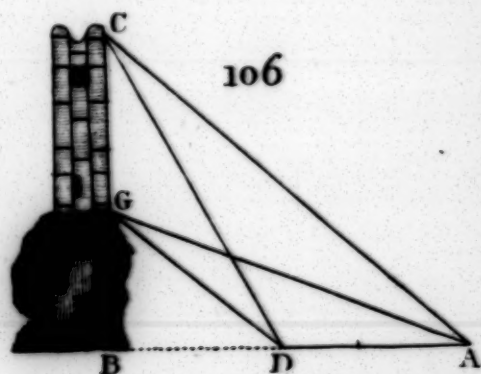
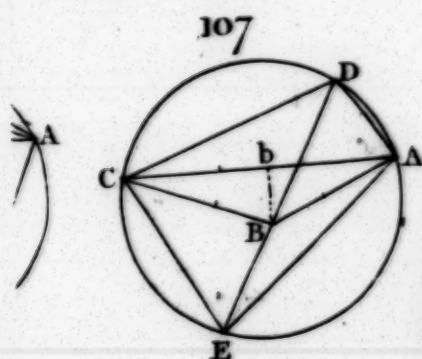
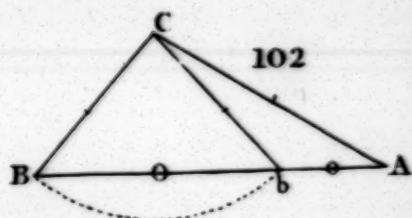
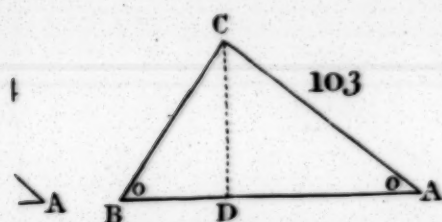
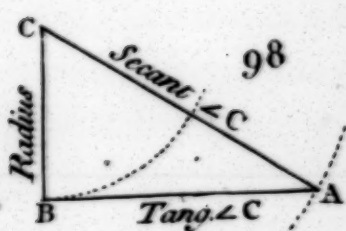
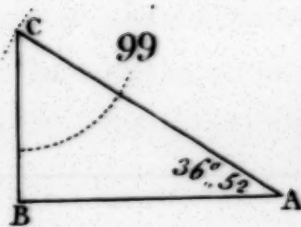
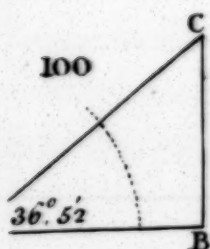
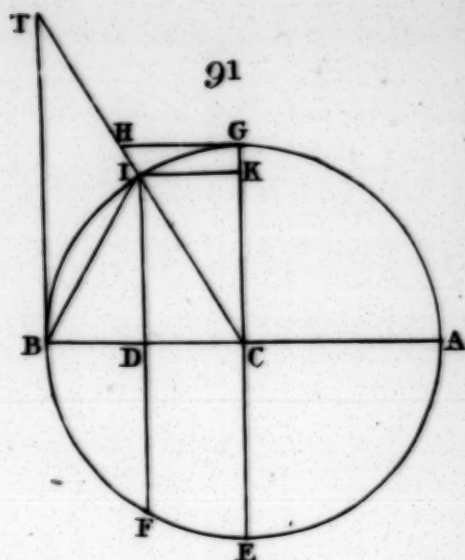
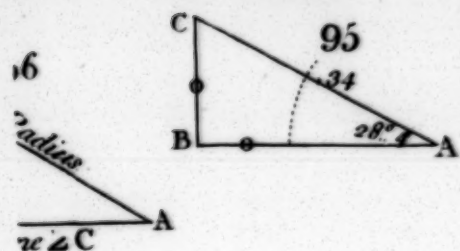
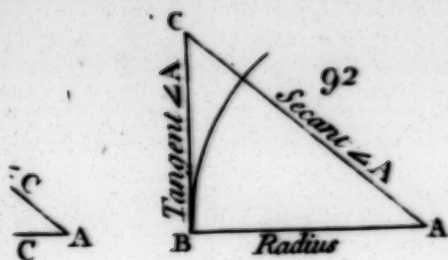
$$\angle ARB = 47^{\circ} 17'$$

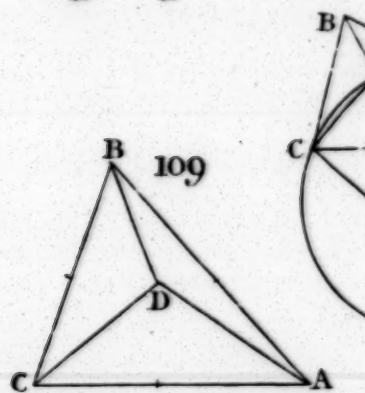
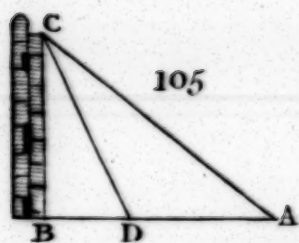
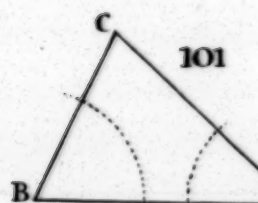
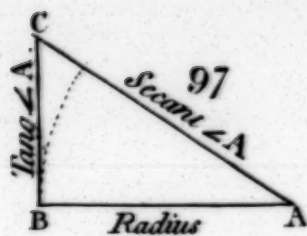
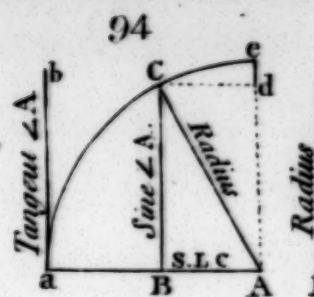
$$\angle BRS = 46 \quad 50$$

$$\angle ASB = 58 \quad 24$$

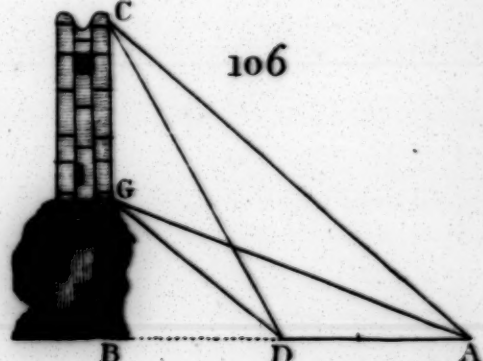
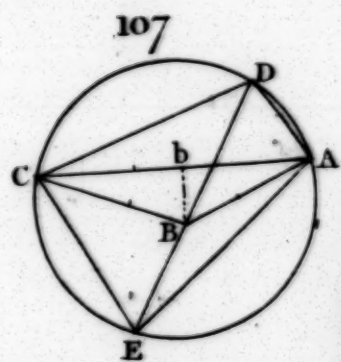
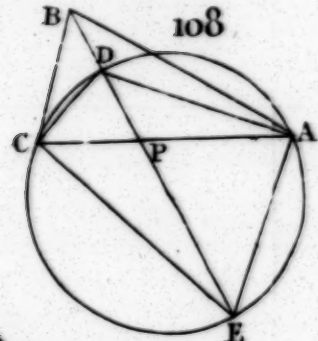
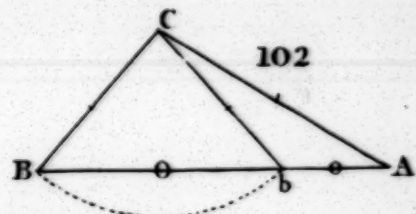
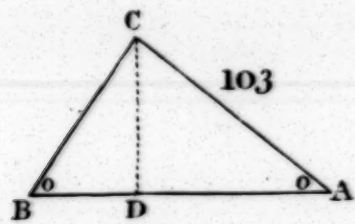
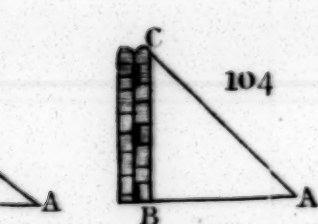
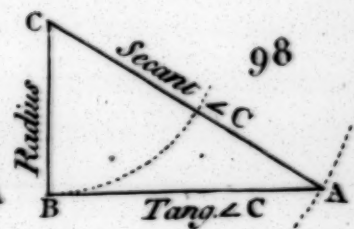
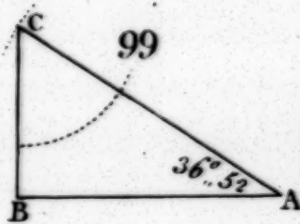
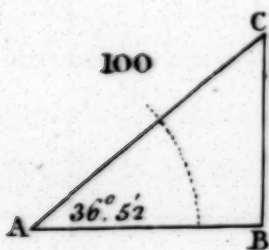
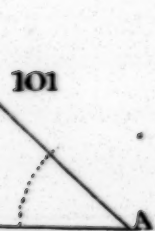
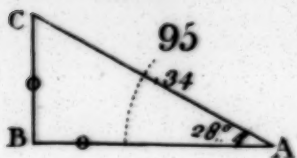
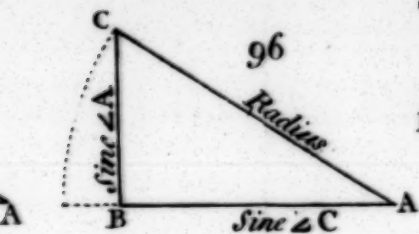
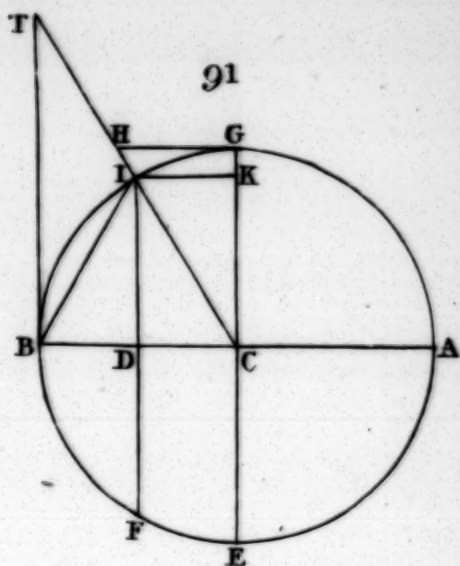
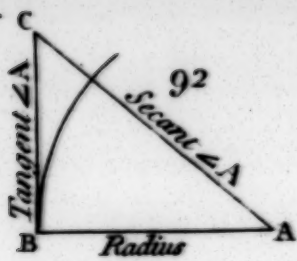
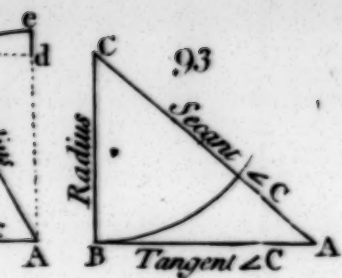
$$\angle ASR = 39 \quad 16$$

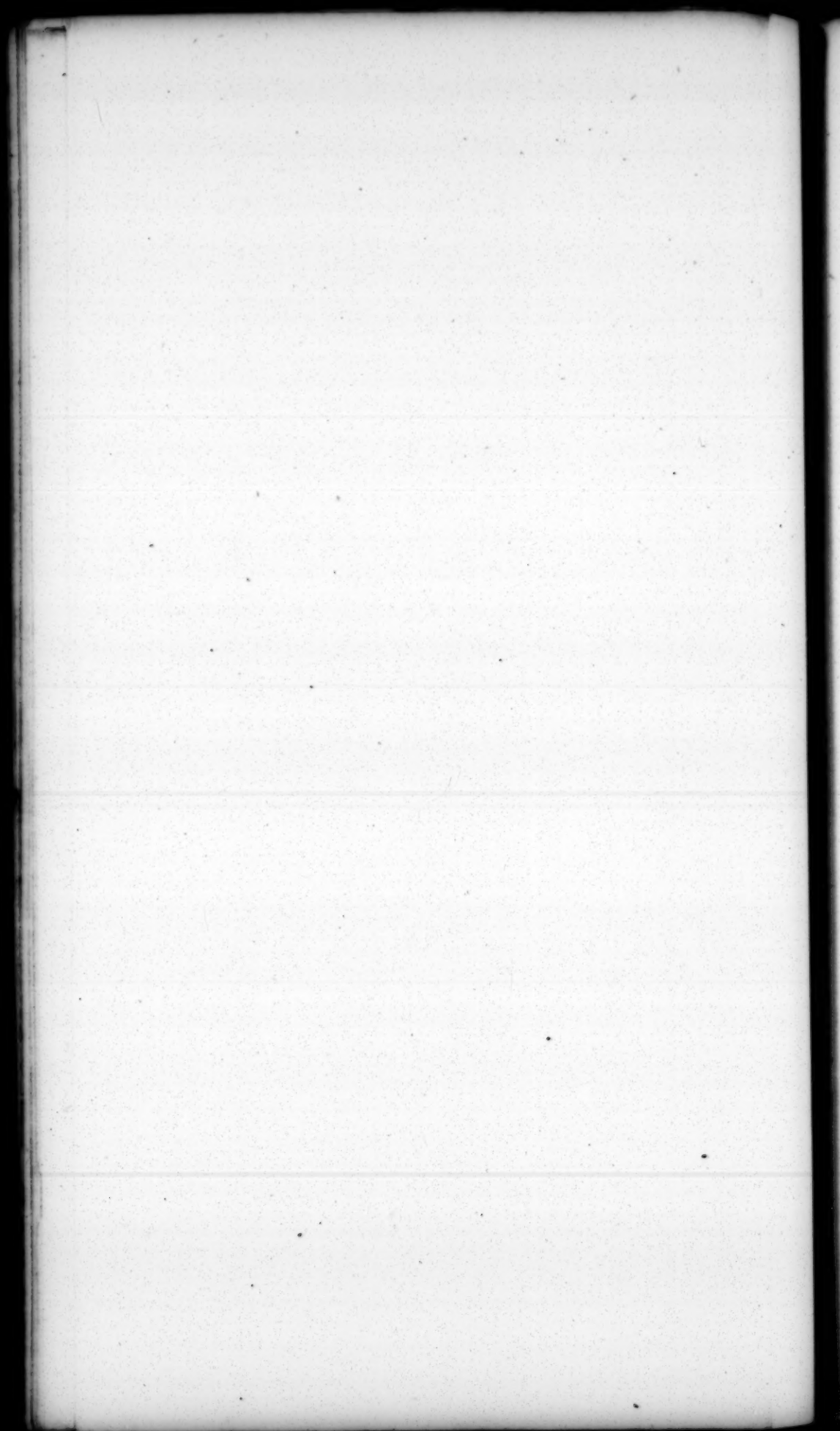
To





Trigonometry.





To find the distance of each battery R and S, from the salient angles A and B.

1. $\angle ARB + \angle BRS = \angle ARS = 94^\circ 7'$ and $\angle RAS = 46^\circ 37'$ by ax. 10.

2. $\angle ASR + \angle ASB = \angle BSR = 97^\circ 40'$ and $\angle RBS = 35^\circ 30'$ by ax. 10.

1. In the triangle ARS are given all the angles, and side RS, to find AR and AS.

As sine $\angle RAS = 46^\circ 37'$	9.86140
-------------------------------------	---------

To RS = 155	2.19033
-------------	---------

So is sine $\angle ARS = 94^\circ 7'$	9.99888
---------------------------------------	---------

To AS = 212.7	2.32781
---------------	---------

As $\angle ARS = 94^\circ 7'$	9.99888
-------------------------------	---------

To AS = 212.7	2.32781
---------------	---------

So is sine $\angle ARS = 39^\circ 16'$	9.80136
--	---------

To AR = 135	2.13029
-------------	---------

2. In the triangle BSR all the angles, and side RS are given, to find BR and SR.

As sine $\angle SBR = 35^\circ 30'$	9.76395
-------------------------------------	---------

To SR = 155	2.19033
-------------	---------

So is sine $\angle BRS = 46^\circ 50'$	9.86295
--	---------

To SB = 194.7	2.28933
---------------	---------

As sine $\angle RBS = 35^\circ 30'$	9.76395
-------------------------------------	---------

To RS = 155	2.19033
-------------	---------

So is sine $\angle RSB = 97^\circ 40'$	9.99610
--	---------

To BR = 264.5	2.42248
---------------	---------

P R O B. IX.

Let A and B represent two churches, whose distance is 5 miles, and I fix upon two stations as C and D, at each of which I can see both churches and the other station; suppose
also

FIG. also a line to be drawn from one church to the other, and from each station to each church; lastly, suppose I observe the angle $\angle ACB$ to be $38^\circ 40'$; the $\angle ACD = 118^\circ 33'$; the the $\angle CDA = 34^\circ 20'$ and the angle $\angle CDB = 48^\circ 27'$; what is the distance of the two stations, and the churches from each station?

113. Draw another figure $abcd$ whose side cd suppose to be any number, as 10, and similar to $ABCD$; now upon supposition that cd is 10, and the small figure similar to the great one, we can by the last problem find ac , bd .

First there is given $AB = 5$ miles,

$$\angle ACB = 38^\circ 40'$$

$$\angle ACD = 118^\circ 33'$$

$$\angle CDA = 34^\circ 20'$$

$$\angle CDB = 48^\circ 27'$$

Hence the following angles are also known.

$$\angle BCD = 79^\circ 53'$$

$$\angle BDA = 14^\circ 07'$$

$$\angle DBC = 51^\circ 40'$$

$$\angle DAC = 27^\circ 07'$$

Assume $dc = 10$, as before, and the small figure being similar to the great one, is to be looked upon as the same.

As $\angle dac = 27^\circ 07'$	9.658778
--------------------------------	----------

Is to $dc = 10$	1.000000
-----------------	----------

So is $\angle acd = 118^\circ 33'$	9.943692
------------------------------------	----------

To $ad = 19.27$	1.284914
-----------------	----------

As sine $\angle dbc = 51^\circ 40'$	9.894546
-------------------------------------	----------

To $dc = 10$	1.000000
--------------	----------

So is sine $\angle bcd = 79^\circ 53'$	9.993194
--	----------

To $bd = 12.55$	1.098648
-----------------	----------

As sine $\angle acd = 118^\circ 33'$	9.943692
--------------------------------------	----------

To $ad = 19.27$	1.284914
-----------------	----------

So is sine $\angle cda = 34^\circ 20'$	9.751284
--	----------

To $ac = 12.37$	1.092506
-----------------	----------

As

As sine $\angle cbd = 51^\circ 40'$	9.894546
To $cd = 10$	1.000000
So is $\angle cdb = 48^\circ 27'$	9.874120
To $bc = 9.54$	0.979574
The sum sides $\left\{ \begin{array}{l} 12.37 \\ 9.54 \end{array} \right\} = 21.91$	
Difference	2.83

From 180°
Take $38^\circ 40'$

141 20, sum of the angles cab, cba .

70 40, half sum	
As sum sides 21.91	1.340642
To their diff. = 2.83	0.451786
So is tangent $70^\circ 40'$	10.454880
To tang. half diff. = $20^\circ 12'$	9.566024
Hence $\angle abc = 90^\circ 52'$ and $\angle dab = 50^\circ 28'$.	
As sine $\angle cba = 90^\circ 52'$	1.999950
To $ac = 12.37$	1.092506
So is sine $\angle acb = 38^\circ 40'$	9.795733
To $ab = 7.731$	0.888289

Now by working in this manner, if we had found $ab = 5$ miles, then it is evident all the sides had been found exactly right; but as it has not been found by similar triangles, the true sides may be found by those already discovered, thus.

ab	AB	ac	AC
As 7.731 : 5 :: 12.37 : 8			
ab	AB	ad	AD
As 7.731 : 5 :: 19.27 : 12.462			
ac	AC	cd	CD
As 12.37 : 8 :: 10 : 6.467			
ab	AB	bd	BC
As 7.731 : 5 :: 12.55 : 8.116			
ab	AB	bc	BC
As 7.731 : 5 :: 9.54 : 6.169			

Hence

FIG.

$$\begin{aligned}
 &\text{Hence } AC = 8 \\
 &\quad AD = 12.462 \\
 &\quad CD = 6.467 \\
 &\quad BD = 8.116 \\
 &\quad BC = 6.169
 \end{aligned}$$

P R O B. X.

There are three market towns, Newark, Mansfield, and Retford, situate in a triangular manner, whose distances from each other are 16, 18, and 20 miles, respectively; it is required to determine a place where a tradesman shall fix his house, so as to ride the fewest miles possible in keeping each market?

114.

Geometrical construction.

Make a triangle whose three sides are given, let N represent Newark, M, Mansfield, and R, Retford. Bisect NR and MR by the perpendicular DF and GI; make the angles DRE and GRH, each equal to 30° , from the centers E and H, with the radii ER, and RH, describe two circles cutting each other in the point P, which is the place required. Draw PR and EH, also NP and MP, and it is done.

1. It is proved from fluxions, which may seen in Simpson's Fluxions, that the angles at P are each equal to 120° .

Numerical solution.

In the triangle NMR, are given NM = 16, MR = 18 and NR = 20, to find the $\angle NRM$; by case 4. oblique triangles.

base sumfides diff. diff. seg.

$$\text{As } 20 : 34 :: 2 : 3.4$$

17, half diff.

Half base	10	10	
Add	1.7	1.7	
	11.7	8.3	
Greater seg.	11.7	Less 8.3	
As MR = 18		1.2552725	
To radius		10	
So is DR = 11.7		1.0681859	
To $\angle DMR = 40^\circ 2'$		9.8129134	
	90		
	$\angle NRM = 49^\circ 28'$		

2. In

P. VIII. TRIGONOMETRY.

177 FIG.

2. In the right angled triangle RDE are given $DR=10$, $\angle DRE=30^\circ$, $\angle DER=60^\circ$, to find ER.

As sine $\angle DER=60^\circ$	9.9375306	114.
To $DR=10$	1.0000000	
So is radius	10.0000000	
To $ER=11.54$	1.0624694	

3. In the triangle RHG are given $GR=9$, $\angle GRH=30^\circ$, and $\angle GHR=60^\circ$, to find HR.

As sine $\angle RHG=60^\circ$	9.9375306
To $GR=9$	0.9542425
So is radius	10.0000000
To $HR=10.39$	1.0167110

Now $\angle NRM=49^\circ 28'$
 $\angle DRE=30^\circ 00'$
 $\angle GRH=30^\circ 00'$
 $\angle ERH=109^\circ 28'$

4. In the triangle ERH are given $ER=11.54$, $HR=10.39$, and the included $\angle R=109^\circ 28'$, to find the angle REH or REO.

11.54	11.54	180°
10.39	10.39	109 28'
Sum 21.93	Diff. 1.15	2) 70 32
		Half sum 35 16

As sum sides 21.93	1.3410386
To their diff. = 1.15	0.0606978
So is tangent $35^\circ 16'$	9.7614638
To tang. half diff. = $1^\circ 44'$	2.4811230
$\angle EHR=37^\circ 0'$, $\angle HER=33^\circ 32'$.	

5. In the right angled triangle REO, are given $RE=11.54$, $\angle REO=\angle REH=33^\circ 32'$, and $\angle ERO=56^\circ 28'$, to find RO.

N

As

FIG.	As radius	10.0000000
	To RE=11.54	1.0624694
114.	So is fine $\angle REO=33^\circ 32'$	9.7422710
	To RO=6.379	0.8047404
	2	

RP=12.758 from Retford.

6. In the triangle RPM, are given RP=12.758, MR=18, and $\angle RPM=120^\circ$, to find MP.

As MR=18

1.2552725

To $\angle MPR=120^\circ$

9.9375306

So is PR=12.758

1.1057826

To fine $\angle PMR=37^\circ 52'$

9.7880407

Then $180^\circ - 120^\circ + 37^\circ 52' = 22^\circ 08' = \angle PRM$.

As fine $\angle MPR=120^\circ$

9.9375306

To MR=18

1.2552725

So is fine $\angle PRM=22^\circ 08'$

9.5760685

To PM=8.672

0.8938104

7. In the triangle NPM, are given PM=8.672, $\angle P=120^\circ$, and NM=16, to find NP.

As NM=16

1.2041200

To $\angle P=120^\circ$

9.9375306

So is PM=8.672

0.8938104

To fine $\angle MNP=25^\circ 04'$

9.6272210

From $180^\circ - 120^\circ + 25^\circ 04' = 34^\circ 56' = \angle NMP$.

As fine $\angle P=120^\circ$

9.9375306

To NM=16

1.2041200

So is fine $\angle PMN=34^\circ 56'$

9.7578687

To NP=10.58

1.0244581

The distance of the house from

Newark	= 10.58
Mansfield	= 8.672
Retford	= 12.758

Answer,

32.010

P R O P.

PROP. XI.

To find the distance of any place from you, by the motion of sound, from the firing of cannon.

Sound, it is found from experiments, flies after the rate of 1142 feet in a second of time; a measured mile is 5280 feet, or 1760 yards, 1056 paces, or 880 fathoms, and therefore a sound requires 4.62 seconds, or 9.24 half seconds to fly a mile.

Hence the time that a sound is going between any two places being known, the distance between those two places may easily be had.

For if you observe the time between first seeing the explosion and hearing the report of the gun, the distance of the place from the observer, where the gun was fired, is easily had thus,

As 4".62 to 1 mile, so is the given time between seeing the explosion and hearing the report, to the distance in miles required.

Example 1.

An engineer was ordered to plant a battery to fire on the enemies, and observing by their firing some guns, he counted eleven seconds between the time of his seeing the flash and hearing the report, how far was the engineer from the enemies guns?

4.62) 11.000 (2.4 miles nearly.

9.24

1760

Or rather thus,

As 4".62 is to 1056 paces, so is 11 seconds to 2514 paces, the distance required.

Example 2.

Being one day ordered to observe how far a battery of cannon was from me, I counted by my watch 17 seconds between the time of seeing the flash and hearing the report; how far was the battery from me?

As 4".62 : 5280 feet :: 17" : 19428 feet, or about $3\frac{1}{2}$ miles.

Or you may come very near the truth by counting the pulsations at the wrist, for it is observed there are 75 pulsations in a minute, or about 19 in a quarter of a minute, or

15 seconds, when the person is in good health and a regular habit of body, and at the same time free from any hurry or flutter in his spirits.

Hence in 6 beats of the pulse sound lies about a mile.

Example.

On seeing the flash of a gun I counted 21 pulsations at the wrist, before I heard the report; how far was the gun from me?

6) $21\frac{1}{2}$ miles, answer.

Besides the foregoing method of taking angles by the theodolite, &c. there is another way, and that by the points of the compass, used principally in navigation, which I will here illustrate by a problem or two; but it will be proper first to give a table of the angles, which every rhumb, or point of the compass makes with the meridian.

TABLE

A TABLE of the angles which every point of the compass makes with the Meridian.

NORTH	SOUTH	Points	Deg. Min.	NORTH	SOUTH.
		$\frac{1}{4}$	2 49		
		$\frac{1}{2}$	5 37 $\frac{1}{2}$		
		$\frac{3}{4}$	8 26		
N. b. E.	S. b. E.	1 0	11 15	N. b. W.	S. b. W.
		$1 \frac{1}{4}$	14 04		
		$1 \frac{1}{2}$	16 52 $\frac{1}{2}$		
		$1 \frac{3}{4}$	19 41		
N. N. E.	S. S. E.	2 0	22 30	N. N. W.	S. S. W.
		$2 \frac{1}{4}$	25 19		
		$2 \frac{1}{2}$	28 07 $\frac{1}{2}$		
		$2 \frac{3}{4}$	30 56		
N. E. b. N.	S. E. b. S.	3 0	33 45	N. W. b. N.	S. W. b. S.
		$3 \frac{1}{4}$	36 34		
		$3 \frac{1}{2}$	39 22 $\frac{1}{2}$		
		$3 \frac{3}{4}$	42 11		
N. E.	S. E.	4 0	45 00	N. W.	S. W.
		$4 \frac{1}{4}$	47 49		
		$4 \frac{1}{2}$	50 37 $\frac{1}{2}$		
		$4 \frac{3}{4}$	53 26		
N. E. b. E.	S. E. b. E.	5 0	56 15	N. W. b. W.	S. W. b. W.
		$5 \frac{1}{4}$	59 04		
		$5 \frac{1}{2}$	61 52 $\frac{1}{2}$		
		$5 \frac{3}{4}$	64 41		
E. N. E.	E. S. E.	6 0	67 30	W. N. W.	W. S. W.
		$6 \frac{1}{4}$	70 19		
		$6 \frac{1}{2}$	73 07 $\frac{1}{2}$		
		$6 \frac{3}{4}$	75 56		
E. b. N.	E. b. S.	7 0	78 45	W. b. N.	W. b. S.
		$7 \frac{1}{4}$	81 34		
		$7 \frac{1}{2}$	84 22 $\frac{1}{2}$		
		$7 \frac{3}{4}$	87 11		
EAST	EAST	8 0	90 00	WEST	WEST.

FIG.

PROP. XII.

As I was walking on the road, I observed Gamston Church bore from N. E. b. N. and having gone $2\frac{1}{2}$ miles N. N. W. the church then bore E. S. E. how far was I from the church at each time of setting?

Construction.

115. With the chord of 60 degrees, describe a circle representing the horizon, draw the Meridian N. S. and at right angles to it, the parallel E. W.

In the circle lay off the N. E. b. N. rhumb Am , the N. N. W. rhumb An , and the E. S. E. Ao ; A is the place of the first observation, make $AB=2\frac{1}{2}$ miles, and draw BC parallel to Ao the E. S. E. and where it cuts the N. E. b. N. as at C , C is the place that represents the church. Produce the lines through A the center, then by *theo. 3. Geo.* the $\angle C = mAo$, (for BC is parallel to Ao) $= \angle dAe$, its opposite, and $\angle B = \angle nAd = \angle pAo$.

Calculation.

In the triangle ABC are given $AB=2\frac{1}{2}$ miles, $\angle A=5$ points = distance between N. N. W. and N. E. b. N. $= 56^\circ 15'$; $\angle C=7$ points = distance between N. E. b. N. and E. S. E. $= 78^\circ 45'$, and $\angle B=4$ points = distance between N. N. W. and W. N. W. $= 45^\circ$ by *theo. 3.* to find AC and BC .

$$\text{As sine } \angle C = 78^\circ 45' \quad 9.9915739$$

$$\text{To } AB = 2.5 \quad 0.3979400$$

$$\text{So is } \angle B = 45^\circ \quad 9.8494850$$

$$\text{To } AC = 1.8 \text{ miles} \quad 0.2558511$$

$$\text{As sine } \angle C = 78^\circ 45' \quad 9.9915739$$

$$\text{To } AB = 2.5 \quad 0.3979400$$

$$\text{So is sine } \angle A = 56^\circ 15' \quad 9.9198464$$

$$\text{To } BC = 2.1 \text{ miles} \quad 0.3262125$$

PROB. XIII.

Grove-Hall bears from Houghton School N. E. b. N. $\frac{1}{2}$ E. distance 4 miles; Cleveland Windmill S. E. b. S. $\frac{1}{2}$ E. distance $2\frac{1}{2}$ miles; what distance is Grove from the Mill, and how do they bear from each other?

Con.

Construction.

With the chord of 60° draw a circle to represent the horizon as the last, the N. E. b. N. $\frac{1}{2}$ E. line = 4 miles = to HG, and S. E. b. S. $\frac{1}{2}$ E. line HC = $2\frac{1}{2}$ miles, join GC, and that is the distance between Grove Hall and the mill; a line drawn parallel to GC through the center H as *mn*, gives the bearing.

Calculation.

In the triangle HGC are given HG = 4, HC = $2\frac{1}{2}$, and included $\angle H = 9$ points = distance between N. E. b. N. $\frac{1}{2}$ E. and S. E. b. S. $\frac{1}{2}$ E. = $101^\circ 15'$, to find GC.

As sum sides = 6.25	0.7958800
---------------------	-----------

To their diff. = 1.75	0.2430380
-----------------------	-----------

So is tang. half sum $\angle s = 39^\circ 22\frac{1}{2}'$	9.9141732
---	-----------

To tang. half diff. = 12 56	9.3613312
-----------------------------	-----------

$39^\circ 22\frac{1}{2}'$

$12\ 56$

$26\ 26\frac{1}{2}' \angle G = \angle mHG.$

As sine $\angle G = 26^\circ 26\frac{1}{2}'$	9.6486394
--	-----------

To HC = 2.25	0.3521825
--------------	-----------

So is sine $\angle H = 101^\circ 15'$	9.9915739
---------------------------------------	-----------

To GC = 4.923 miles	0.6951170
---------------------	-----------

From the distance between the N. and N. E. b. N. $\frac{1}{2}$ E. = $39^\circ 22\frac{1}{2}'$ take $\angle G = \angle mHG = 26^\circ 26\frac{1}{2}'$ remains the $\angle NHm = 12^\circ 56'$, therefore Grove-Hall bears from the mill N. $12^\circ 56'$ E. or N. b. E. $1^\circ 41'$ E. distance 4.923 miles, or 5 miles nearly.

P R O P. XIV.

There is a church and tower whose distance from each other is known to be 20 miles; a ship at anchor sees a windmill in a right line with the tower, they bearing from her N. W. b. W. and the church then bore N. W. b. N. $\frac{1}{2}$ N. after this she weighs anchor, and sails upon a W. b. N. course 25 miles, and then finds the mill and church in a line bearing from her N. E. b. N. required the distance of the ship from the church and tower, and windmill, in both stations?

Construction.

117. With the chord of 60° describe a circle to represent the horizon, draw the meridian N, S, and at right angles to it, the parallel E. W. In the circle lay off the W. b. N. line SH = 25, and the other lines of bearing as *per* question, and the N. E. b. N. line mn, which being done, S represents the ship in her first station, H in her second station, C the church, T the tower, and M the windmill; then will the line SMT cut HC at right angles in M, and HC is parallel to mn.

Calculation.

In the right angled triangle HMS, are given HS = 25; $\angle$ HSM = 2 points = distance between W. b. N. and N. W. b. W. = $22^\circ 30'$, to find SM and HM.

As radius	10 000000
To SH = 25	1.3979400
So is $\angle$ HSM = $22^\circ 30'$	9.5828397
To HM = 9.6 miles	0.9807797
As radius	10.0000000
To SH = 25	1.3979400
So is $\angle$ MHS = $67^\circ 30'$	9.9656153
To MS = 23.09	1.3635553

In the right angled triangle MCS are given MS = 23.09 $\angle$ MSC = $1\frac{1}{2}$ points = distance between N. W. b. W. and N. W. b. N. $\frac{1}{2}$ N. = $28^\circ 7\frac{1}{2}'$, and $\angle$ C = $61^\circ 52\frac{1}{2}'$, to find SC and MC.

As sine $\angle$ MCS = $61^\circ 52\frac{1}{2}'$	9.9454298
To SM = 23.09	1.3635553
So is radius	10.0000000
To SC = 26.19	1.4181255
As sine $\angle$ MCS = $61^\circ 52\frac{1}{2}'$	9.9454298
To SM = 23.09	1.3635553
So is sine $\angle$ S = $28^\circ 7\frac{1}{2}'$	9.6733865
To MC = 12.34	1.0915120

In

In the right angled TCM are given TC=20, MC=12.34' to find the angles and TM.

As TC=20 1.3010300

To radius 10.0000000

So is MC=12.34 1.0915120

To $\angle$ CTM=38° 7' 9.7904820
 90 0

$\angle$ TCM=51° 53'

As radius 10.0000000

To TC=20 1.3010300

So is $\angle$ TCM=51° 53' 9.8958398

To TM=15.73 1.1968698

In the right angled triangle HMT are given TM=15.73 and HM 9.6, to find the angles and TH.

As HM=9.6 0.9807797

To radius 10.0000000

So is TM=15.73 1.1968698

To tang. $\angle$ H=58° 42' 10.2160901
 90 00

$\angle$ MTH=31° 18'

As sine $\angle$ MTH=31° 18' 9.7156015

To HM=9.6 0.9807797

So is radius 10.0000000

To TH=18.41 1.2651782

The several distances as follows.

		miles.	
From the ship	to windmill	} in the 1st station=	23.09
	to church		26.19
	to tower		38.82
From the ship	to windmill	} in the 2d station=	9.6
	to church		21.94
	to tower		18.41

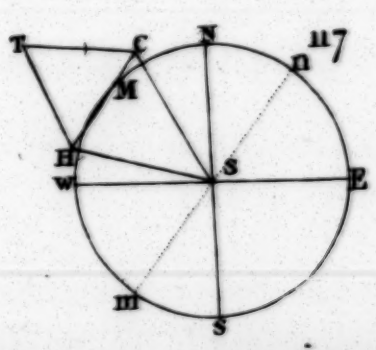
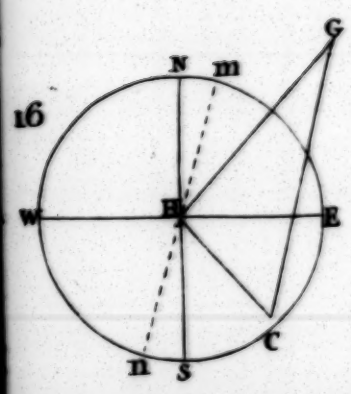
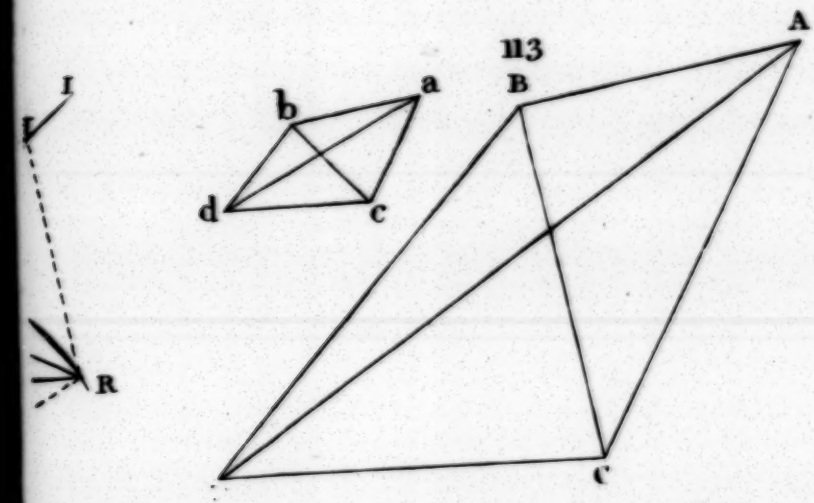
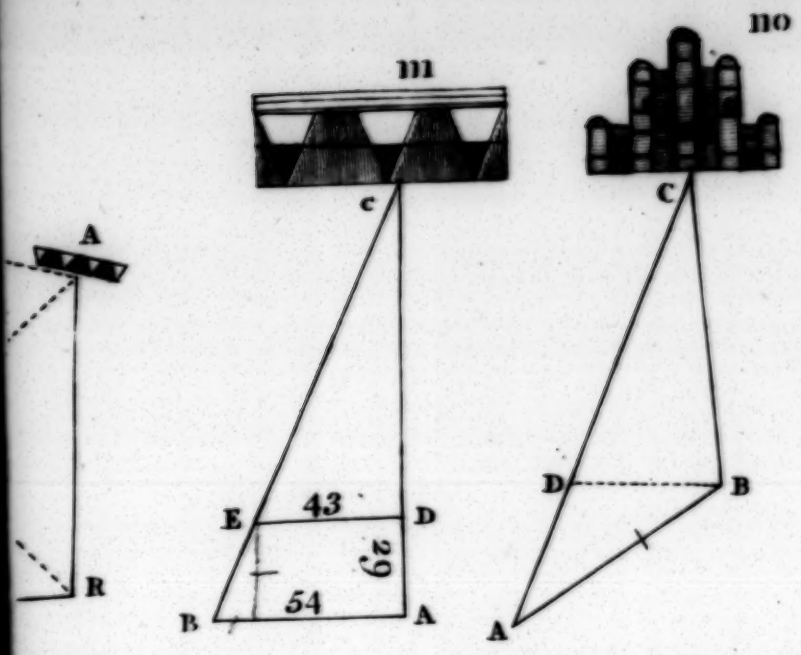
P A R T

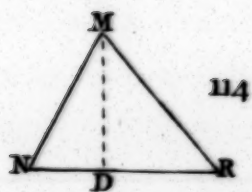
P A R T IX.
O F
F O R T I F I C A T I O N.

MILITARY architecture, or fortification, teaches to fortify a town or place, by the help of lines, and angles, &c. and requires the assistance of arithmetic and geometry to execute it, by means of putting the place in such a posture of defence, that every one of its parts defends; and when it is executed so, that every part of the work can be seen from, and defended by the other, it is called a *fortified* place; but if it has no more than a simple wall round it, then it is called an *inclosed*, and not a fortified place; and as every one of the parts of a fortified place defends, so it is defended by some other parts, by means of ramparts, parapets, moats, &c.

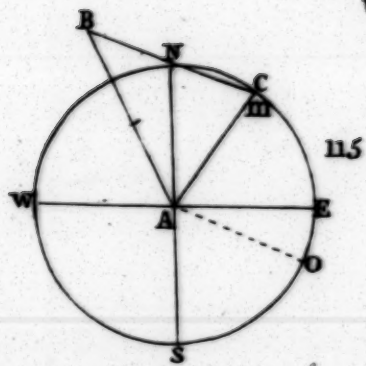
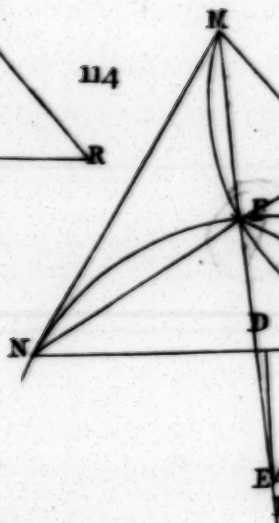
A *rampart* is a body of earth, raised about a town to cover the buildings from those in the field, and made capable of resisting the cannon of an enemy; it is formed into bastions, curtains, &c. and ought to be sloped on both sides, and to be broad enough to allow room for the marching of waggons and cannon, besides that allowed for the parapet, which is raised on it: its thickness is generally about 20 or 24 feet, and its height not above six; it is encompassed with a ditch, and is sometimes fortified in the inside, if not, it has a berme; it is upon the rampart that soldiers continually keep guard, and pieces of artillery are planted there for the defence of the place.

The *parapet*, or breastwork is an elevation of earth designed for covering the soldiers from the enemies cannon or small shot. The thickness of the parapet is about 20 feet; it is six feet high on the inside, and four or five on the outside. It is raised on the rampart and has a slope above called the superior slope, and sometimes the glacis of the parapet. The exterior slope or talus of the parapet is that facing the country; there is a stair or step three feet thick, and about two and a half high, called *banquette* or foot bank for the soldiers, who defend the parapet, to mount upon, that they may better discover



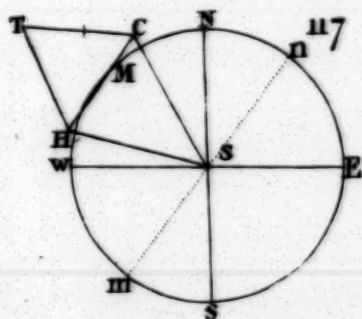
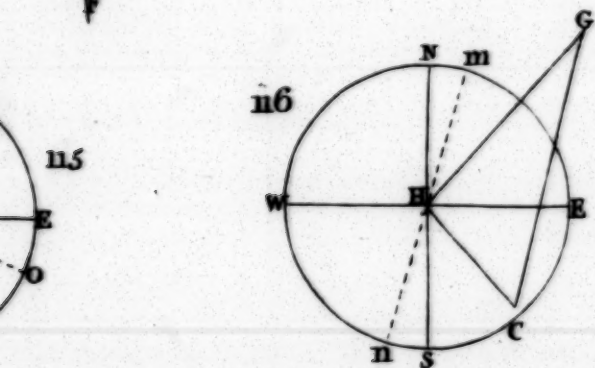
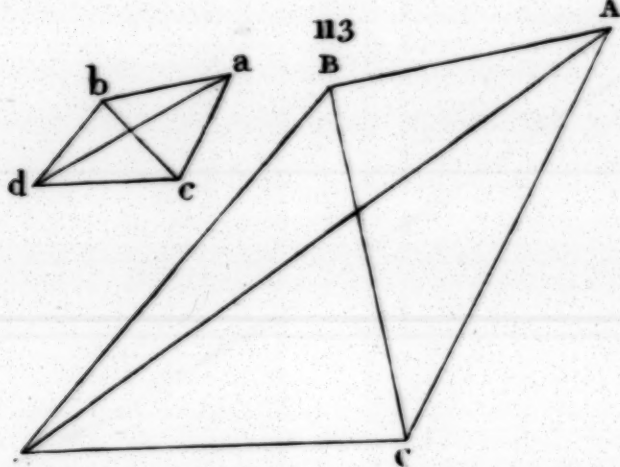
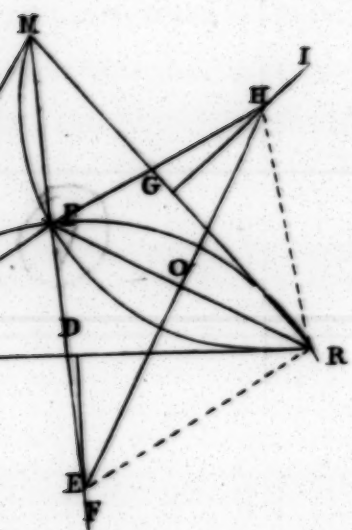
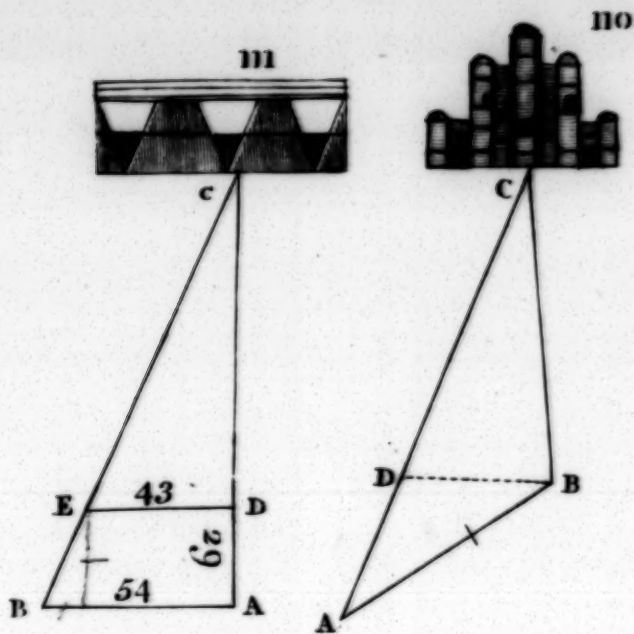
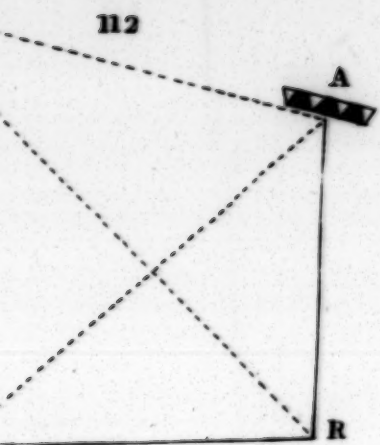


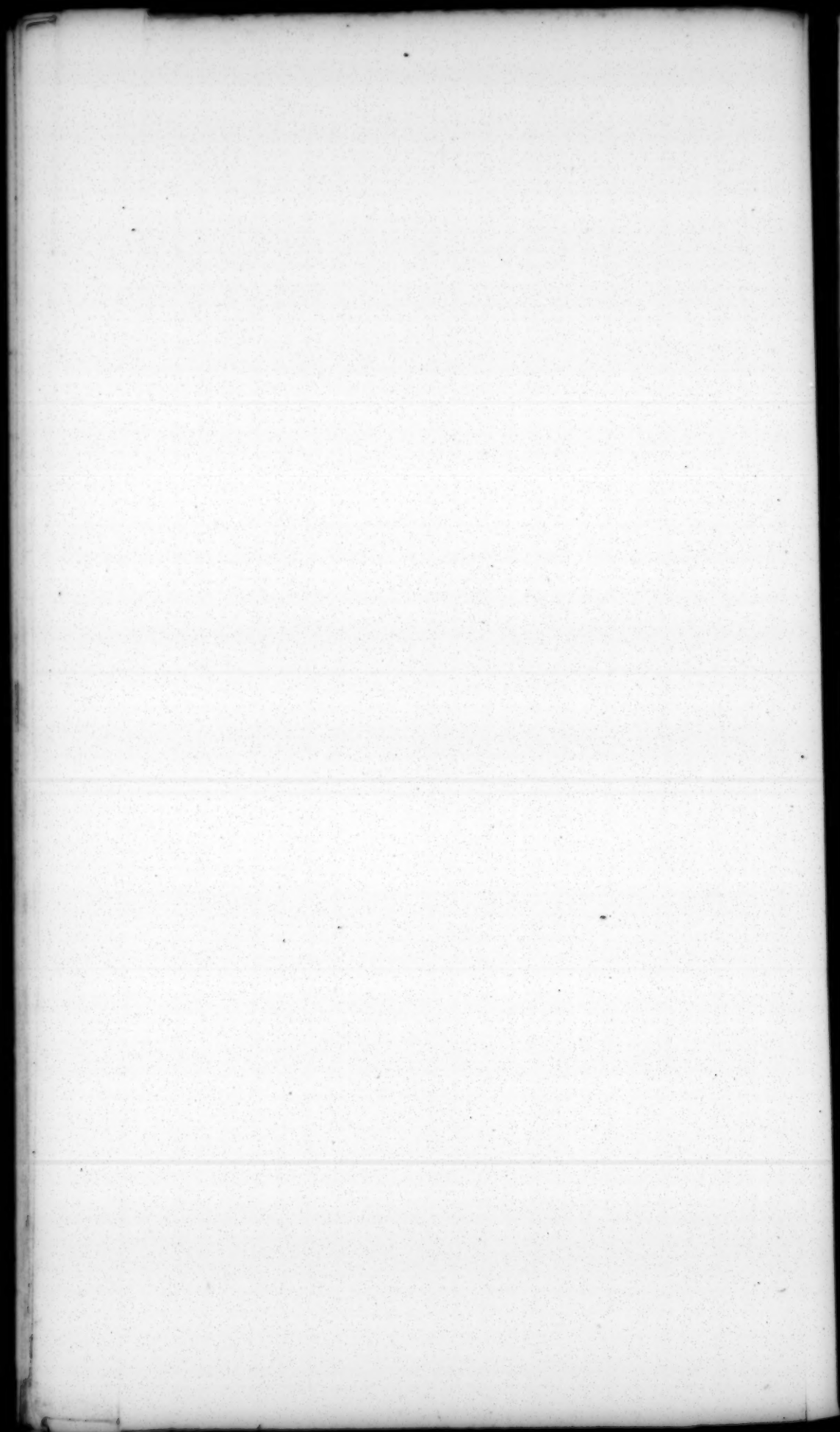
114



115

Trigonometry.





discover the country, ditch, and counterscarp, and fire as occasion requires. Upon the outward edge of the rampart is left a little way or passage of about 3 or 4 feet wide, to mount upon, to receive the earth which may roll from the parapet, and keep it from falling into the ditch either of itself or from the shock of the enemy's cannon; this space is called the *berme*.

The *moat* or *ditch* is a channel made about a place to avoid surprizes. The edge of the ditch towards the place, next the rampart is called the *scarp*, and the opposite side, that towards towards the field is called *counterscarp*, which is commonly made round over-against a point of a bastion, in order to have more room in the covered way. All moats must be well flanked, and must be made wider than any ladder, tree, &c. can reach across them. A dry moat round a large place, with a strong garrison, is preferable to one of water. The deepest and broadest moats are accounted the best; the ordinary breadth about 40 yards, and depth about 10, 12 or 15 feet.

The *covered way* is a way left upon the counterscarp, 8 or 10 feet broad. It has a parapet raised on a level, together with its banquetts and glacis or esplanade; this glacis is large and loses itself insensibly towards the field. The greatest effort in sieges is to make a lodgment on the covered way, because the besieged usually pallisade it along the middle, and undetermine it on all sides. When a place is well constructed, and skilfully defended, says the late truly ingenious Mr. *Robins*, the taking of the counterscarp is but a small step towards the possession of the place; indeed the rashness and precipitancy of the director of the approaches, hath often intimidated a weak and ignorant governor; but when the attacks have been thus eagerly hurried on against a place commanded by a brave and knowing officer, he has taken such advantages of these incautious steps, as have made them too fatal to be copied by any pretending to prudence or humanity.

Line of circumvallation is a trench bordered with a parapet, thrown up quite round the besieger's camp, by way of security against any army that may attempt to relieve the place, as well as to prevent desertion. This trench ought to be at the distance of cannon shot from the place; it is usually 12 feet broad and 7 feet deep, and at small distances is flanked with redoubts, and other small works, or with field-forts, raised on the most proper eminences. It ought never to be drawn at the foot of a rising ground, lest the enemy seizing
on

on the eminence, should erect batteries of cannon there, and so command the line.

A *battery* is a place raised a little on purpose to plant cannon on to play upon the enemy, fixed on platforms of strong planks to support the wheels of the carriages that they may not sink in the ground: the open spaces are called *embrasures*, through which the muzzles of the cannon are put, and the distances between the *embrasures* are called *merlons*, which are nearly 9 feet wide within, and 6 without, or the cannon are planted about 15 feet distant from one to another, that the parapet may be strong, and the gunners have room to work. A battery of *mortars* is different from that of cannon, for it is sunk into the ground, and has no *embrasures*.

A *cavalier* is a heap of earth raised in different shapes, situated ordinarily in the gorge of a bastion, bordered with a parapet, and cut into more or less *embrasures*. They are a double defence for the faces of the opposite bastion: they defend the ditch, break the besiegers galleries, command the traverses in dry moats, scour the salient angle of the counter-scarp, where the besiegers have their counter-batteries, and *enfilade* or scour the enemies trenches, or oblige them to multiply their parallels: they are likewise very serviceable in defending the breach and the retrenchments of the besieged, and can very much incommode the intrenchments which the enemy makes, being lodged in the bastion.

A *redoubt* is a small square fort, without any defence but in front, used in trenches, lines of circumvallation, contravallation, and approach, as also for the lodgings of *corps de gard*, and to defend passages; their face consists of from 60 to 90 feet; the ditch round them about 8 or 9 feet broad and deep, their parapets have the same thickness; they are built within musket shot of the out-works of the place to defend it.

Out-works are all the parts of a fortification detached from the body of the place, and made without side of the ditch of a fortified place, which serve for its defence, not only to cover the body of the place, but also to keep the enemy at a distance, and prevent his taking advantage of the cavities and elevations usually found in the places about the counter-scarp, which might serve them as lodgments to facilitate the carrying on their trenches and planting their batteries against the place: such as *tenailles*, horn-works, counter-guards, *ravelins*, half-moons, &c.

Half-

Half-moons are placed before flanked angles, because they have a re-entrant angle, either in right or crooked lines in the form of a crescent. Those which are before a gate or curtain are called *ravelins*, and have only two faces; and all these tenailles, counter-guards, &c. should be surrounded with a good ditch half the breadth of the ditch of the place.

The name of *half-moon*, says a French writer, is generally confounded with that of *ravelin*; but here, by a half-moon, must be understood a kind of detached bastion, made just beyond the ditch, overagainst the *point* of the bastion to cover it, so called because its gorge is an arch in the shape of a crescent.

A *ravelin* is a little out-work, placed upon the angle of the counterscarp, over-against the curtains of the place, and made of two faces and two demigorges, to cover not only the flanks, but also the bridges and gates, and to defend the half-moons, which I said before, are before the angles of the bastion, when it is too acute.

A *traverse* is made of earth in the form of a parapet, made facing or flanking towards the enemy, to hinder him from passing a narrow place, as also to cover one's self from being *enfiladed*: they are made how you please, and as many different ways as you think proper.

Rules of good Fortification by the great marshal Turenne.

1. Places commanded by high grounds are less strong than those that are not, and cannot make a long defence against an enemy who knows how to make use of the advantage.

2. That place that has most ground inclosed with fewest bastions is the best; and it naturally follows the greatest bastions are the strongest.

3. The body of the place is to be considered as well as the out-works. Upon which you are to observe, that a place, though strong by its out-works, is little worth, and cannot hold out a siege in form, if the body of the place is not fortified likewise, as well as the ground will allow.

4. Every part of the place must be sufficiently strong to resist the force of the enemy's cannon. Let there be no part of the wall but may be seen from top to bottom, from one or more places of the town; this is called flanking: it must not be out of musket-shot; that in case of an attack, it is of great advantage to the sustainers, to keep as good a fire on it as possible.

5. The

5. The ramparts must be so wide as to afford you a good parapet cannon-proof, a good banquette, and room enough for your artillery

6. Let your bastion be contrived as wide as possible; it has already been said the largest are the best, because they give you greater room to retrench yourselves, and more fire for your defence. Two hundred men may be sufficient for each bastion.

7. Let the gorge have at least 70 yards, the wider the better. The flanks must be long at least 30 or 35 yards. Let the flanked angle be about 90 degrees or more, but never never less than sixty. The flanking angles to be as close as may be; the least to be of 150 degrees.

8. The *curtain* is the space between the two bastions, or that which joins them. They serve to cover the houses and inside of the place. To be good they should be in a straight line; the others are defective, in that they hinder the flanks from seeing, and defending each other.

9. The curtains then ought to be defended with two flanks; but if necessity will let you have but one, you must plant palissades before them, and an advanced ditch. Let your line of defence go from the flanking angle, or from some part of the curtain, to the point of the opposite bastion, which must not exceed 240 yards; which is the ordinary range of a firelock.

10. The prolonged curtains are the best, because they enlarge the place, and lessen the number of bastions; provided nevertheless they be short enough for the defence of the place, and according to the rules of good fortification.

11. The simple curtain has generally 140 or 160 yards in length; it should never exceed 170, nor be less than 80 yards, to be within the rule of defence.

12. The prolonged curtain should never be more than 260 or 270 yards in length.

13. A fortress ought to be equally strong every where, and to command all the place round about it.

14. Such works, as are nearest to the center of the place, ought always to be higher than those that are farther off.

15. The irregular which come the nearest to the regular fortifications are the best; for those whose situation will permit you to make every part of the work according to the rules of art, and in just geometrical proportions are preferable; but where the ground is irregular, it prevents art from giving those

those geometrical proportions to the different parts of the work.

Some advise, for the defence of a place, to have as many men in garrison, as there are geometrical paces round it, that is to say, a man for every five feet. Others appoint 200 for every bastion; which is most followed, and comes to pretty near the same thing.

That number may suffice for the ordinary duty of a garrison; but there must be more to sustain a siege, and the more, the better the defence.

The greatest strength of any fortress, says Mr. *Robins*, is the well securing its flanks, for the enemy cannot approach to the rampart of the place if the flanks are well screened from the batteries of the enemy. A piece of cannon covered by an orillon, in the common manner, cannot be seen by the enemy, till he has gotten over the greatest part of the ditch, or is mounting the breach, in either of which places it is impossible for him to raise a counter-battery; and the more complete the artifice is, by which the flank is screened, the greater will be the space, in which the enemy will be thus exposed.

Another way of securing the flanks, says the abovesaid Mr. *Robins*, is by interposing the entering angle of the counterscarp, (or of the ravelin) between them, and the counter-batteries. This practice is described by *Errard of Barleduc*, and is said to be the invention of the count of *Lynar*. And though some authors, who were ignorant of the advantages hereby proposed, have severely censured the having any part of the ditch hidden from the flank, a circumstance which must necessarily attend this construction; yet the greatest genius, (*Coehorn*) who ever applied himself to the study of this art, has thought it worthy of his imitation; the celebrated fortress of *Berghen-op-Zoom*, having its flanks in part covered by this artifice.

An explanation of the terms of a regular fortified place.

NMAEG, a bastion.

AE the face.

AB the outer side of the polygon.

IK the inner side.

IR the inner radius.

AR the outer radius.

AI the capital.

GH the curtain, cortin.

IG the demi-gorge.

- FIG. GN the gorge.
 IH the length of curtain.
 EG the flank.
 AH the line of defence.
 $\angle ARB$ the angle at the center.
 $\angle IKQ$ the angle of the polygon.
 $\angle ABG (= \angle CAD = \angle CBD)$ the diminished angle.
 $\angle EAM$ the angle of the bastion, or flanked angle.
 $\angle ADB = \angle EDF$ the exterior flanking angle, or angle of the tenaille.
 $\angle EGH = \angle FHG$ the angle of the flank, or angle of the curtain.
 $\angle AEG = \angle BFH$ the shoulder-angle, or *epaule*.
ae the straight part of the counterscarp.
bnxa a ravelin.
 $\angle bnx$ salient angle }
bn the face } of the ravelin.
ba the demi-gorge }
cdm the counterscarp of the ditch before the ravelin.
 P, P, P, place of arms.
 Hv, or *st*, retrenchment of the flank, or platform of the casemate.
 Arch *ut* retired flank.
 Arch *sF* the orillon.

These are the principal lines and angles of a regular fortification, which I come now to shew how they may be calculated.

To fortify a square.

PROBLEM I.

Given the outer side of a square the face and normal, to fortify such square, that is, to draw the master-line.

118.

Example.

I let the outer side $AB = 360$ yards, the face $AE = 100$, and normal $CD = 45$ yards, it is required to find the diminished angle CAD , or its equal CBD , the angle of the center, of the polygon, the curtain, &c.

Geometrical construction.

Make a square whose outward side $AB = 360$ yards, bisect it in C ; on C make the normal $CD = 45$ yards, and perpendicular to AB ; from A and B , through D , draw the lines ADH

P. IX. FORTIFICATION. 193

ADH and BDG, for the lines of defence, make the faces AE and BF each equal to 100 yards, draw EF, and make EH and FG each equal to EF, draw the curtain GH, and and the flanks EG, FH, and you have one front completed. FIG.

Again, bisect another side, Ae, and draw the normal, perpendicular to it equal to 45, its given length, and proceed exactly in the same manner as before for every front till the whole be finished, and you will have a square truly fortified.

Calculation.

Divide the whole circle or 360° by the number of sides of a regular polygon, the quotient is the angle at the center, which taken from 180° , leaves the angle of the polygon.

$$4)360^\circ (90^\circ, \text{ the angle of the center.}$$

$$\text{And } \begin{array}{r} 180^\circ \\ 90 \\ \hline \end{array}$$

90, the angle of the polygon.

1. In the right angled triangle ACD, right angled at C, are given AC=180 yards, and CD=45, to find the diminished angle CAD, or its equal CBD.

As AC=180	2.25527	118.
To CD=45	1.65321	120.
So is radius	10.00000	

To tang. $\angle CAD = 14^\circ 02'$	9.39794	
	—	

From half the angle of the polygon = $45^\circ 00'$	45 00	
'Take the diminished angle CAD =	14 02	

Rem. $\angle IAH$ half the flanked angle =	30 58	2
	—	

The flanked angle, or angle of the bastion = $61^\circ 56' = \angle EAM$

But $\angle CAD = \angle CBD$ each = $14^\circ 02'$

Then from $180^\circ 00'$

Take $\left\{ \begin{array}{l} \angle CAD = 14^\circ 02' \\ \angle CBD = 14^\circ 02' \end{array} \right\}$	28 04	
	—	

Rem. $\angle ADB = \angle EDF = 151^\circ 56'$, the exterior flanking angle, or angle of the tenaille.

O

The

FIG. 120. The hypotenuse AD is found = 185.5 yards by *cor.* to *theo. 11. Geom.*

Then from 185.5

Take 100 the face AE

Rem. ED = 85.5

2. In the triangle EDF, are given the angles $\angle EFD = 14^\circ 02'$, $\angle EDF = 151^\circ 56'$, and the side DE = 85.5 yards, to find EF, or its equal EH.

As sine $\angle EFD = 14^\circ 02'$ 9.38468

To DE = 85.5 1.93196

So is sine $\angle EDF = 151^\circ 56'$ 9.67255

To EF = EH = 165.9 2.21983

To EH = 165.9

From EH = 165.9

Add AE = 100

Take ED = 85.5

Line of defence = 265.9 = AH DH or DG = 80.4

3. In the isosceles triangle GDH, are given all the angles and side DH, to find the curtain GH.

As sine $\angle DGH = 14^\circ 02'$ 9.38468

To DH = 80.4 1.90525

So is sine $\angle GDH = 151^\circ 05'$ 9.67255

To the curtain GH = 156 2.19312

To find the angle of the flank and shoulder angle, or angle of the epaule.

Since the triangle EFG is isosceles, and the angle FEG = $14^\circ 02'$, for EF is parallel to AB, *theo. 3. Geom.*

Therefore from 180° 00'

Take $\angle EFG =$ 14 02

165 58

Half is 82 59 = $\angle EGD$

To which add 14 02 = $\angle FGH$

$\angle EGH =$ 97 01, the angle of the flank,
or angle of the curtain.

To

To $97^{\circ} 01'$
Add $\angle GHE = 14^{\circ} 02'$

FIG.
118.
1204

$\angle AEG = 111^{\circ} 03'$, the shoulder angle, or angle of the epaule.

4. In the triangle EDG are given the angles EGD, $EGD = 82^{\circ} 59'$, and $28^{\circ} 04'$ respectively, and side $DE = 85.5$, to find the flank EG.

As sine $\angle EGD = 82^{\circ} 59'$ 9 99673

To $DE = 85.5$ 1.93196

So is sine $\angle EDG = 28^{\circ} 04'$ 9.67255

To the flank $EG = 40.5$ 1.60778

5. In the oblique triangle AIH are given the angle $A = 30^{\circ} 58'$ ($= 54^{\circ} - 14^{\circ} 02'$) $\angle H = 14^{\circ} 02'$, and $\angle I = 135^{\circ}$, and line of defence $AH = 265.9$, to find the lengthened curtain IH, or its equal GK.

As sine $\angle AIH = 135^{\circ}$ 9.84948

To $AH = 265.9$ 2.42471

So is sine $\angle IAH = 30^{\circ} 58'$ 9.71141

To the lengthened curtain $IH = 193.5$ 2.28664

From $IH = 193.5$

Take $GH = 156.0$

To $IH = 193.5$

Add $IG = 37.4$

Demigorge $IG = 37.4$

Inside $IK = 230.9$

6. In the oblique triangle AIH are given the angles and side AH, to find the capital AI.

As sine $\angle I = 135^{\circ}$ 9.84948

To $AH = 265.9$ 2.42471

So is sine $\angle AHI = 14^{\circ} 2'$ 9.38468

To the capital $AI = 91.2$ 1.95991

As sine $\angle R = 90^{\circ}$ 10.00000

To $IK = 231$ 2.36361

So is $\angle I = 45^{\circ}$ 9.84948

To inner radius $KR = IR = 163.3$ 2.21309

To fortify a regular pentagon.

PROB. II.

119. *Given the outward side, face, and normal of a regular pentagon, to fortify it.*

Example.

Let the outward side $AB=360$ yards, the face $AE=100$, and normal CD 54 yards, to find the curtain, flank, &c.

Geometrical construction.

Make a pentagon whose outward side is 360 yards, and bisect it in C ; on C make the normal CD equal to 54 yards, and perpendicular to AB ; from A and B , through D , draw the lines of defence ADH and BDG ; set off the faces AE and BF each equal to 100 yards, and draw EF , which will be parallel to AB , make EH and FG each equal to EF , and draw the curtain GH , and the flanks EG , FH , and you have one front completed; proceed thus round the polygon, and it will be fortified.

N. B. If you describe a circle with black lead (which may be rubbed out with bread, after your work is done) with the outward radius, and apply the given outward side of the polygon to that circumference, as often as the polygon has sides, and those points, as A , B , &c. be joined, you will have the polygon constructed.

Calculation.

$5)360^\circ(72^\circ$, the angle at the center.

180°

72

108 , the angle of the polygon $= \angle ABO$.

54 , half the angle of the polygon.

1. To find the outward or exterior radius of any polygon.

Say, As the sine of the angle at the center :

To the length of its outward side ::

So is sine of half the angle of the polygon :

To the exterior or outward radius of that polygon required ; that is, in this case,

As

As sine $\angle ARB = 72^\circ$ 9.97821

To $AB = 360$ 2.55630

So is sine $\angle ABR = 54^\circ$ 9.90796

To $AR = 306.2$ radius outer side 2.48605

2. In the right angled triangle ACD , right angled at C , there are given $AC = 180$, $CD = 54$ yards, to find the diminished angle CAD , or its equal CBD .

As $AC = 180$ yards 2.25527

To $CD = 54$ 1.73239

So is radius 10.00000

To tang. $\angle CAD = 16^\circ 42'$ 9.47712

From RAB half angle of the polygon = $54^\circ 00'$

Take diminished angle $CAD =$ 16 42

$\angle IAH$ half the flanked angle 37 18

2

$\angle EAM$ the flanked angle or of bastion = $74^\circ 36'$

To $EAM = 74^\circ 36'$ the angle of the bastion

Add $\angle$ center = $72^\circ 00'$

The $\angle ADB = 146^\circ 36' = \angle EDF$ the exterior flanking angle, or angle of the tenaille.

The hypotenuse AD is found = 187.9 yards by *cor. to theo.*

11. *Geom.* from which take the face $AE = 100$, remains $ED = 87.9$ yards.

3. In the isosceles triangle EDF , there are given the angles EFD , $EDF = 16^\circ 42'$ and $146^\circ 36'$ respectively, and the side $DE = 87.9$, to find EF , or its equal EH .

As sine $\angle EFD = 16^\circ 42'$ 9.45842

To $DE = 87.9$ 1.94398

So is sine $\angle EDF = 146^\circ 36'$ 9.74074

To $EF = 168.4$ 2.22630

To $EH = 168.4$

Add $AE = 100$

From $EH = 168.4$

Take $DE = 87.9$

Line of defence $268.4 = AH$ DH = DG = 80.5

FIG.

119. 4. In the isosceles triangle are given the angles and $DH=120.80.5$, to find the curtain GH .

As fine $\angle DGH=16^{\circ} 42'$	9.45842
To $DH=80.5$	1.90579
So is fine $\angle GDH=146^{\circ} 36'$	9.74074
To the curtain $GH=154.2$	2.18811

To find the angle of the flank EGH , and shoulder angle AEG .

Since $\angle EFG =$	$16^{\circ} 42'$ by <i>thec. 4. Geom.</i>
Then from	$180^{\circ} 00'$
Take $\angle EFG =$	$16 \quad 42$
	163 18

Half	$81 \quad 39 = \angle EGD = \angle FGE$
Add	$16 \quad 42 = \angle FGH$

$\angle EGH$ flanked $\angle =$	$98 \quad 21$ or angle of the curtain.
Add	$16 \quad 42 = \angle GHE$

$\angle AEG =$	$115 \quad 03$, the shoulder angle.
----------------	--------------------------------------

5. In the triangle EDG are given the angles EGD , $EDG = 81^{\circ} 39'$, and $33^{\circ} 24'$ (=twice $16^{\circ} 42'$) and side $DE=87.9$, to find the flank EG , or its equal FH .

As fine $\angle EGD=81^{\circ} 39'$	9.99537
To $DE=87.9$	1.94398
So is fine $\angle EDG=33^{\circ} 24'$	9.74074
To the flank $EG=48.9$	1.68935

6. In the oblique triangle AIH are given the angles and side AH , to find IH , or its equal GK , the lengthened curtain.

As fine $\angle AIH=126^{\circ}$	9.90796
To $AH=268.4$	2.42878
So is fine $\angle IAH=37^{\circ} 18'$	9.78246

To the lengthened curtain $IH=201$	2.30228
------------------------------------	---------

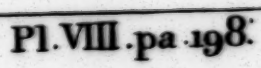
From IH , the lengthened curtain =	201
--------------------------------------	-----

Take GH , the curtain =	154.2
---------------------------	-------

IG , the demigorge =	46.8
the lengthened curtain =	201.

IK , the inner side =	247.8
-------------------------	-------

To



IX.

H=

angle

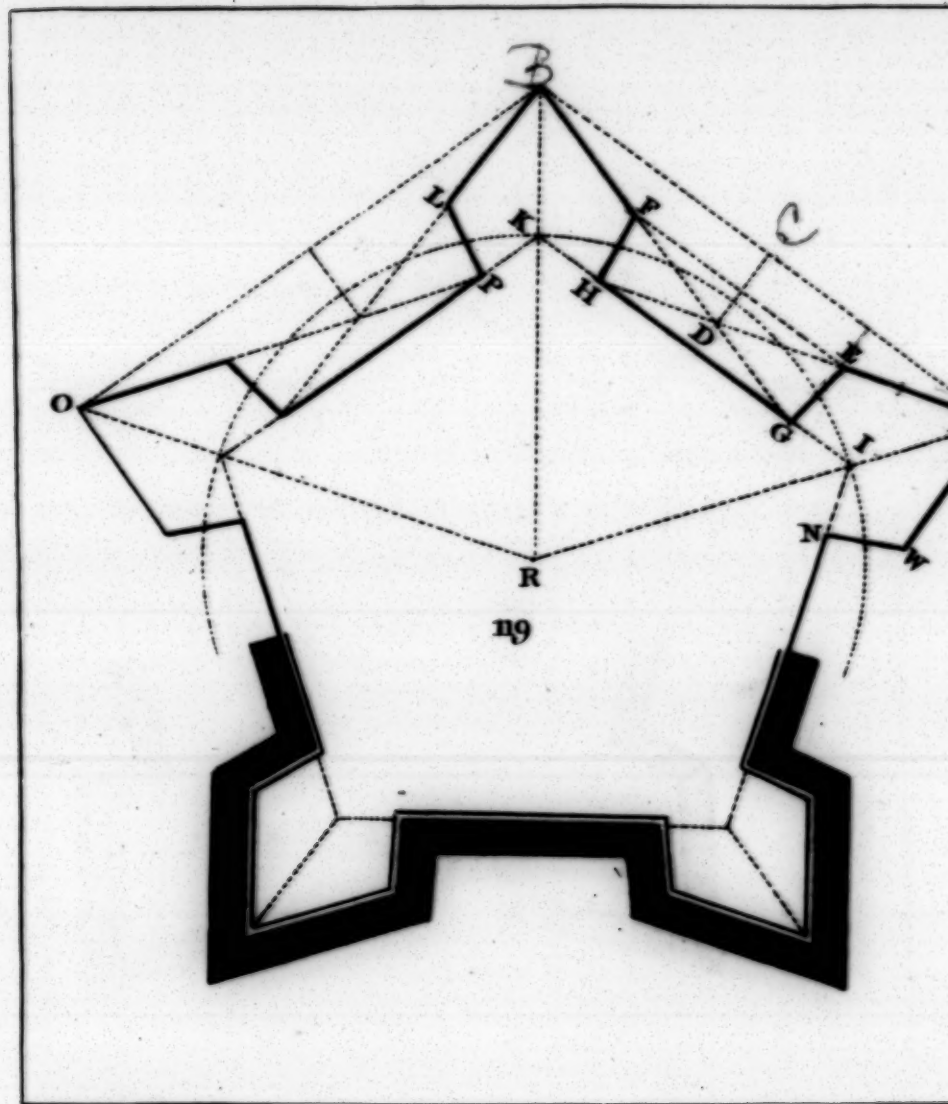
in.

DG

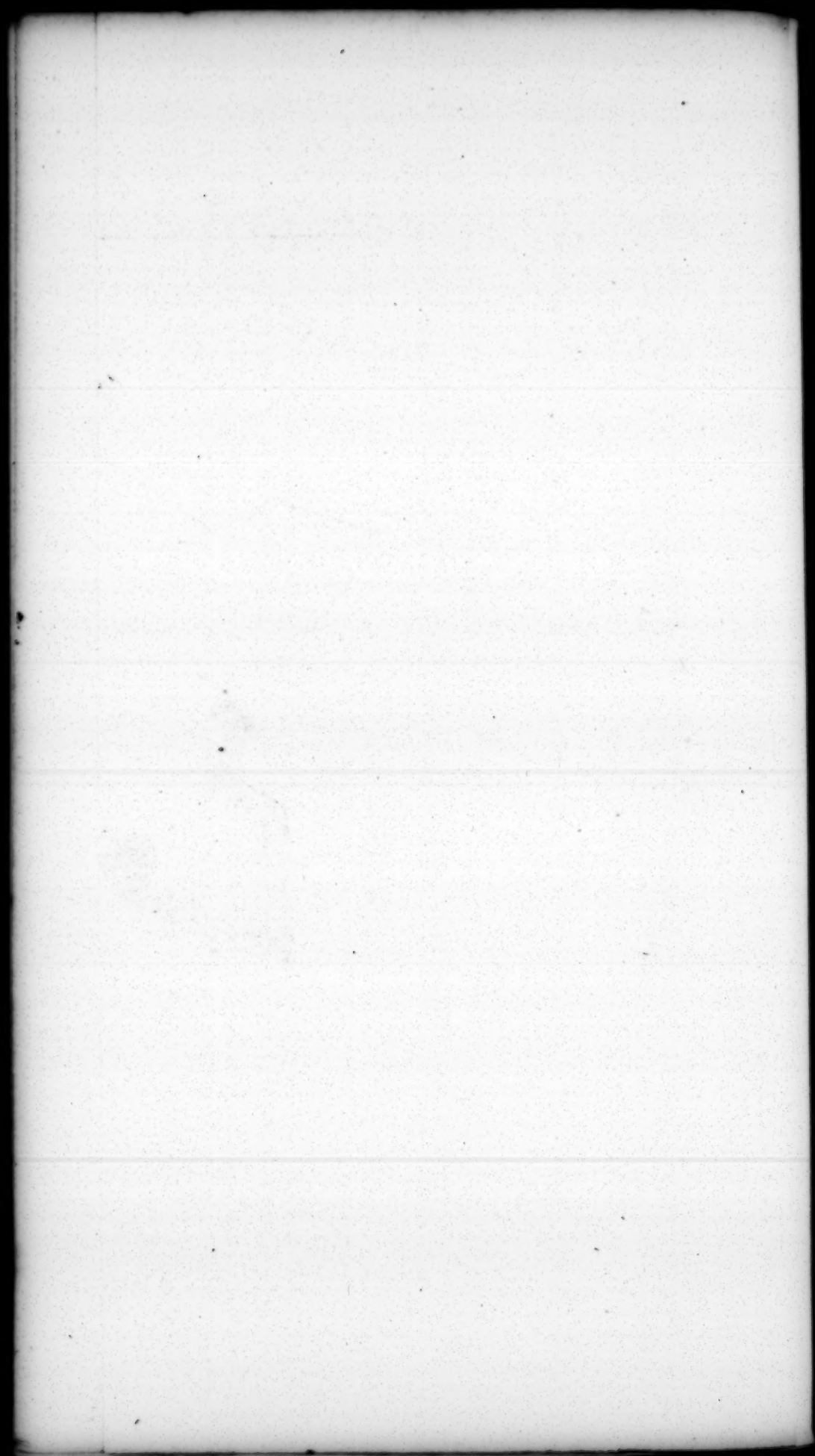
7-9,

and
rain.

To



Fortification.



To find the capital AI.

As sine $\angle AIH = 126^\circ$	9.90796
To AH = 268.4	2.42878
So is sine $\angle AHI = 16^\circ 42'$	9.45842
To capital AI = 95.34	1.97924
As sine $\angle R = 72^\circ$	9.97821
To IK = 247.8	2.39410
So is sine $\angle I = 126^\circ$ or $54^\circ \angle \text{pol.}$	9.90796
To KR = IR = 210.8 inner radius	2.32385

To find the length of the counterscarp aeh.

120.

The ditch being generally supposed 40 yards over, with the radius of 40 yards, describe the round part of the counterscarp before the salient angle A, of the bastion; and if the counterscarp be produced, it will meet the shoulder F; draw Ae perpendicular to ae, and join cF. Then,

In the triangle AEF, there are given the face AE = 100 yards, EF = EH = 168.4 yards, and the included angle AEF = $163^\circ 18'$ the supp. to $\angle HEF = 16^\circ 42'$ to find $\angle AFE$, and AF.

Sum $\angle s = 16^\circ 12'$	AE + EF = 268.4
	EF - AE = 68.4
Half sum = $8^\circ 21'$	
As sum sides = 268.4	2.42878
To their diff. = 68.4	1.83505
So is tangent half sum = $8^\circ 21'$	9.16665
To tangent half diff. = $2^\circ 08'$	8.57292
Whence $8^\circ 21' - 2^\circ 08' = 6^\circ 13' = \angle AFE$. Then	
As sine $\angle AFE = 6^\circ 13'$	9.03458
To AE = 100	2.00000
So is sine $\angle AEF = 163^\circ 18'$	9.45842
To AF = 205 yards	2.42384

FIG.
120.

In the right angled triangle AeF , right angled at e , there is given $AF=265$ yards, and $Ae=40$, to find eF , and angles eAF , AFe .

By *cor. to theo. 11. Geom.* $\sqrt{AF^2 - Ae^2} = eF = 261$ yards.

As $eF=261$ 2.41664

To $Ae=40$ 1.60206

So is radius 10.00000

To tangent $\angle AFe = 8^\circ 43'$ 9 18542

Whence $90^\circ - 8^\circ 43' = 81^\circ 17' = \angle eAF$.

To find a the length of the counterscarp.

$\angle AFE = 6^\circ 13'$

$\angle AFe = 8 43$

$\angle sFA = 14 56$

Also $sF = sE = \text{half } EF = 84.2$ yards, to find aF .

In the right angled triangle Fsa , there are given the angles and side, sF , to find aF and sa .

First $90^\circ - 14^\circ 56' = 75^\circ 04' = \angle saF = \angle ban$.

As sine $\angle saF = 75^\circ 4'$ 9.98507

To $sF = 84.2$ 1.92531

So is sine $\angle sFa = 14^\circ 45'$ 9.41110

To $sa = 22.4$ yards 1.35134

As sine $\angle saF = 75^\circ 4'$ 9.98507

To $sF = 84.2$ 1.92531

So is radius 10.00000

To $aF = 87.1$ yards 1.94024

From $eF = 261$

Take $aF = 87.1$

$ea = 173.9$ yards, for the straight part ea of the counterscarp.

To

To find the arch eb , half the circular part before the bastion.

120.

The outward angle bAC , in this case, is $= 108^\circ$, the angle of the polygon, and $\angle FAC = \angle AFE = 6^\circ 13'$ by *theo. 4. Geom.*

$$\text{To } \angle bAC = 108^\circ$$

$$\text{Add } \angle FAC = 6 \ 13'$$

$$\angle bAF = 114 \ 13$$

$$\text{Take } \angle eAF = 81 \ 17$$

$$\angle eAb \ 32 \ 56 = \text{arch } eb.$$

Or. 32.933° . then by *prop. 3. Trig.*

$$.01745$$

$$40 = \text{radius } Ae.$$

$$.69800$$

$$\text{Mult. } 32.933$$

$$\text{Prod. } 22.987234 \text{ yards} = 23 \text{ yards very nearly} = \text{arch } eb.$$

To find the several parts of the ravelin bnx .

The capital an , is supposed generally 100 yards, and the faces produced to meet those of the bastions in a point v , about 6 or 8 yards from the shoulder E .

In the right angled triangle EsD , there are given the sides $Es = sF = 84.2$, and $ED = 87.9$ yards, to find Ds .

$$\text{By cor. theo. 11. } \sqrt{ED^2 - Es^2} = Ds = 25.2 \text{ yards.}$$

$$\begin{array}{r} \text{yards} \\ \text{To } sa = 22.4 \\ \text{Add } sD = 25.2 \end{array}$$

$$\text{Da} = 47.6$$

$$\text{Add } an = 100$$

$$\text{Side } Dn = 147.6$$

$$\text{To } ED = 87.9$$

$$\text{Add } Ev = 6$$

$$Dv = 93.9$$

$$\text{From } 90^\circ - \angle CAD = 16^\circ 42' = \angle HEF, \text{ remains } 73^\circ 18' = \angle ADC = \angle vDC = \angle vDn.$$

In the triangle vDn , there are given $vD = 93.9$, $Dn = 147.6$ yards, and the included angle $vDn = 73^\circ 18'$ to find the $\angle vnd$.

From

FIG.
120.

From	180° 00'	$vD = 93.9$
Take	73 18	$Dn = 147.6$
	106 42	Sum = 241.5
Half sum	53 21	Diff. = 53.7
As sum sides = 241.5		2.38291
To their diff. = 53.7		1.72997
So tang. half sum $\angle s = 53^\circ 21'$		10.12841
To tang. half diff. = $16^\circ 38'$		9.47547
From $53^\circ 21'$		
Take 16 38		

$$\begin{array}{r} 36 \\ 2 \end{array} 43 = \angle vnd = \angle bna.$$

73 26 = $\angle bnx$, or the salient angle of the ravelin.

The $\angle sc F$ is equal to $\angle ban$, and has been found = $75^\circ 04'$.

In the triangle abn , given the side $an = 100$ yards, and the angles, to find bn , the face of the ravelin, and ba , the demi-gorge.

From	180° 00'	
Take $\left\{ \begin{array}{l} \angle ban = 75^\circ 4' \\ \angle bna = 36^\circ 43' \end{array} \right\}$	.111 47	
	68 13	
$\angle abn =$		
As sine $\angle abn = 68^\circ 13'$		9.96782
To $na = 100$		2.00000
So is $\angle ban = 75^\circ 4'$		9.98507
To $bn = 104$ the face of the ravelin		2.01725
As sine $\angle abn = 68^\circ 13'$		9.96782
To $na = 100$		2.00000
So is sine $\angle bna = 36^\circ 43'$		9.77659
To $ba = 64.38$ the demigorge.		1.80877

To

To find the counterscarp of the ditch before the ravelin.

Draw mn , rn , each equal to 24 yards, perpendicular to the counterscarps, and since they are parallel to the faces of the ravelin, they are also perpendicular to the faces; therefore the four angles at n , are equal to four right angles.

For $\angle bnr = 90^\circ$, and $\angle xnm = 90^\circ$, therefore $\angle rnm = (180^\circ - \angle bnx = 180^\circ - 73^\circ 26' =) 106^\circ 34' = 106.566^\circ = \text{arch } rm$; then by prop. 3. Trig.

$$\begin{array}{r} .01745 \\ 24 = nr, \text{ radius.} \end{array}$$

$$\begin{array}{r} \text{Mult. } .41880 \\ 106.566 \end{array}$$

Prod. = 44.6298408 yards = length of the arch rm .

Draw bd perpendicular to the counterscarp cr , then $dr =$ the face $bn = 104$ yards, and $\angle bcd = \angle abn = 68^\circ 13'$, and its complement is $\angle cbd = 21^\circ 47'$.

In the right angled triangle bdc , right angled at d , there are given the angles bcd , cbd , and side $ba = rn = 24$ yards, to find bc and cd .

As sine $\angle bcd = 68^\circ 13'$	9.96782
To $bd = 24$	1.38021
So is radius	10.00000
To $bc = 25.8$ yards	1.41239
As radius	10.00000
To $bc = 25.8$	1.41239
So is sine $\angle cbd = 21^\circ 47'$	9.56948
To $cd = 9.59$ yards	0.98187

To construct the orillons and retired flanks.

121.

Since the body of the place in the French fortifications is often made with *orillons*, and *retired flanks*, it will be proper to shew how they are constructed.

The *orillon* covers part of the flank, which, for that reason, is taken inwards, and therefore it is properly called *retired* or *covered flank*, and the line Hv or st , is called *retrenchment of the flank*, or *platform of the casemate*. The orillon is a round-
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FIG.

120.

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From	180° 00'
Take $\left\{ \begin{array}{l} \angle ban = 75^\circ 4' \\ \angle bna = 36^\circ 43' \end{array} \right\}$	.111 47
$\angle abn =$	68 13

As sine $\angle abn = 68^\circ 13'$	9.96782
-------------------------------------	---------

To $na = 100$	2.00000
---------------	---------

So is $\angle ban = 75^\circ 4'$	9.98507
----------------------------------	---------

To $bn = 104$ the face of the ravelin	2.01725
---------------------------------------	---------

As sine $\angle abn = 68^\circ 13'$	9.96782
-------------------------------------	---------

To $na = 100$	2.00000
---------------	---------

So is sine $\angle bna = 36^\circ 43'$	9.77659
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The *orillon* covers part of the flank, which, for that reason, is taken inwards, and therefore it is properly called *retired* or *covered flank*, and the line Hv or st , is called *retrenchment of the flank*, or *platform of the casemate*. The orillon is a round-
ing

FIG. ing of earth lined with a wall raised on the shoulder of those
121. bastions that have casemates to cover the cannon in the retired flank, and serve as a screen between the enemy's direct fire and a part of the flank, where the cannon placed there are loaded with grape or case-shot, to scour along the ditch.

To find the length of the arch vt, which forms the retired flank.

Find the flank FH, as has been directed = 48.9; then divide it into three equal parts, and make Hs equal to two of them, then sF the orillon will be one of those parts; from the flanked angle A, draw sA, and produce it to t, making st equal to 10 yards; the line of defence EH must be produced to v, making Hv = st = 10 yards, join tv and make thereon an equilateral triangle vrt, then r is the center from which the arch vt, the retired flank, is described.

The face AE = 100

EH = 168.4

AH = 268.4

In the triangle AHs there is given the side AH = 268.4, and Hs = (two thirds of the flank HF =) 32.6 yards, and the included $\angle AHS = \angle FGE = 81^\circ 39'$, to find the angles AsH, HAs, and side As.

AH = 268.4	From 180° 00'
Hs = 32.6	$\angle H = 81 \quad 39$
Sum 301.0	Sum $\angle s = 98 \quad 21$
Diff. 235.8	Half sum = 49 $10\frac{1}{2}$
As sum sides = 301	2.47856
To their diff. = 235.8	2.37254
So is tang $49^\circ 10\frac{1}{2}'$ half sum	10.06351
To tang. half diff. = $42^\circ 12'$	9.95749
Whence $\angle AsH = 91^\circ 22'$ and $\angle HAs = 6^\circ 59'$.	
As sine $\angle HAs = 6^\circ 59'$	9.08486
To Hs = 32.6 yards	1.51321
So is sine $\angle AHS = 81^\circ 39'$	9.99537
To As = 265.3	2.42372
Hence As + st = 275.3 = At	
And AH + Hv = 278.4 = Av.	

In

In the triangle vAt , there are given the sides Av , At , and included $\angle A = 6^\circ 59'$, to find the angle Avt , and side vt . FIG. 121.

From	$180^\circ 00'$	$275.3 = At$
Take $\angle A =$	$6 \quad 59$	$278.4 = Av$
Sum	<u>$173 \quad 01$</u>	<u>553.7, sum.</u>
half sum	$86 \quad 30\frac{1}{2}$	3.1 , diff.
As sum sides $= 553.7$		<u>2.74327</u>
To their diff. $= 3.1$		0.49136
So is tang. of $86^\circ 30\frac{1}{2}'$		<u>11.21455</u>
To tang. half diff. $= 5^\circ 14'$		8.96264
Hence $\angle Avt = 81^\circ 16'$, and $\angle Atv = 91^\circ 44'$.		
As sine $\angle Avt = 81^\circ 16'$		<u>9.99493</u>
To $At = 275.3$		2.43980
So is sine $\angle vAt = 6^\circ 59'$		<u>9.08486</u>
To $vt = 33.8$ yards		<u>1.52973</u>

But $vr = vt$ by construction, for vrt , is an equilateral triangle, and each angle $=$ to $60^\circ =$ arch vt ; therefore to find the retired flank vt , proceed thus by *prop. 3. Trig.*

Mult. $.01745$
by $33.8 = vr$ radius.

Prod. $.589810$
 60°

35.388600 yards, length of the arch vt , the retired flank.

To find the orillon sF.

Make Fx perpendicular to FB the face of the bastion, at F the end thereof; then bisect sF in z , and raise a perpendicular at z , and where it cuts Fx , the intersection x is the center to describe the orillon sF .

From $\angle BFH = 115^\circ 03'$, the shoulder angle
Take $\angle BFx = 90$

$\angle xFz = 25 \quad 03'$; hence $\angle zxF = 64^\circ 57'$

But

FIG.

But Fz is one third part of the flank $FH=16.3$ yards, its half $=8.15=Fz$.

In the right angled triangle Fzx , there are given all the angles and side Fz , to find Fx , the radius of the orillon.

$$\text{As sine } \angle xzF = 64^{\circ} 57' \quad 9.95709$$

$$\text{To } Fz = 8.15 \quad 0.91115$$

$$\text{So is radius} \quad 10.00000$$

$$\text{To } Fx = 9 \text{ yards nearly} \quad 0.95406$$

Whence twice $\angle xzF = 64^{\circ} 57'$ is $129^{\circ} 54' = 129.9^{\circ}$ the orillon's arch; then by *prop. 3. Trig.*

$$\begin{array}{r} .01745 \\ 99 \text{ degrees,} = Fx \text{ radius} \end{array}$$

$$\begin{array}{r} .15705 \\ 129.9 \text{ degrees} \end{array}$$

Prod. 20.400795 yards, the length of the arch sF , or the orillon.

To fortify a regular hexagon.

120.

P R O B. III.

122.

Given the outward side, normal, and face of a regular hexagon, to fortify it.

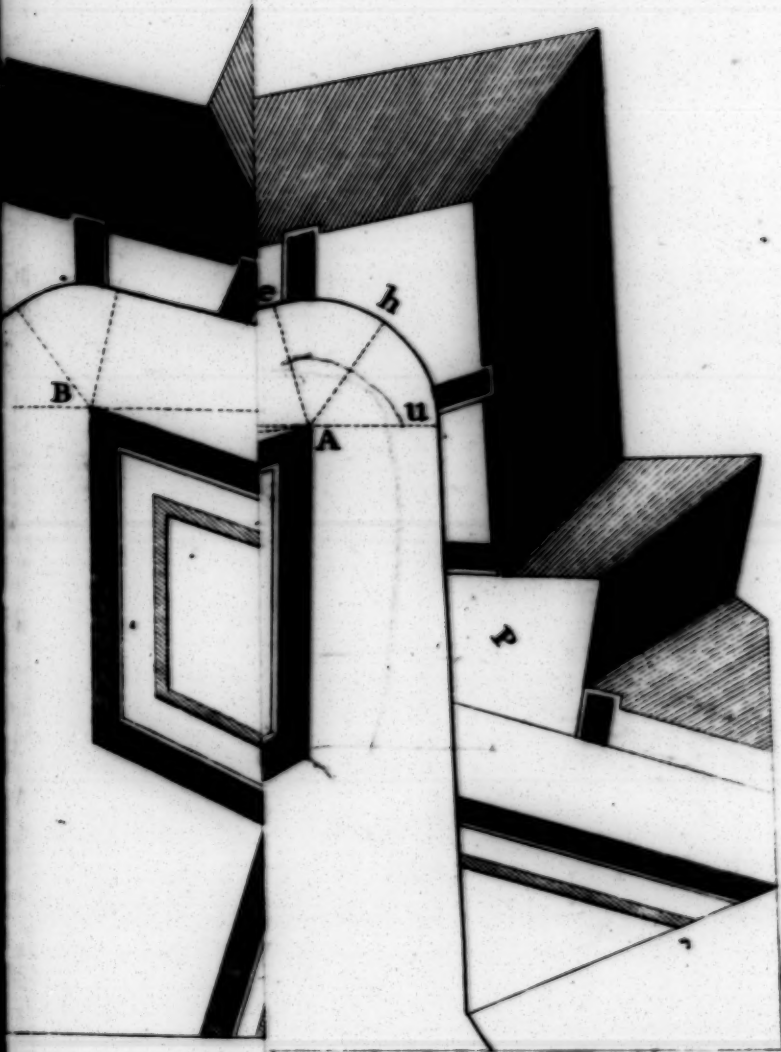
Example.

Let the exterior side be 360 yards, the normal 60 yards, and face 100 yards, to find the rest.

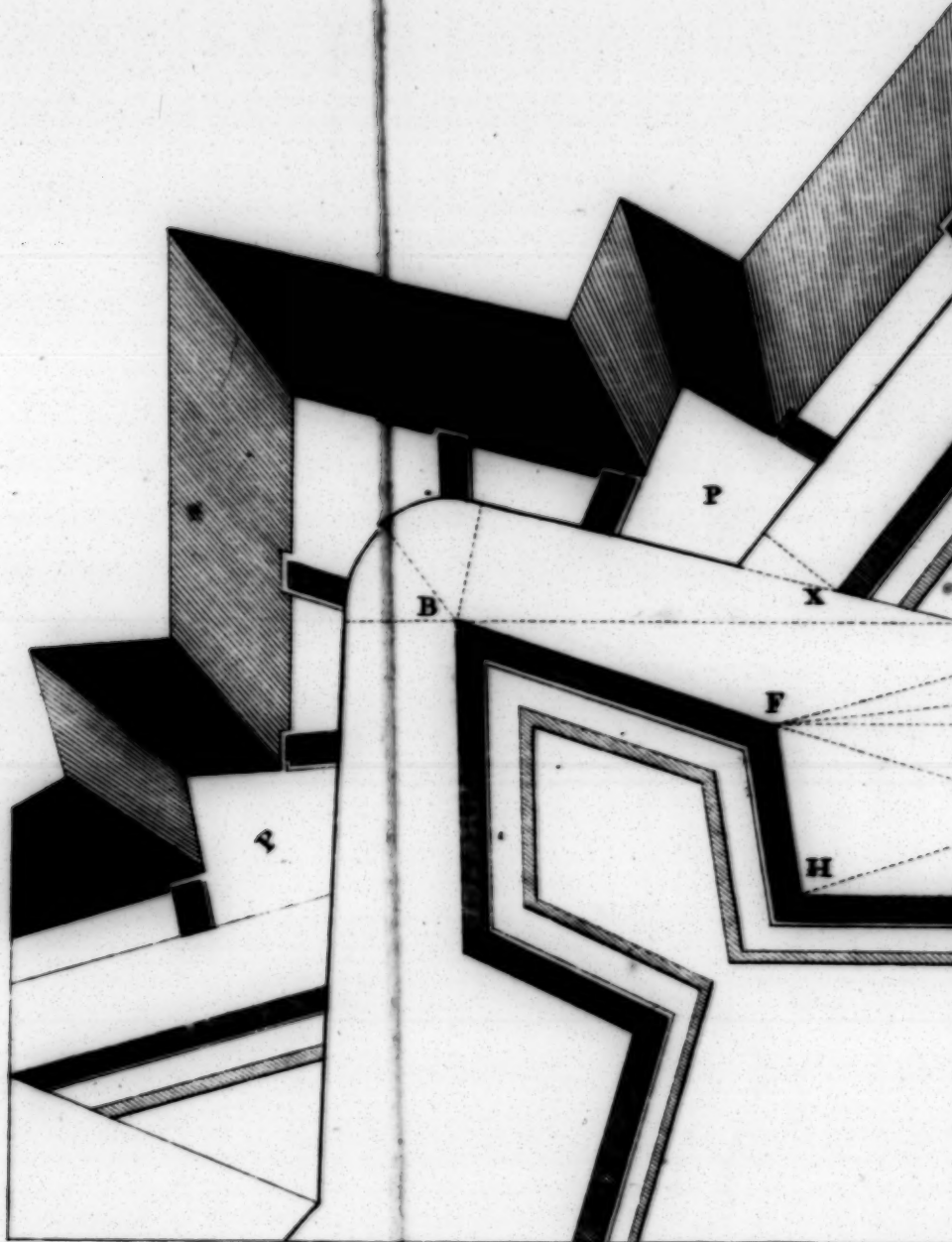
Geometrical construction.

With the exterior side as a radius $= 360$ yards, describe a circle; then take the outward side $AB = 360$ yards, and apply it to the circumference six times, then bisect AB in C , and on C make the normal $CD = 60$ yards, perpendicular to AB , from A and B , through D , draw the lines ADH , BDG the lines of defence, set off the faces AE , BF , each equal to 100 yards, and draw EF ; make EH , and FG each equal to EF , and draw the curtain GH , and the flanks EG , FH , you have then one front completed: proceed thus round the polygon, and the hexagon will be fortified.

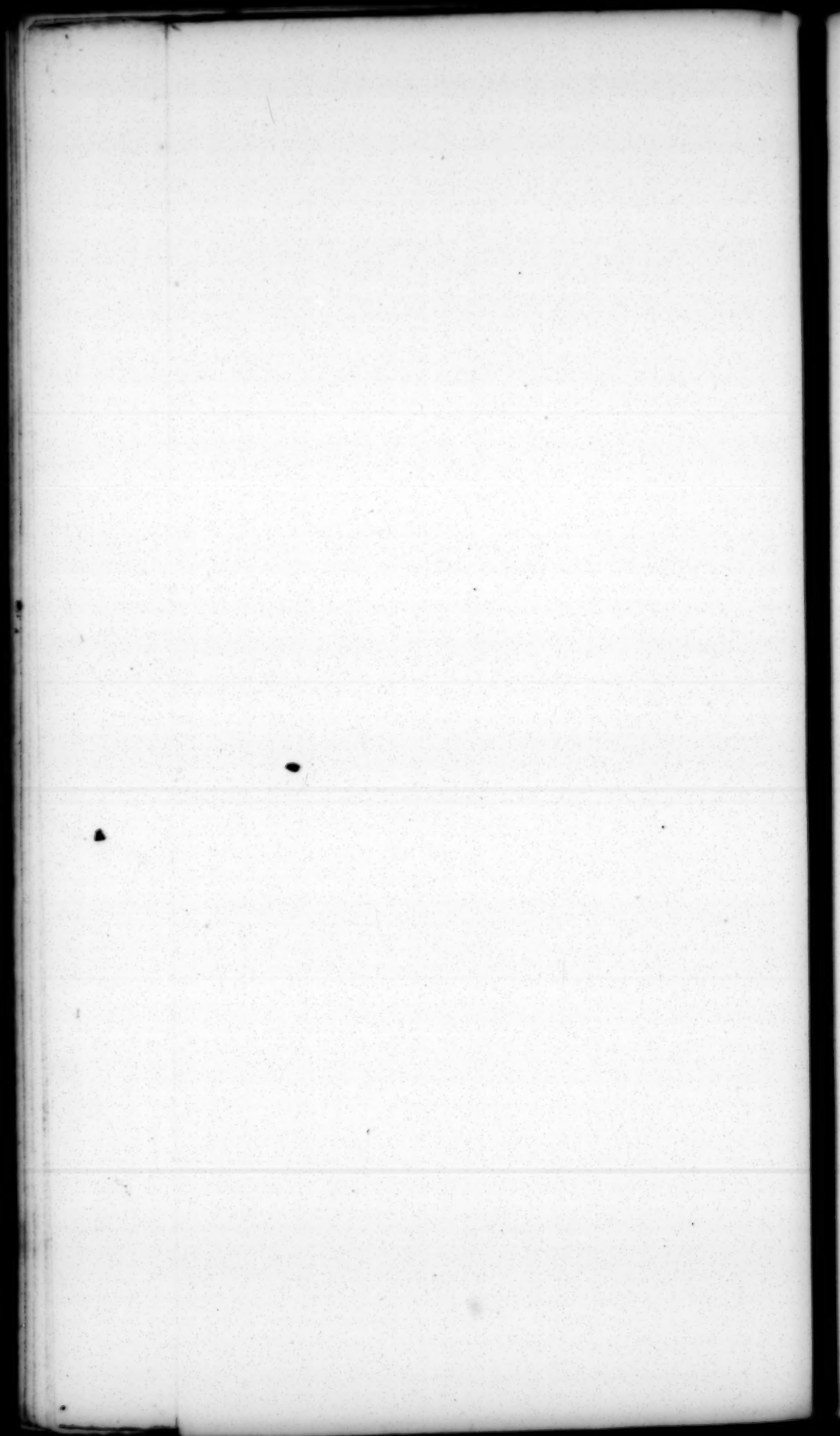
Or



Pl. IX. pa. 206



Fortification.



Or thus, after Mr. Vauban's manner of fortifying.

120.

122.

Divide the outward side AB into seven equal parts, and allow two of them for the faces AE, BF, and make the lines EH, FG, each equal to the line EF, to have the curtain GH, and the flanks EG, FH; and if the like be done round the hexagon, you will have the master-lines of the curtains and bastions, to which in the inside must be added a rampart about 24 yards broad, which its parapet of 6; and on the out-side a ditch 32 or 36 yards broad over-against the flanked angle and 40 yards broad over-against the shoulder. Not forgetting a covered way of 10 yards in breadth, and an esplanade or glacis of 40.

Calculation.

6)360°(60°, angle of the center.

Add 120, angle of the polygon.

The radius of a circle circumscribing a regular hexagon, is always the length of the side of that hexagon. For

As sine 60° : 360 yards :: (sine $\frac{120^\circ}{2}$ =) 60° : 360 yards, the radius of the exterior side.

In the right angled triangle ACD, there are given AC, =180 yards, and CD=60, to find the diminished angle CAD, or its equal CBD.

As AC=180

2.25527

To CD=60

1.77815

So is radius

10.00000

To tang. $\angle CAD = 18^\circ 26'$

9.52288

The angles CAD, and CBD are each = $18^\circ 26'$ their sum = $36^\circ 52'$ taken from 180° leaves $143^\circ 8' =$ the exterior flanking angle, viz. ADB=EDF or angle of the tenaille.

From $\angle RAB$, half the angle of the polygon = $60^\circ 00'$

Take $\angle CAD$, the diminished angle = $18^\circ 26'$

$\angle RAH$, half the flanked angle

41 34

doubled is $\angle EAM$, the flanked angle or angle of the bastion = $83^\circ 08'$. And by *cor. theo. 2. Geom.* the hypotenuse AD = 189.73 yards; from which take the face AE = 100, leaves ED = 89.73 yards.

In

Fig.

In the triangle EDF, there are given the angles $\angle EFD = 18^\circ 26'$, $\angle EDF = 143^\circ 8'$ and the side $DE = 89.73$ yards, to find EF, or its equal EH.

$$\text{As sine } \angle EFD = 18^\circ 26' \quad 9.49996$$

$$\text{To } DE = 89.73 \quad 1.95293$$

$$\text{So is sine } \angle EDF = 143^\circ 8' \quad 9.77811$$

$$\text{To } EF = EH = 170.2 \quad 2.23108$$

$$\text{To } EH = 170.2 \text{ yards}$$

$$\text{Add } AE = 100$$

$$AH = 270.2, \text{ the line of defence} = BG.$$

$$\text{From } EH = 170.2$$

$$\text{Take } DE = 89.73$$

$$DH = 80.47 = DG.$$

In the isosceles triangle GDH, there are given the angles $\angle DGH$, and $\angle GDH = 18^\circ 26'$, and $143^\circ 8'$, also the side $DH = 80.47$, to find the curtain GH.

$$\text{As sine } \angle DGH = 18^\circ 26' \quad 9.49996$$

$$\text{To } DH = 80.47 \quad 1.90563$$

$$\text{So is sine } \angle GDH = 143^\circ 8' \quad 9.77811$$

$$\text{To the curtain } GH = 152.6 \text{ yards} \quad 2.18378$$

$$\text{From } 180^\circ 00'$$

$$\text{Take } 18 \ 26 = \angle EFG$$

$$161 \ 34$$

$$80 \ 47 = \angle EGF$$

$$\text{Add } 18 \ 26 = \angle FGH$$

$$\angle EGH = 99 \ 13, \text{ the angle of the curtain,}$$

$$\text{Add } 18 \ 26$$

$$\angle AEG = 117 \ 39, \text{ the shoulder angle.}$$

In the triangle EDG are given the angles $\angle EGD$, $\angle EDG = 80^\circ 47'$, and $36^\circ 52'$, also the side $DE = 89.73$, to find the flank EG.

As

As sine $\angle EGD = 80^\circ 47'$ 9.99435

To $DE = 89.73$ 1.95293

So is sine $\angle EDG = 36^\circ 25'$ 1.77811

To the flank $EG = 54.54$ yards 1.73669

In the triangle AIH , are given the angles $AIH = 120^\circ$, $IAH = \angle RAB - CAD = 41^\circ 34'$, and side $AH = 270.2$, to find the lengthened curtain IH , or its equal GK .

As sine $\angle I = 120^\circ$ 9.93753

To $AH = 270.2$ 2.43168

So is $IAH = 41^\circ 34'$ 9.82183

To the lengthened curtain $IH = 207$ yards 2.31598

From $IH = 207$

Take $GH = 152.8$

Demigorge $IG = 54.2$

To $IH = 270$

Add $IG = 54.2$

$IK = 261.2$ yards, the inner side.

To find the capital AI .

As sine $\angle I = 102^\circ$ 9.93753

To $AH = 270.2$ yards 2.43168

So is sine $\angle AHI = 18^\circ 26'$ 9.49996

To the capital $AI = 98.6$ yards 1.99411

The inner radius $= 261.2$ yards is always equal to the inward side, in a regular hexagon.

If my reader would find the length of the counterscarp aeb , the circular part before the bastion, the several parts of the ravelin, the counterscarp of the ditch before the ravelin, the retired flank, and the orillon for a hexagon, &c. he must proceed in the same way as is shewn before in the pentagon, only remembering to take its proper line of defence, diminished angle according to the proposed polygon; for *fig. 120.* represents a part of any regular polygon. I shall here set down the results of the same for the hexagon, and leave the operation for the learner's exercise.

FIG. 210

FORTIFICATION.

P. IX.

120.

Of the several parts of the counterscarp aeb.

121.

yards

Line AF = 267

Given Ae = 40

eF = 264

sF = 85.1

aF = 88.27

ae = 157.7

Arch eb = 31.7

Angles AFE = 6° 48'

∠AEF = 161 34

∠eAF = 81 23

∠sFa = 15 25

∠saF = 74 35

∠baF = 126 48

Arch eb = 45 25

Of the several parts of the ravelin bnx.

yards

sa = 23.4

Ds = 28.4

Da = 51.8

Dn = 151.8

Dv = 95.73

bn = 103.5

ba = 64.26

∠ADC = 71° 34' = vDn

∠Dnv = 36 46 = ∠bna

∠bnx = 73 32

∠saF = 74 35 = ∠ban

∠abn = 68 39 = ∠bcd

Of the counterscarp of the ditch before the ravelin.

yards

bc = 25.76

cd = 9.38

Arch mr = 44.58, &c.

∠vnm = 106° 28'

∠bnx = 73 32

∠bcd = 68 39

∠cbd = 21 21

Of the retired flank.

yards

As = 266.8

At = 276.8

Av = 280.2

Chord vt = 37.97

Arch vt = 39.75, retired flank.

st = Hv = 10 given

Hs = 36 36

∠AsH = 91° 29'

∠HAs = 7 44

∠Avt = 80 59

∠Atv = 91 27

Of the orillon sF.

Chord sF = 18.18

Fz = 9.09

Fx = 10.26

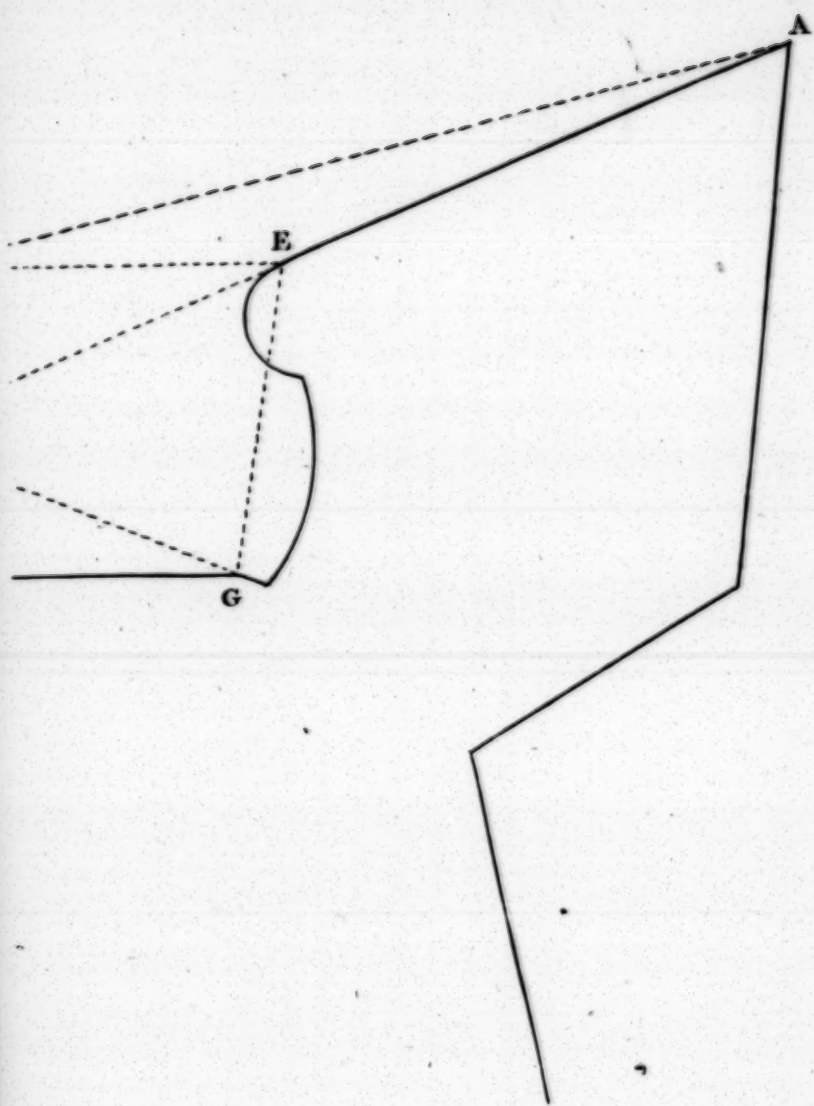
Orillon sF = 22.32, &c.

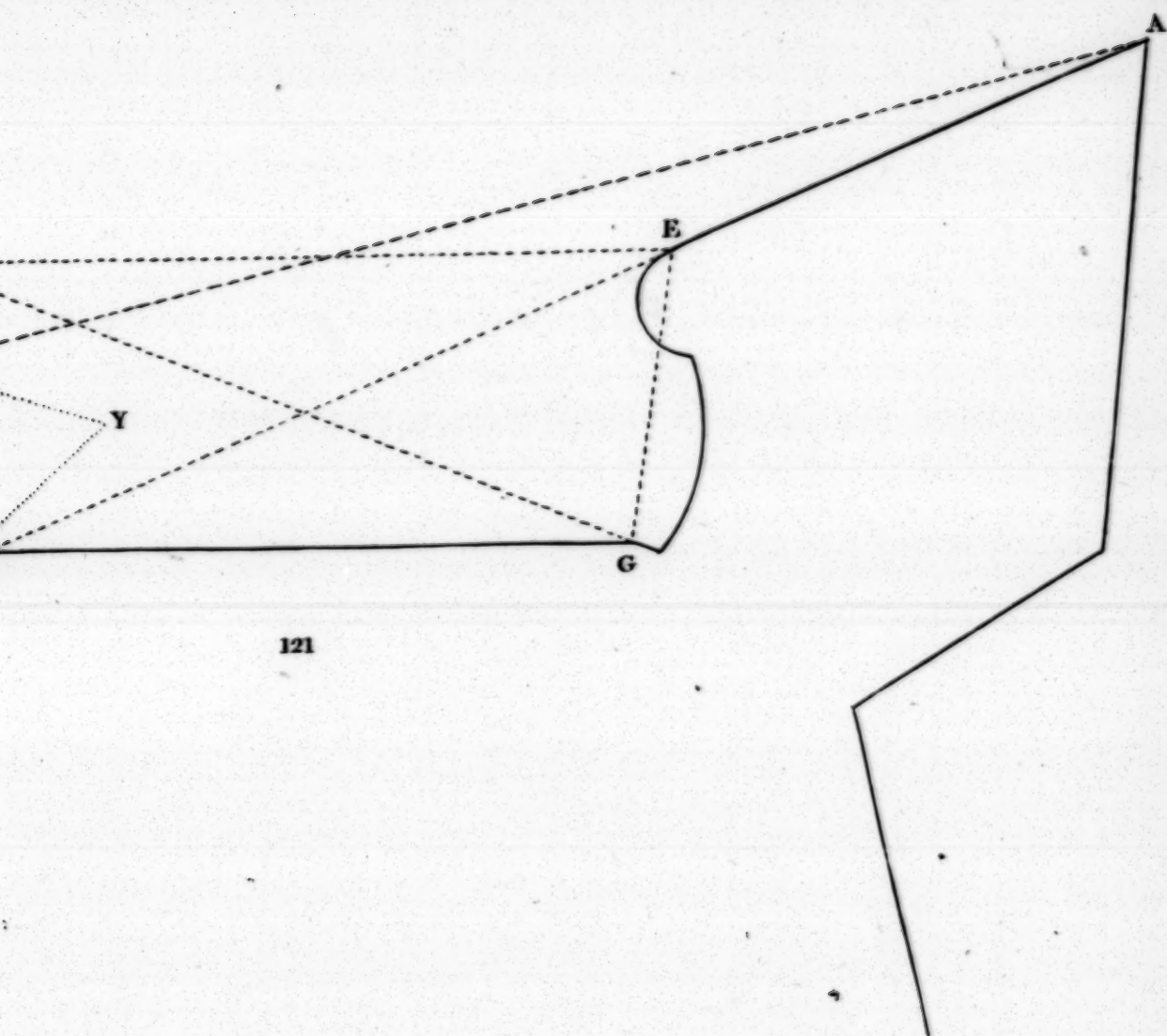
∠xFz = 27° 39'

∠zxF = 62 21

Arch sF = 124 42

These





121

Pl.X. pa. 210.

P. 12

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P.IX. FORTIFICATION.

211

FIG.

These are the lines and angles of the several parts of the counterscarp, ravelin, counterscarp of the ditch before the ravelin, the retired flank, and orillon for a fortified regular hexagon, which I calculated from trigonometry, in the same manner as I have done in the pentagon; and I have here inserted them to assist the reader in calculating the same.

To fortify a regular octagon.

121.

PROB. IV.

123.

Given the outward side, face, and normal of a regular octagon, to fortify it.

Example.

Let the outward side be 360 yards, the face 100, and the normal 68 yards, to find the rest.

Geometrical construction.

With the outward radius=470.3 yards describe a circle; apply to its circumference the given side AB=360 yards, eight times, and join A, B, &c. and you will have a regular octagon; bisect AB in C, and make the normal CD=68 yards, perpendicular to AB; from A, B, and through D, draw ADH, BDG, the lines of defence; set off the faces AE, BF, each equal to 100 yards, and draw EF; make EH and FG each equal to EF, and draw the curtain GH, also the flanks EG, FH, and you have a front completed: proceed thus round the polygon, and the octagon will be fortified.

Calculation.

8)360°(45°, the angle of the center.
And 180°-45°=135°, the angle of the polygon.

Half= 67 30'

As sine ∠ARB=45°

9.84948

To AB=360 yards

2.55630

So is sine ∠ABR=67° 30'

9.96561

To AR the exterior radius=470.3 2.67243

In the right angled triangle ACD, there are given AC=180 yards, and CD=68, to find the diminished angle CAD.

FIG.

120.

123.

As AC=180 yards

2.25527

To CD=68

1.83251

So is radius

10.00000

To tang. $\angle CAD=20^{\circ} 42'$

9.57724

From $\angle BAR=67^{\circ} 30'$ Take $\angle CAD=20^{\circ} 42'$

$\angle IAH = 46^{\circ} 48'$, half the flanked angle, whence the flanked angle, or angle of the bastion $EAM=93^{\circ} 36'$.

The $\angle CAD=\angle CBD$ by *theo. 4. Geom.* each equal to $20^{\circ} 42'$, whence $\angle ADB=\angle EDF=138^{\circ} 36'$, the exterior flanking angle, or angle of the tenaille.

By *Cor. theo. 11. Geom.* the hypotenuse $AD=192.4$ yards, from which take the face $AE=100$, remains $ED=92.4$.

In the triangle EDF , there are given the angle. $EFD=20^{\circ} 42'$, $\angle EDF=138^{\circ} 36'$ and side DE , to find EF , or its equal EH .

As sine $\angle EFD=20^{\circ} 42'$

9.54835

To $ED=92.4$

1.96567

So is $\angle EDF=138^{\circ} 36'$

9.82040

To EF , or $EH=172.9$

2.23772

To $EH=172.9$ Add $AE=100$

 $AH=272.9$, line of defence.
From $EH=172.9$ Take $DD=92.4$

 DH or $DG=80.5$

In the isosceles triangle GDH , there are given the $\angle DGH=20^{\circ} 42'$, $\angle GDH=138^{\circ} 36'$, and side $DH=80.5$, to find the curtain GH .

As sine $\angle DGH=20^{\circ} 42'$

9.54835

To $DH=80.5$

1.90579

So is sine $\angle GDH=138^{\circ} 36'$

9.82040

To the curtain $GH=150.5$ yards

2.17784

From

$$\begin{array}{r} \text{From} \quad 180^{\circ} 00' \\ \text{Take} \quad 20 \quad 42 \\ \hline 159 \quad 18 \end{array}$$

$$\begin{array}{r} \text{Half} = \quad 79 \quad 39 = \angle \text{EGD.} \\ \text{Add} \quad 20 \quad 42 = \angle \text{FGH.} \\ \hline \end{array}$$

$$\begin{array}{r} \angle \text{EGH} = 100 \quad 21, \text{ the angle of the curtain.} \\ \text{Add} \quad 20 \quad 42 \\ \hline \end{array}$$

$$\angle \text{AEG} = 121 \quad 03, \text{ the shoulder angle.}$$

In the triangle EDG, are given $\angle \text{EDG} = 41^{\circ} 24'$, $\angle \text{EGD} = 79^{\circ} 39'$, and side $\text{DE} = 92.4$, to find the flank EG.

$$\begin{array}{r} \text{As sine } \angle \text{EGD} = 79^{\circ} 39' \quad 9.99287 \\ \hline \end{array}$$

$$\begin{array}{r} \text{To } \text{ED} = 92.4 \quad 1.96567 \\ \hline \end{array}$$

$$\begin{array}{r} \text{So is sine } \angle \text{EDG} = 41^{\circ} 24' \quad 9.82040 \\ \hline \end{array}$$

$$\begin{array}{r} \text{To the flank } \text{EG} = 62.1 \quad 1.79320 \end{array}$$

To find the lengthened curtain IH.

In the triangle AIH, are given $\angle \text{AIH} = 112^{\circ} 30'$, $\angle \text{IAH} = 46^{\circ} 48'$, to find IH the lengthened curtain.

$$\begin{array}{r} \text{As sine } \angle \text{AIH} = 112^{\circ} 30' \quad 9.96561 \\ \hline \end{array}$$

$$\begin{array}{r} \text{To } \text{AH} = 272.9 \quad 2.43600 \\ \hline \end{array}$$

$$\begin{array}{r} \text{So is sine } \angle \text{IAH} = 46^{\circ} 48' \quad 9.86270 \\ \hline \end{array}$$

$$\text{To the lengthened curtain } \text{IH} = 215.3 \quad 2.33309$$

From IH = 215.3, the lengthened curtain

Take GH = 150.5, the curtain.

$$\text{IG} = 64.8 = \text{demigorge.}$$

$$\text{To } \text{IH} = 215.3$$

$$\text{Add } \text{IG} = 64.8 = \text{HK}$$

$$\text{IK} = 280.2 = \text{the inward side.}$$

$$\begin{array}{r} \text{And as sine } \angle \text{AIH} = 112^{\circ} 30' \quad 9.96561 \\ \hline \end{array}$$

$$\begin{array}{r} \text{To } \text{AH} = 272.9 \quad 2.43600 \\ \hline \end{array}$$

$$\begin{array}{r} \text{So is sine } \angle \text{AHI} = 20^{\circ} 42' \quad 9.54835 \\ \hline \end{array}$$

$$\begin{array}{r} \text{To the capital } \text{AI} = 104.4 \quad 2.01874 \end{array}$$

As sine $\angle R = 45^\circ$ 9.84948

To $IK = 280.1$ 2.44231

So is sine $\angle RIK = 67^\circ 30'$ 9.96561

To the inner radius $KR = 366$ nearly 2.56344

The following table is computed in the same manner as the foregoing, where you have in one view the chief lines and angles of all regular polygons, from the square to the decagon, according to the maxims of good fortification. The use of this table is great, for as the lines and angles are all given from a square to a decagon, so it will be easy to draw the master lines of the same, from a scale of equal parts and protractor.

A TABLE of the lines and angles of a fortified polygon from the square to the decagon, calculated from the outward side, face, and normal, according to marshall de Vauban's measures.

Names of the lines and angles.	IV. Square.		V. Pentagon.		VI. Hexagon.		VII. Heptagon.		VIII. Octagon.		IX. Nonagon.		X. Decagon.	
	Yards.	Feet.	Yards.	Feet.	Yards.	Feet.	Yards.	Feet.	Yards.	Feet.	Yards.	Feet.	Yards.	Feet.
Face.														
Normal.	45	54	54	64	64	76	64	76	63	72	72	86	78	92
Outward side.	360	360	360	360	360	360	360	360	360	360	360	360	360	360
Line of defence.	265.9	268.4	268.4	270.2	270.2	271.6	271.6	272.8	272.8	274.3	274.3	276.4	276.4	276.4
Cur aim.	156	154.2	154.2	152.8	152.8	151.8	151.8	150.5	150.5	149.4	149.4	147.3	147.3	147.3
Plank.	40.5	48.9	48.9	54.5	54.5	58.3	58.3	62.1	62.1	65.9	65.9	71.6	71.6	71.6
Demigorge.	37.4	46.8	46.8	54.2	54.2	60.3	60.3	64.7	64.7	68.2	68.2	70.5	70.5	70.5
Capital.	91.2	95.3	95.3	98.6	98.6	100.9	100.9	104.4	104.4	108.4	108.4	115.6	115.6	115.6
Inward side.	211	247.9	247.9	261.2	261.2	272.4	272.4	280	280	285.8	285.8	288.5	288.5	288.5
Radius of outward side.	254.5	306.2	306.2	360	360	414.8	414.8	470.3	470.3	526.2	526.2	582.5	582.5	582.5
Radius of inward side.	163.3	210.9	210.9	261.2	261.2	313.9	313.9	365.9	365.9	417.8	417.8	466.9	466.9	466.9
Angle of the center.	90°	72°	72°	60°	60°	51° 26'	51° 26'	45°	45°	40°	40°	36°	36°	36°
Angle of the polygon.	90	108	108	120	120	128 34	128 34	135	135	140	140	144	144	144
Angle of the curtain.	97 1'	98 21'	98 21'	99 13'	99 13'	99 47	99 47	100 21'	100 21'	100 54'	100 54'	101 43'	101 43'	101 43'
Angle of the shoulder.	111 3	115 3	115 3	117 39	117 39	119 21	119 21	121 3	121 3	122 42	122 42	125 9	125 9	125 9
Ang. of bastion or flanked ang.	61 56	74 36	74 36	83 8	83 8	89 26	89 26	93 36	93 36	96 24	96 24	97 8	97 8	97 8
Diminished angle.	14 2	16 42	16 42	18 26	18 26	19 34	19 34	20 42	20 42	21 48	21 48	23 26	23 26	23 26
Outward flanking angle.	151 56	146 36	146 36	143 8	143 8	140 32	140 32	138 36	138 36	136 24	136 24	133 8	133 8	133 8

A TABLE of Mr. de Vauban's Method of fortifying.

Of the body of the place.	{	Base of the rampart,	25 or 30 yards
		Base of the parapet,	6
		Breadth of the ditch,	40
Of the tenaille.	{	Distance from the orillon of the bastion,	6 yards
		Base of the rampart of the the face and flank,	14
		Base of the rampart of the curtain,	10
		Base of the parapet,	6
Of the Half-moon.	{	Base of the rampart,	20 yards
		Base of the parapet,	6
		Breadth of the ditch,	24
Of the covered way, traverses and places of arms.	{	Breadth of the covered way,	10 yards
		Demigorge of the places of arms at the re-entrant angles,	20
		Faces of the places of arms,	24
		Length of the traverses at the re-entrant angles,	10
		Length of the traverses at the saliant angles,	9
		Base of the traverses,	6

PROBLEM.

To draw the profile of a regular fortification, or to lay down the heights and breadths of all the parts of such a work from the maxims of good fortification.

1. The profile of the rampart, ditch, covered way, and glacis.

First draw the ground line or level of the field AR, then from a scale of equal parts,

Make AH = 30 yards, the base of the rampart.

HM = 40 yards, the breadth of the ditch.

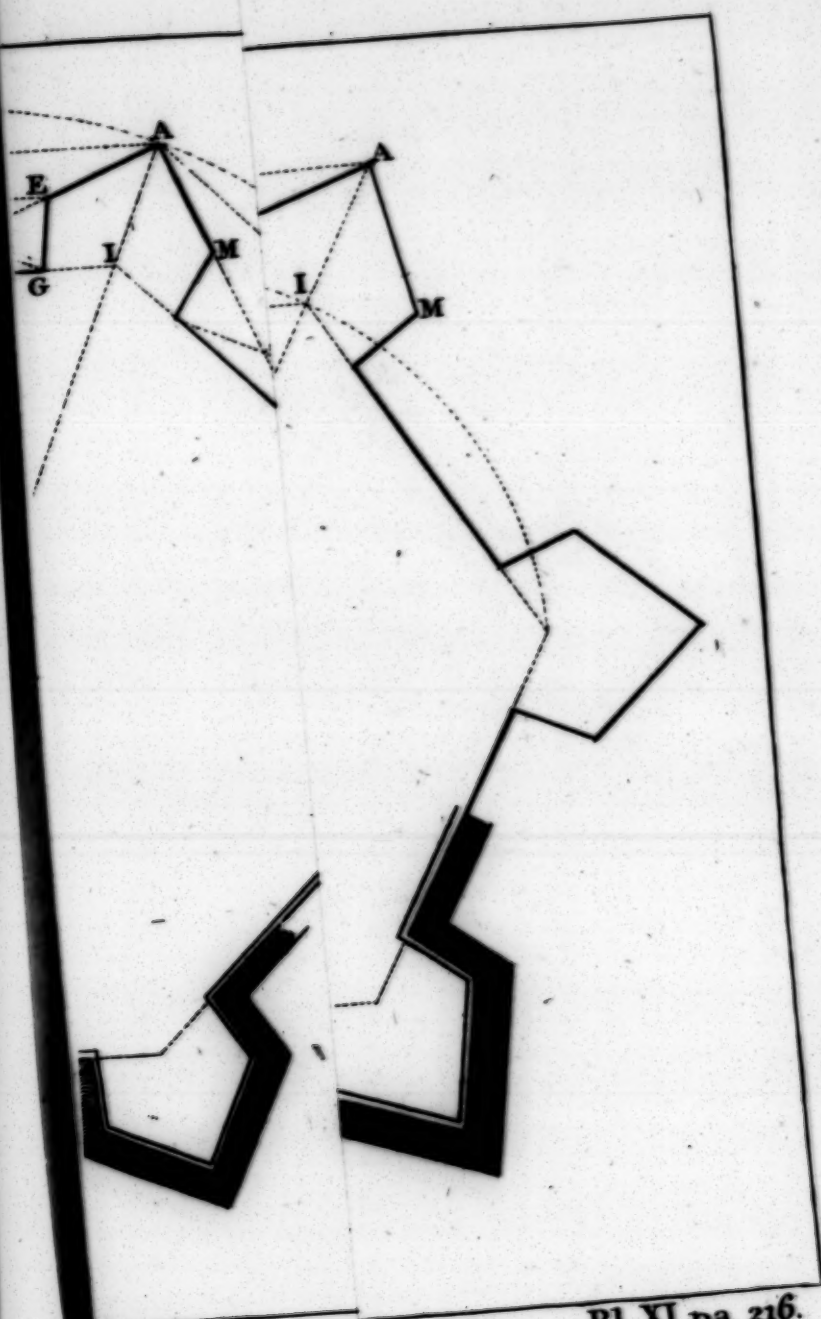
MP = 10 yards, the covered way.

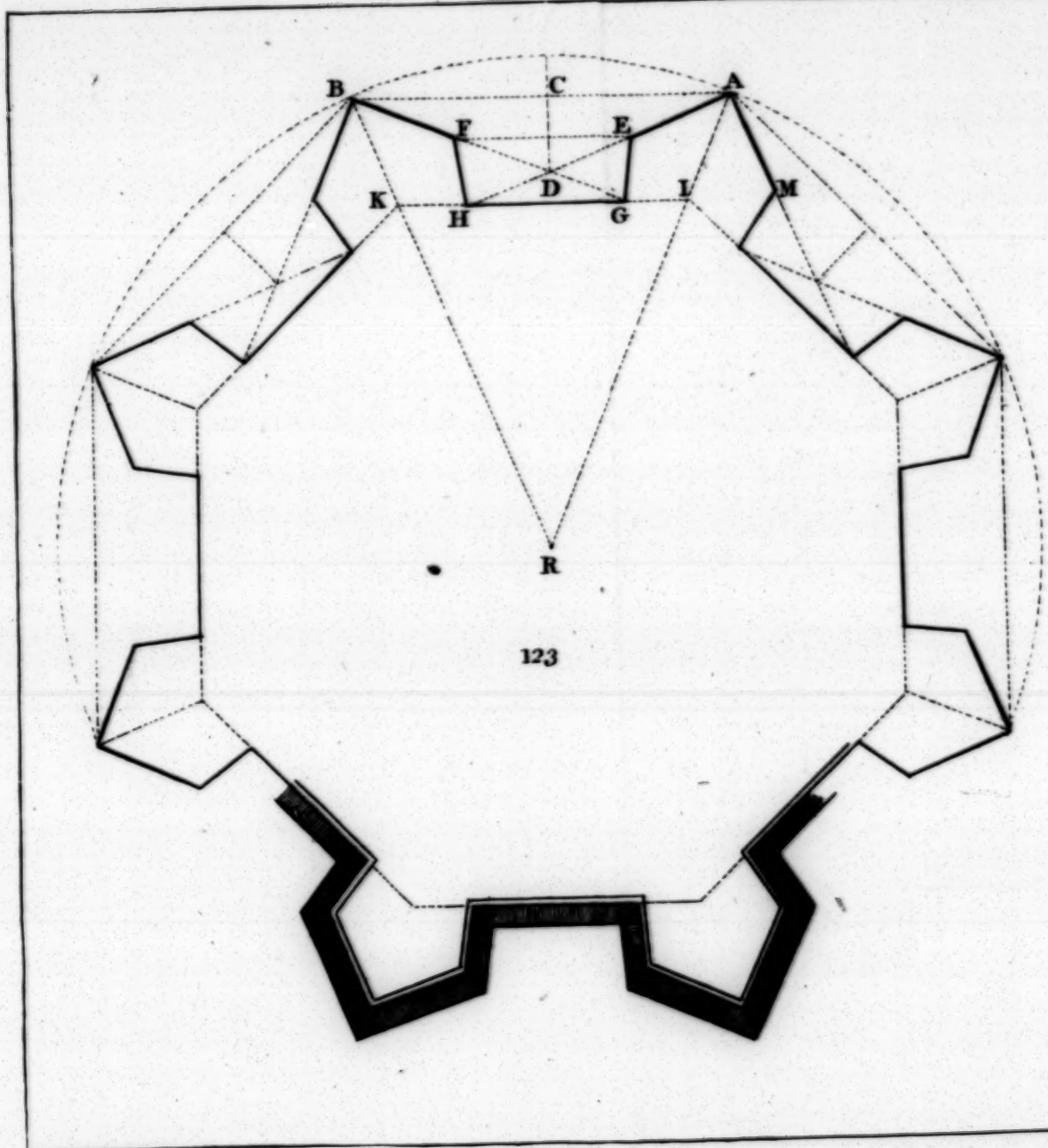
PR = 40 yards, the breadth of the glacis.

HI = $1\frac{1}{2}$ the berme, or fore land.

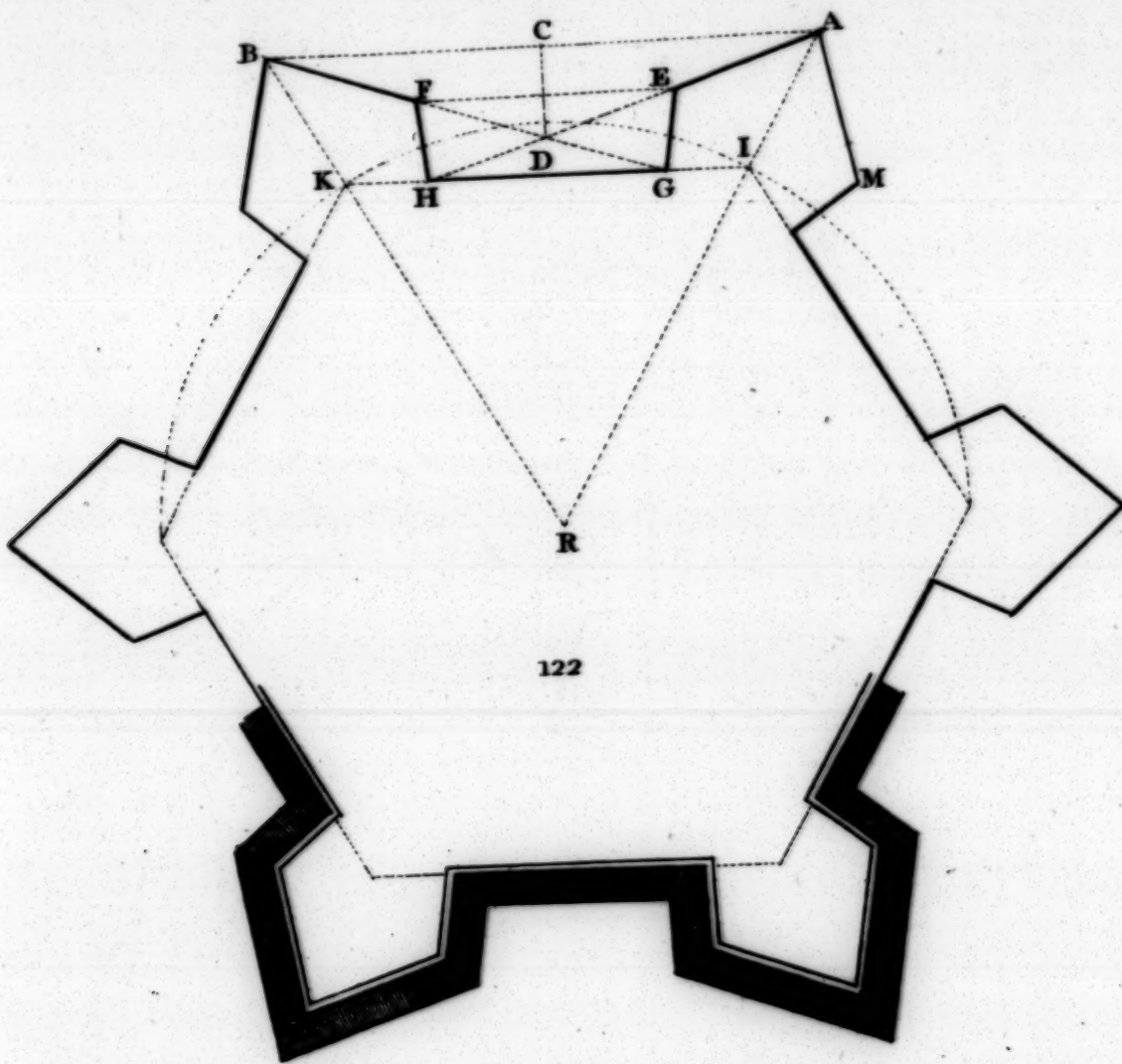
Then from the points A, H, I, M, P, draw as many perpendiculars to the line AR, and

Make





Fortification.



122

Make AS, HG=6 yards, the height of the rampart.

IK, MO=5 yards, the depth of the ditch.

PQ=2 yards, the height of the esplanade or glacis.

Draw QR, the glacis, which insensibly loses itself towards the field for 30 or 40 yards, also SG, and KO. Then

Make SB=6 yards, the inward slope } of the rampart.

GT=6 yards, the outward slope }

FT=6 yards, the base of the parapet.

KL=5 yards, the inward slope } of the ditch.

NO=5 yards, the outward slope }

From F raise the perpendicular FD, and make it equal to 124. 2 yards for the height of the parapet, also at P raise PQ perpendicular equal to 2 yards, for that of the glacis; add a banquette or foot-bank to equal parapet, at least 2 feet high, and a yard broad. DE the glacis of the parapet is drawn to the point of the counterscarp, and ends where HT produced cuts DM.

II. Another sort of profile with a *false braye*.

125.

Draw the profile of the rampart, ditch, covered way, as before directed. Then

Make GH=6 yards, for the space } of the *false braye*.

HL=6 yards, the breadth }

HI=2 yards, the height }

LM=1 yard, the berme.

The next as before.

Another profile through the face of a bastion, across the 126. ditch, through the covered way, and glacis.

Draw AR to represent the ground line, or level of the field, as before. Then

Make AB= 3 yards

BC=10 yards

CD= 2 yards

DE= $\frac{1}{2}$ yard

EF= $6\frac{1}{2}$ yards

FG= $4\frac{1}{2}$ yards

GH= 3 yards

Hh= 4 yards

hm=40 yards, the breadth of the ditch.

mL= 4 yards

LM= 5 yards

MO= $2\frac{1}{2}$ yards

OR=40, or 50 yards.

Then

FIG.

126. Then from the points A, B, C, D, E, F, G, &c. draw as many perpendiculars to the line AR, and

Make $Bb = 3$ yards

$Cc = 3\frac{2}{3}$ yards

$Dd = 4\frac{1}{3}$ yards

$Ee = 5\frac{2}{3}$ yards

$Ff = 4\frac{2}{3}$ yards

$bl, mK = 6$ yards

$Mn = \frac{2}{3}$ yard

$No = 1\frac{1}{4}$ yard

$OP = 2\frac{2}{3}$ yards

Draw the lines $Ab, bc, cd, ef, fg, gh, Ln, \&c.$ also de parallel to AG, equal to one yard, and make go the same, draw cd, ng , and it is done.

To represent the profile in perspective.

Having drawn the profile according to the above directions, draw Aa what length you please, and from a , draw ar parallel to ab , rs parallel to bc , st parallel to cd , tx parallel to de , ux parallel to ef , &c. which being done through the whole figure, you will have a second profile, which shaded as you see in the plan, will shew the heights and breadths in perspective.

Where ab , the inward } slope of the rampart.

FG , the outward }

bc , the walk of the rampart.

ef , the inside } of the parapet:

fg , the crown }

cd , the slope } of the foot-bank.

de , the top }

GH , the berme.

HI , the scarp.

KL , the counterscarp.

IK , the bottom of the ditch.

LM , the covered way.

eq , the slope } of the banquette, or foot-bank.

qs , the top }

op , the inside of the parapet.

pR , the slope of the glacis.

Note. The breadth of the ditch and glacis are not laid down in their true proportion, it would make the plate too long here.

Having taught how to fortify regular polygons both by construction and calculation, whether they be fortified with or without orillons and retired flanks, I shall say no more of this subject, but conclude it with a short essay upon encampments.

As

An Essay upon Encampments, by Mr. Rosand, a lieutenant-colonel and engineer in the Bavarian army.

This gentleman published a Treatise of Fortification in the year 1733, from which the following extract is a translation.

The colonel asserts, that he had always seen forty or fifty paces allowed for a squadron to encamp, and as much for the interval or space; that he has even seen one hundred paces allowed for the front of the camp of every battalion, and as much for its interval or space. This practice, he says, may be looked upon as an invariable rule, if the general chuses to give battle with intervals equal to the fronts of the different troops, of his army. But in whatever method he determines his order of battle, the camp of every respective corps, and its interval, must be proportional to the front.

It follows therefore, from the principles, laid down, with regard to the extent or front of the camp, that there ought to be, before every corps, battalion, and squadron; an open ground, for forming an army in order of battle. Wherefore, if we are obliged, says he, to encamp in disadvantageous posts, the first thing, we are to attend to, is to lay out the ground in such a manner, that the troops may have an easy communication, and move without any other obstacle.

The order of battle being commonly a right line, on the side next the enemy, and in the same line, if the ground permits, the colours and standards are placed. This is the principal line, or to express it in the terms of military fortification, the master line of the camp, on which all the rest depend.

After having explained the principles on which the front of a camp is formed, let us proceed to speak of its depth. This is determined by the depth of the camps of the battalions and squadrons which we may reckon at forty-eight or fifty yards. The second line must have the same space before it as the first, quite clear, to draw the troops up in a line of battle.

The distance from the head of the camp, or first line, to the second is commonly three or four hundred paces, sometimes five hundred, if the ground be spacious enough; but the distance should never be less than two hundred paces, because otherwise the rear of the camp of the first line would interfere with the front of the camp of the second line.

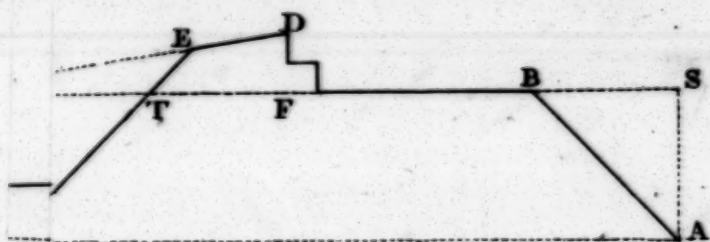
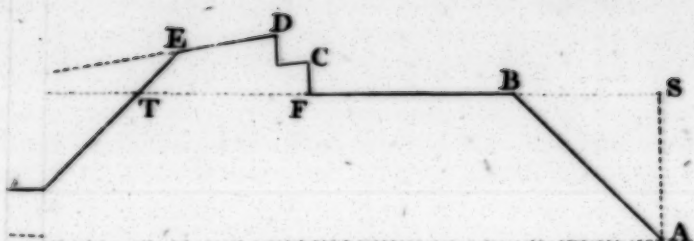
In case of an attack, it is extremely beneficial to have a large space of clear ground before the camp, because the
troops

troops of the second line may march out through their intervals, and have an opportunity of forming themselves behind those of the first, in order to support them; and this is an advantage which ought never to be neglected, when it can be done. It sometimes happens that an intrenchment is made before the front of the camp: in this case, there should be no obstacle or hindrance to the communication of the troops between the camp and intrenchment.

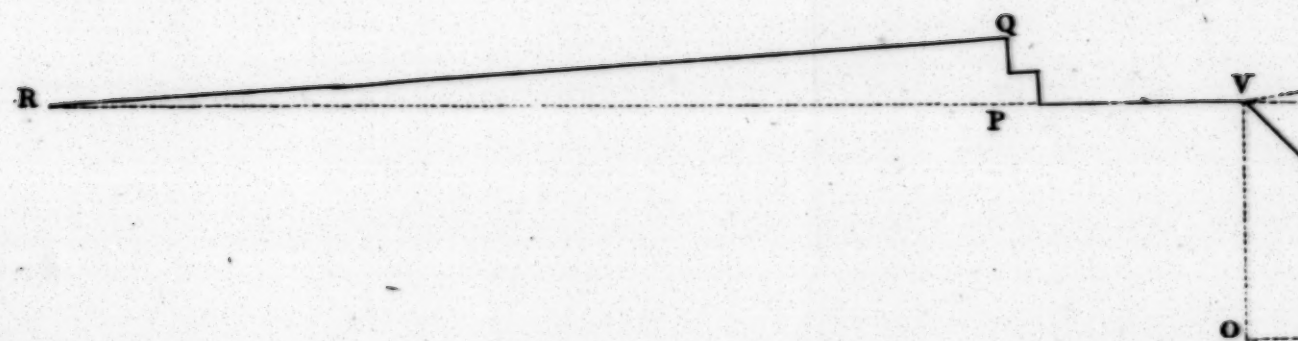
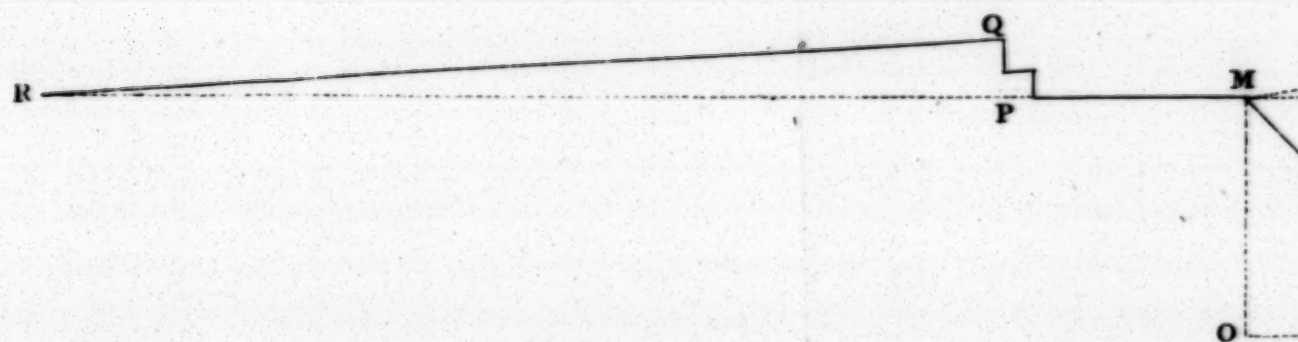
In countries, such as Hungary, and the provinces bordering on the Danube, where the Germans make war upon the Turks, the officers in general make use of tents; but in Flanders, Italy, &c. which are often the seat of war, there are many villages and houses, wherein the principal officers, as lieutenant-generals, generals, field-marsbals, &c. have their quarters. The quarter-masters of the army appoint each a house in the villages contained within the extent of the camp. Brigadiers may even lodge in a house, according to military law, provided there be one at the rear of their brigade; but inferior officers, as colonels, and all below them, must encamp at the rear of their respective corps.

Care is always taken to lodge or dispose the general officers on the side of the army which they command, that is, those who command the right, on the right; those who command the left, on the left; and those who command the center, on the center.

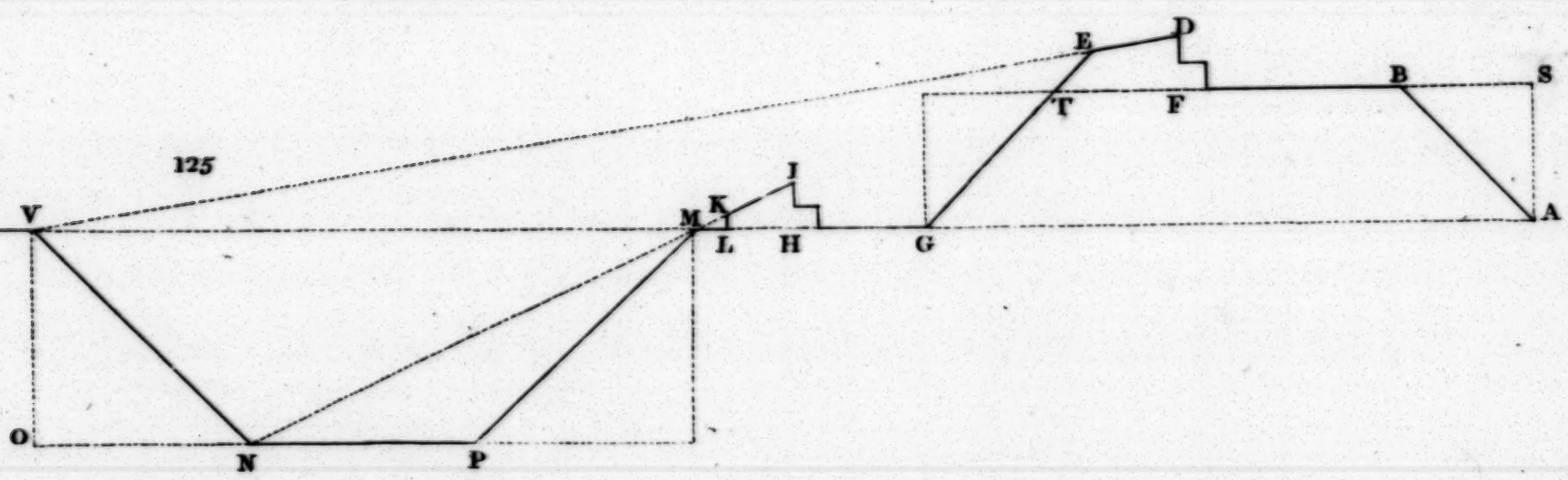
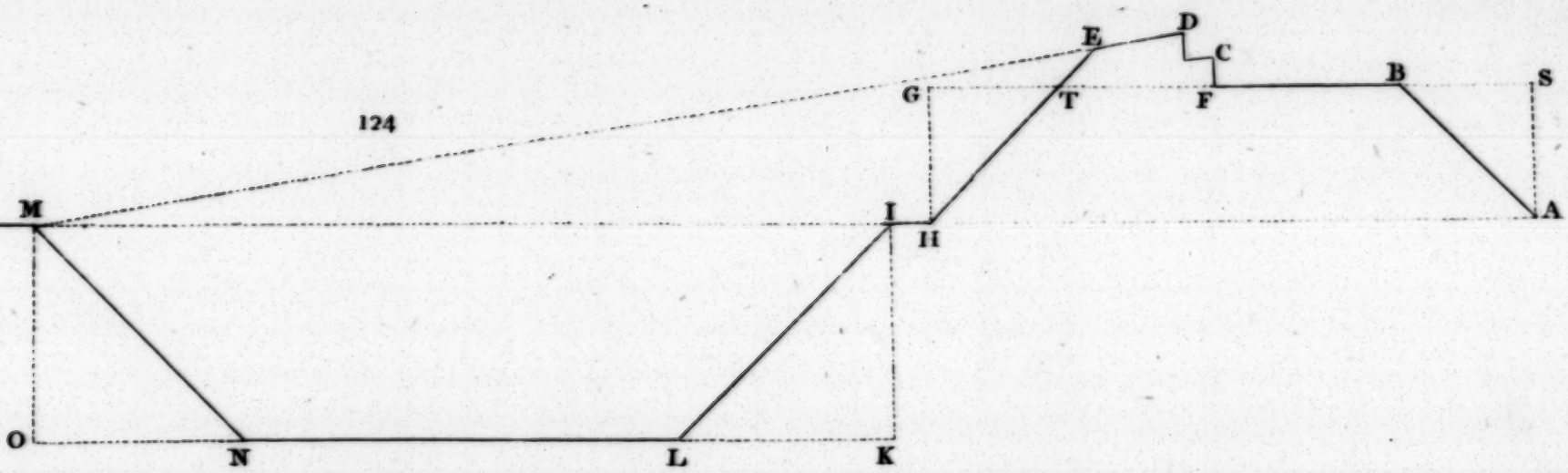
These are the principal rules laid down by authors for forming encampments, and being founded on the nature of things, and the present practice of military operations in Europe, are commonly observed by the greatest generals.



a. 220.

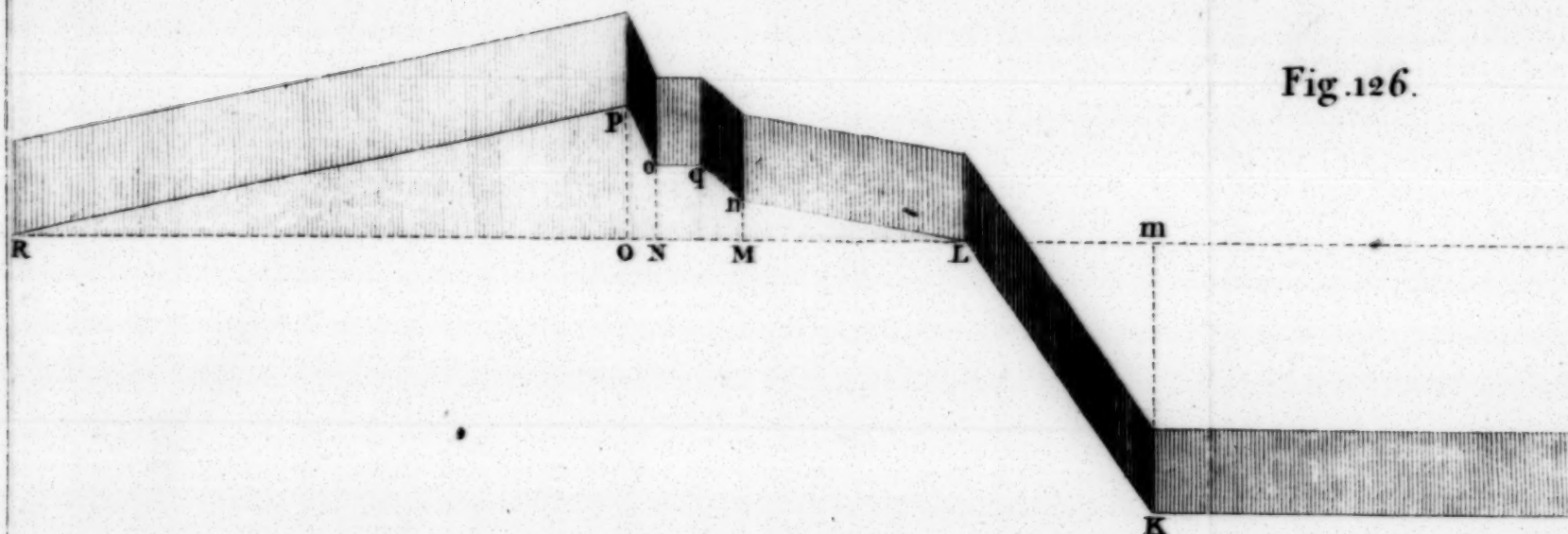


Fortification .



Profile through the Face of

Fig. 126.



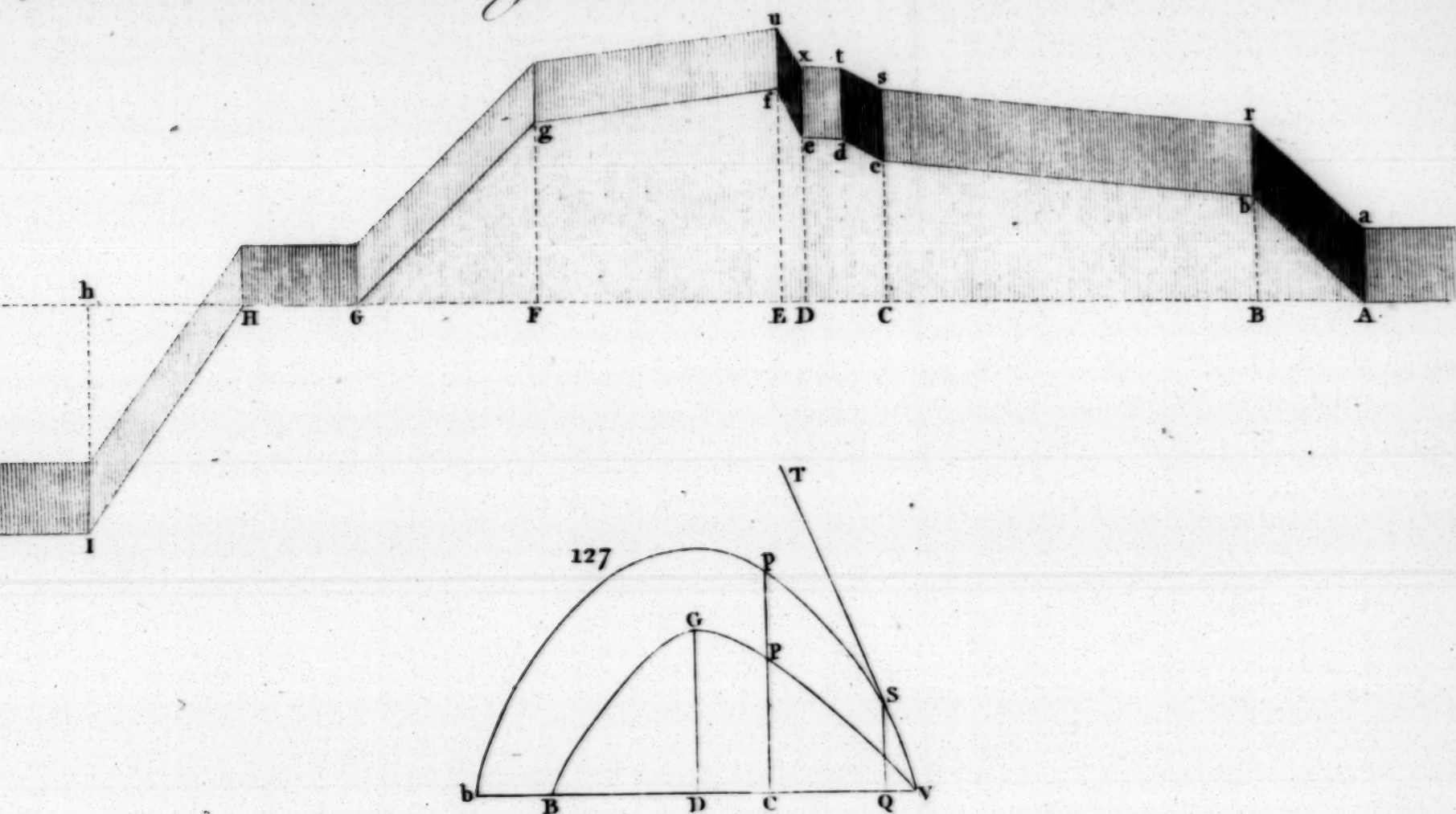
bA, inner
 gG, outward
 e, f, inside
 f, g, Crown
 c, d Slope
 d, c, Top
 GH, the Berm

} Slopes of the Rampart, bc. its Walk.
 } of the Parapet.
 } of the Foot Bank.

IH, the Scarp
 KI, Counterscarp
 IK, bottom of the Ditch
 Ln, Cover'd way
 nq, Slope
 qo, the Top
 op, inside of the Parapet
 pR, Slope of the Glacis.

} of the Foot Bank.

of a Bastion, its Ditch & Glacis.



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P A R T X.
O F
G U N N E R Y.

GUNNERY is the application of the doctrine of projectiles, and is a science absolutely necessary to a martial people; it shews the properties, use, and composition of gun-powder; gives the different sizes, figures, charges, carriages of cannon, mortars, pedreros, &c. how to point, prove and manage them; it shews the art of throwing balls, bombs, carcasses, cartouches, granadoes, &c. the manner of nailing up cannon, so as to render them useless to an enemy, and widening the touch-hole; fixes rules for the making of batteries, pontons, bridges, and constructing mines, countermines, &c. and all those engines and fireworks necessary for taking and defending fortified towns.

It is necessary for an able gunner to be well acquainted with arithmetic, to keep an accurate account of all remarkable shots, to be ready at the finding the requisite charges of powder fitting for any piece required; the weight of all sorts of shot; to be acquainted with the extraction of roots; to understand so much of geometry and trigonometry as to be expeditious in taking heights and distances, to level, construct batteries and platforms, to know the names and different parts of cannon, their different ranges, the powder for proof and service, the point blank on a level, utmost random; the number of horses or men to draw the pieces on sudden emergencies; the best method for the arrangement of artillery, according to the nature and circumstances of the place; to know the nature and use of the fuses for bombs, and the necessary instruments relating to cannon; to be expert in constructing mines and their galleries; the most useful fireworks for incommoding an enemy such as bombs, granado-balls, powder-sacks, torches, thundering hedgehogs, thundering barrels, tarred faggots, &c.

DEFI-

DEFINITIONS.

I. Angle of $\left\{ \begin{array}{l} \text{elevation} \\ \text{depression} \end{array} \right\}$ according as it is $\left\{ \begin{array}{l} \text{above} \\ \text{below} \end{array} \right\}$ the horizontal line, is the angle which the line of direction makes with the horizon.

II. The $\left\{ \begin{array}{l} \text{amplitude} \\ \text{or random} \end{array} \right\}$ is the distance between the point of projection and the object aimed at.

III. The *impetus* is the greatest height that any cannon ball can ascend, when projected perpendicularly to the horizon; and is always just one half of the greatest random upon a horizontal plane.

The greatest random must be found by trials, which is necessary to be known.

IV. When any body is projected in any direction whatever, which is not perpendicular to the plane of the horizon, it will in its motion describe a curve line, called a parabola, unless so far as the resistance of the air hinders it. The curve AEB is the parabola. *Figure II.*

V. If the body projected meet with no resistance, it would throw it in the direction AC, but gravity acts every instant, and draws the body from the direction AC, as soon as it leaves the engine, in the curve AB.

VI. The direction AC, of any engine, is a tangent at the point where the engine is placed, to the curve AEB, which the projected body describes by its flight; for the curve AEB is described by the projected body in the same time that it would have fallen through height CB.

VII. If AC is the velocity and direction of a projected body, then AB is the horizontal velocity.

VIII. BAC is the angle of direction, or the angle for elevating your piece, and the greater the elevation is, the higher the ball will rise; whence a body thrown up with a given velocity from A, the place where your piece is fixed, will rise to a height equal to a fourth part of the parameter belonging to that point.

IX. The greatest random is half the parameter, and when the utmost or greatest random is mentioned, it must be understood that the piece is always laid at the elevation of 45 degrees, twice the sine whereof is radius; and likewise ranges made equally distant above and below 45°, are equal; for the sines of all double arches are equal to the sines of their double complements &c. The random of a ball made at an angle of

45 degrees, is equal to the square of the time of its description in seconds multiplied by $16\frac{1}{2}$ feet.

X. when any object's distance on a horizontal plane is less than the piece's greatest random, it may be hit then by two elevations, each whereof is the complement of the other to 90 degrees.

XI. Since the versed sine of 180 degrees is equal to twice the radius, it is manifest, that the greatest altitude of a perpendicular projection is one fourth of the parameter, or half of the greatest horizontal random, and consequently the impetus is half of the greatest horizontal random.

XII. The velocity of the impulse moved in a second, is always a mean proportional between the parameter and $16\frac{1}{2}$ feet; that is, equal to the square root of the product of $16\frac{1}{2}$ feet into the parameter.

XIII. The velocity of a project in any point of its curved line made by its flight, is such, as a body acquires in falling down a fourth part of the parameter belonging to that point.

XIV. In projects, cast with the same velocity, if the velocity be defined by the number of feet moved in a second, the parameter will be found by dividing the square of the velocity by $16\frac{1}{2}$ feet.

XV. The utmost random on an inclined plane, and consequently, the least force that can reach an object, is made when the axis of the piece makes equal angles with the zenith and the object; so that there needs no more to be done, but to let the axis of the piece, properly charged, bisect the angle between the object and zenith; and the projections are equal at elevations equally distant from this line above and below; and this is true, whether the projections be made on planes above the level of your piece, or below it.

XVI. The general rule for hitting any object, is to get your distance from it, as near as you possibly can, the greatest random of your piece you must know from experiments, then

As the greatest random is to radius,

So is the distance of the object from you, to the sine of double the angle of elevation, half whereof is the angle of elevation required.

XVII. When any object is so near you as you can take aim with your eye, as you may always in firing (what is vulgarly called) point *blank*, and in battering walls; then *dispart* your piece, that is, set such a mark as a piece of wood hollowed on one side to fix on the cannon's convexity, on or near the muzzle ring, that a sight line taken on the top of the
base

base ring against the touch-hole, by the mark set at or near the muzzle may be parallel to the cannon's cylinder, its top being just as high above the piece as the metal is thicker at the breech than at the mouth, this *sight-piece* serves for guiding your eye in taking aim, and by levelling the upper part of the breech, and the upper part of the sight, with a given object, the bullet discharged in that direction will be carried towards that object, only lower just half the diameter of the cannon at its breech, therefore always level the piece half the diameter of the breech higher, the ball then will hit the object; provided no accident vary the direction.

XVIII. Cannons are best named from the weight of the ball they carry.

As a cannon carrying a ball weighing

30 *lb.* is a thirty pounder.

24 *lb.* is a twenty-four pounder.

16 *lb.* is a sixteen pounder.

&c.

&c.

XIX Mr. *du Metz*, lieutenant-general of the late *French* king's forces, and lieutenant of the artillery in *Flanders*, observed by which the pieces being fired at random shot, at an elevation near 45° , and charged as follows, it appeared that

		fathoms
A 24 pounder charged with 16 <i>lb.</i> of powder, carried		2250
16 pounder	10 $\frac{1}{2}$ <i>lb.</i>	2020
12 pounder	8 <i>lb.</i>	1870
8 pounder	5 $\frac{1}{4}$ <i>lb.</i>	1660
4 pounder	2 $\frac{1}{4}$ <i>lb.</i>	1520

But notwithstanding, with what care soever the above experiments were made, many accidents will occur so as to render the distance of the shot uncertain; and though not so absolutely to be depended on, yet so much regard is to be paid to these experiments, as they give pretty nearly the true extent of the shot.

XX. M. the marshal *de Vauban* was a great improver in the practical management of artillery, especially that called by him *batterie à ricochet*, or rebounding shot, which he fired with small quantities of powder, and elevated the piece so, that the ball in his descent might just go clear of the parapet of the enemy, and drop into their works; by this means, the bullet coming to the ground in a small angle, and consequently with a small velocity, it either bounds or rolls along in the direction it was fired in; and therefore if the piece be placed in a

line with the battery, it is intended to silence, or the front it is to sweep, each shot rakes the whole length of the battery or front, and has infinitely more chance of disabling the defendants, dismounting their cannon, killing and laming all it meets with, and creating great disorder. This invention is said to render the defences of the enemy useless, and which has brought the attack of places to great perfection.

But this has been almost wholly confined to cannon, unless the name *ricochet* be allowed to the bombs thrown with the *hawitzer**, which strictly corresponds in effect with the *ricochet*, since their use is to roll in shells among bodies of men, and there do great execution. The governors of the school of artillery at *Straßburg*, thinking however the bombs fired by way of *ricochet* might be used against fortifications with great advantage, made several experiments on the subject in 1723, which are related by M. *Belidor*, in his *Bombardier François*, from which I think it proper in this place to give the following account from which this is extracted.

“ To fire bombs *en ricochet*, mortars of 8 inches diameter are made use of, mounted upon cannon carriages. The batteries erected for this purpose are placed upon the continuation of the branches of the covered way, because bombs there do so great execution, that it is scarce possible the troops should keep possession of it. They break palisades, barrs and intrenchments which are made in the re-entrant or entering place of arms, and create much more disorder than bullets; for they are not only bigger and heavier, but after making many bounds, burst where they stop, without entering the ground, and their splinters are always fatal. Besides, these mortars may be used with greater expedition than cannon, for there is nothing to do, but to put the powder into the chamber, the bomb upon it, and fire; and as this may be done in *three* or *four* minutes, a battery of two mortars, used this way, will throw 30 or 40 bombs in an hour. I leave any one to judge (adds M. *Belidor*) if a covered way was flanked with such batteries, what garrison could maintain

* A *hawitzer* is a kind of mortar, which is to be fired horizontally like a cannon, and has a carriage with wheels, the same as cannon; these sorts of pieces are 8 inches in the bore, above 3 feet long, and about ten hundred weight. It is used to throw bombs into the platform of a bastion, or the middle of a party of men. The *English* and the *Dutch* were inventors of this kind of mortar; the first seen in France, were taken at the battle of *Nerwind*, which the marshal *de Cambray* gained against the Allies in 1693.

their ground, what advantage the besiegers would there have in making a vigorous attack, and with how much facility they might advance their works.

“ As the bombs must be prevented from entering the ground when they fall, because they would not then roll or bound, the mortars ought never to be pointed above 12 degrees; but any angle may be taken between 8 and 12, which is found most convenient, with respect to the charge the mortar is fired with, and the distance of the place where the bomb is intended to begin to bound. The experiments made at *Straßburg* may serve as rules on this subject, which are as follow.

“ A battery was erected 70 fathoms or 140 yards from the salient angle of the covered way of the half moon of the polygon belonging to this school; a mortar pointed at 9 degrees above the horizontal line, and charged with three quarters of a pound of powder, threw the bombs upon the glacis at 2, 4, 6, 8 fathoms from the parapet of the covered way, where rising with a bound, they went forward, falling first in the part or branch between the two traverses, and then in the re-entrant place of arms against a small intrenchment which had been thrown up there on this occasion.

“ The piece was afterwards elevated at 10 degrees, with the same charge, and after five or six times firing in this manner, it was observed that the bombs fell in the place of arms of the salient angle, and there rising with a bound, fell again, as in the former experiment, in the part between the two traverses, and then into the re-entrant place of arms. The mortar was at last pointed at 11 degrees, and still fired with the same charge; after discharging it 5 or 6 times, it was observed, that the bomb still fell in the part or branch between the two traverses, and rising, went from thence over the covered way. From all these experiments, it was concluded, that the most advantageous manner of firing by way of ricochet is, when the mortar is so levelled that the bombs may fall on the crown of the covered way, or the salient place of arms, by which means they never fail of producing considerable effects.

“ In order to try if the fuse would not be extinguished, either by the fall of the bomb, or its rolling *en ricochet* in the ricochet, several were discharged with the fuse lighted, all which succeeded, the fuse burning till it was quite consumed.

With regard to the manner of charging mortars, *M. Le Blond*, in his *Elements of War*, written for the use of count

de Brionne, says, the officer who directs the charging of the mortar, having ascertained the proper quantity of powder causes it to be put into the chamber of the mortar, and then to be covered with a wad, well beat down with the rammer, over these are put two or three shovelfuls of earth, which are also well beat or rammed down; after which the bomb is placed upon this earth, as near the middle of the mortar as possible, with the fuse or touch-hole, uppermost; more earth is then put in, and pressed down close all round the bomb, with a wooden knife about a foot long, so as to keep the bomb firm in the situation it is placed in. All this being done, the officer points the mortar, that is, gives it the inclination necessary to carry the bomb to the place designed. When the mortar is thus fixed, the fuse is opened, the picker is also passed in the touch-hole of the mortar to clear it, and it is then primed with the finest powder. This done, two soldiers, taking each one of the lint-stocks, the first lights the fuse, and the other fires the mortar. The bomb, thrown out by the explosion of the powder, is carried to the place intended, and the fuse, which ought to be exhausted at the moment of the bomb's falling, sets fire to the powder it is charged with; this immediately, by its explosion, bursts the bomb into splinters, which are thrown off circularly round the point the bomb falls upon, and do considerable damage on every side.

XXI. A twenty-four pounder has been fired five or six times in an hour for twenty four hours together, but great care must be taken to cool it, for the piece will grow softer, and the touch-hole will be widened; the hotter a cannon grows, the greater is the velocity of the ball, and it recoils with greater violence upon the carriage, because the powder takes fire quicker and in a greater quantity; yet it may happen to shorten the random of the ball, if the cannon recoiling falls again before the bullet is out. Therefore the platform must be firm and solid, for if the platform be shaken with the first impulse of powder, it is sure the piece will likewise be shaken, and this will certainly alter its direction, and so render the shot uncertain: in order to prevent this, platforms are usually made very firm to a considerable depth backwards, so that the piece is well supported, both in the beginning of its motion, and through a great part of its recoil; and Mr. *Robins* has computed from experiments, that a piece ten feet long, carrying a twenty-four pounder, charged with sixteen pounds of powder, the bullet will be out of the piece before it has recoiled half an inch.

XXII. When cannon cannot very readily be taken away, or to render them useless to an enemy, the usual way is to nail them up, which is driving a large nail into the touch-hole, and leaving no part of it to be taken hold on to draw it out, and this renders them quite useless: pebbles or any hard stones will do, if nails are not at hand.

XXIII. Red-hot bullets are often made use of to fire towns, &c. these sorts of bullets are made flame or red-hot on a grate contrived on purpose, and then put into the mortar, or piece, with a pair of tongs, the piece being wadded before with turf; for the powder must be covered with something of this nature, or else the hot bullet would set fire to the powder, and render the force ineffectual; then as soon as the bullet is arrived at the wad of turf, the piece is immediately fired, for it cannot well be rammed down with wad, for fear the powder in the mean time should take fire and do mischief. Seldom heavier than six, eight or ten pound bullets are used this way, for they would be troublesome to manage.

XXIV. There is another engine of metal made use of on particular occasions, called a *petard*, pretty much resembling a high crown hat, with narrow brims; this is filled with very fine powder, the engine is fixed to a plank, bound fast down with ropes, going through handles, which are round the rim of its mouth; it is well primed. Its use is to break down gates, drawbridges, portcullises, barriers, &c. it also is for breaking through the enemies galleries, and to frustrate their mines; the officers who are to manage the *petard*, when made use of, are in a dangerous post, few ever returning safe from the expedition, if we may credit *M. St. Remy*, the garrison incessantly firing on them.

XXV. Bombs are hollow balls of cast iron, filled with powder, old nails, pieces of iron, &c. they are designed rather for firing houses, magazines, annoying a garrison, than any execution by their projecting force; being shot out of mortars, some of the larger sort weigh four or five hundred weight. In adjusting the fuse of bombs, I have inserted at the conclusion of this work some problems in order to proportionate them to the time of their flight. The fuses of bombs or grando shells, are they which make the whole powder or composition in the shell take fire, to do the designed execution; they are round and thicker at one end than at the other, filled with a composition of sulphur, salt-peter, and the best powder, and is designed to burn so long and no longer, as is the time of the motion of the bomb from the mouth of the mortar

mortar to the place where it is to fall; so that the fuse must be contrived either from the nature of the wildfire, or the length of the pipe that contains it, to burn just that time.

XXVI. When the bomb is charged, the fuse is put into it through the touch-hole, and after it is fired it sets fire to the powder in the bomb; the fuses must be charged with great care for answering the end they are designed, that nothing may prevent their firing the powder in the middle of the bomb; and as they are generally charged long before they are used, the ends are well secured, lest the composition get wet, or fall out; and when they are used, the small end is opened or cut off, and put into the bomb, but the bigger end is never opened till the bomb be in the mortar, and just a going to be fired.

XXVII. The lower part of the shell is the thickest part of it, which is necessary it should be so, that being the heavier, it may always fall upon it, and never on the fuse; if the bomb be set fire to by the fuse before it reaches the designed place, it will burst in the air, and then it may be as fatal to that persons that fired the mortar, as those against whom it was intended; and to remedy this inconvenience, the fuse is made to last as long as the bomb is going its utmost random, if thrown at a great distance; but when near, part of the fuse is burned out before discharged.

XXVIII. There are pieces called *howitzers*, which are about eight inches in the bore, and above three feet long; they are used to throw bombs in the platform of a bastion, or the middle of a parcel of men.

XXIX. The best battering pieces are mostly twenty-four pounders, they being found abundantly sufficient for making breaches; moreover they are better manageable, and take less ammunition than cannons that are larger. The modern way of making breaches, is first cutting off the whole wall as low as possible, before its upper part is attempted to be beat down.

XXX. Balls strike with greater force against rough stones than smooth ones, and the greater the velocity the greater the force, for the weight of the ball in pounds multiplied by the velocity, gives the force of the ball at that place where the velocity was taken; as for instance, if a piece carrying a 24 lb. ball, and its velocity be 500 feet in a second, when it arrives near an object it is designed to hit, the force of this ball will then be 12000 lb. or 5 ton, 7 hundred, and 16 pounds.

XXXI. Small bodies with great velocities are more proper to tear in pieces; and great bodies with small velocity, to sh ke or move the whole.

XXXII. That part of a cannon next us when it stands ready to fire, is called by the artillery people the *breech* or *coyle*; the round knob at the end of it is called the *cascabel*: from this end I shall here set down the several parts of a cannon.

1. The cascabel.
2. The cascabel astragal.
3. The neck of the cascabel.
4. The breech and breech mouldings, is all the metal, behind the touch-hole.
5. The base ring and resistance.
6. The vent and vent astragal, is the next cornish or ring to the vent.
7. The charging cylinder.
8. The first reinforced ring, is that cornish next the vent astragal.
9. The first reinforced astragal.
10. The vacant cylinder.
11. The dolphins.
12. The trunnions are two knobs of metal by which the cannon hangs in the carriage.
13. The second reinforced ring.
14. The second reinforced astragal.
15. The chace.
16. The neck astragal.
17. The neck and muzzle ring.
18. The muzzle mouldings or freeze.
19. The face of the piece.
20. The bore or hollow part.

XXXIII. I need not say any thing with regard how to make cartridges, to fix the hand-granades, powder-chests, powder-pots, powder-tubs, &c. these are better learned in the field: but proceed to the solution of some of the most useful problems in practical gunnery.

PROJECTIONS on the plane of the Horizon.

PROPOSITION I.

Given the utmost random of any piece of artillery, to find the angle of elevation to hit an object at a known distance.

RULE I.

As the impetus (or half the greatest random) is to half the given distance of the object from you, so is radius to the sine of double the angles of elevation. Or,

RULE II.

As the greatest random is to the object's given distance, so is radius to the sine of double the angles of elevation.

Example.

Suppose the greatest random of a cannon was 6840 feet, how must the said piece be elevated to hit an object a mile off?

As impetus 3420 feet 3.53402

Is to half a mile = 2640 feet 3.42160

So is radius 10.00000

To sine of $50^{\circ} 31'$, or $129^{\circ} 29'$ 9.88758

Therefore $\frac{50^{\circ} 31'}{2} = 25^{\circ} 15\frac{1}{2}'$ the angle of the lower elevation.

And $\frac{129^{\circ} 29'}{2} = 64^{\circ} 44\frac{1}{2}'$, the angle of the higher elevation.

PROP. II.

Given the greatest random of any piece, to find what random the said piece makes at any other given elevation.

RULE.

As radius to the sine of double the given angle of elevation, so is the greatest random to the random required.

Example.

A piece whose greatest random is known to be 6840 feet, and the engineer found that by elevating his piece, $25^{\circ} 15\frac{1}{2}'$ he struck an object on the plane of the horizon, pray what distance was the object from him?

Q 4

As

As radius	10.00000
To sine of twice $25^{\circ} 15'$	9.88751
So is 6840 feet	3.83505
To 5280 feet, the distance required	3.72256

PROP. III.

By knowing the distance of any object from you and the piece's elevation, to find the impetus to hit that object.

RULE.

As the sine of twice the angle of elevation is to radius, so is the given distance of the object to the greatest amplitude, half whereof is the impetus required.

Example.

Suppose an object a mile distant from me, and by elevating the piece $64^{\circ} 44\frac{1}{2}'$, I can strike that object, required the impetus?

Twice $64^{\circ} 44\frac{1}{2}'$ is $129^{\circ} 29'$ its sup. to 180° is $= 50^{\circ} 31'$	
As sine of $50^{\circ} 31'$	9.88751
Is to radius	10.00000
So is 5820 feet (1 mile)	3.72263
To 6840	3.83512

Therefore $\frac{6840}{2} = 3420$ feet, the impetus required.

PROP. IV.

Given the angle at the lower elevation, and the object's distance from you, to find the altitude of projection.

RULE.

As radius to the tangent of the lower elevation, so is a fourth part of the object's distance, to the ball's altitude required.

Example.

Suppose at $25^{\circ} 15'$ elevation, I can hit an object a mile off, what height does the ball fly at?

As radius	10.00000
To tangent of $25^{\circ} 15'$	9.67360
So is 1320 feet	3.12057
To 623 feet nearly, the alt. required	<u>2.79417</u>

PROP. V.

Given the altitude of projection, and the piece's impetus, to find how far an object is from you.

RULE I.

From the impetus subtract the ball's altitude, multiply this remainder by the altitude, the square root of the product is a fourth part of the distance required.

Example.

Suppose the height of a ball projected be 623 feet, and the piece's impetus 3420 feet, required the distance of the object?

From 3420
Take 623

2797
623

8391

5594
16782

1742531, whose square root is 1320 feet, and 1320×4
= 5280, the distance required.

Or thus.

RULE II.

To the log. of the difference of the impetus and ball's altitude, add the log. of the ball's altitude; half their sum answers to a fourth of the distance required.

From 3420
Take 623

To log. of 2797
Add log. 623

3.44669
2.79448
2)6.24117
3.12058

Sum = 1320

4
5280, the distance required.

PROP.

PROP. VI.

The distance of an object and the ball's altitude being given, to find the piece's elevations.

RULE.

As half the given distance is to twice the ball's height, so is radius to the tangent of $\left\{ \begin{array}{l} \text{lower} \\ \text{higher} \end{array} \right\}$ elevation.

Example.

Suppose an object a mile distant from me, and the ball's altitude is 623 feet, what are the angles of elevation?

As 2640 feet	3.42160
Is to 1246 feet	3.09551
So is radius	10.00000
To tangent $25^{\circ} 16'$ lower elevation	9 67391
And $64^{\circ} 44'$ is the higher direction	

PROP. VII.

Given the angle of elevation, together with its amplitude, to find the amplitude at any other given elevation.

RULE.

As the sine of twice the first elevation, is to its given amplitude, so is the sine of twice the other elevation, to the respective amplitude required.

Example 1.

Suppose at an elevation at $30^{\circ} 20'$ the given distance be 2700 feet, what distance will the said piece throw a ball, which is elevated at $39^{\circ} 48'$?

Twice $30^{\circ} 20' = 60^{\circ} 40'$, and twice $39^{\circ} 48' = 79^{\circ} 36'$.

As sine $60^{\circ} 40'$	9.94040
To 2700 feet	3.43136
So is sine $79^{\circ} 36'$	9.99280
To 3046 feet the distance required	3 48367

Example

Example 2.

A cannon elevated at 45° threw a ball 6840 feet, how far will it throw the said ball when elevated at 15° ?

Sine of double 45°	10.00000
Is to 6840	3.83505
So is sine of 30° double 15°	9.69897
To 3420 feet	3.53402

And *vice versa*, to find one elevation, from two amplitudes and a given elevation.

If you know what distance a given charge of powder will effect, and would know what charge will be sufficient to hit an object at any given distance, within your cannon's reach, without altering the elevation, observe the following

P R O P. VIII.

The charge of powder, and the distance of projection made from that charge, being known, to find what charge of powder will do to hit an object at a known distance, without altering the piece's elevation.

R U L E.

As the given distance is to its respective charge of powder, so is the known distance, to its respective charge of powder.

Example.

Suppose a 24 pounder, charged with 14 pounds of powder, will throw a ball 6840 feet, what quantity of powder must the said piece be charged with, to throw the said shot so as to hit an object 8900 feet from you, without altering the piece's elevation?

given. dist. lb. dist.

As 6840 : 14 :: 8900 : 10 $\frac{7}{10}$ lb. required.

N. B. This is only true when the velocity is supposed to be in the subduplicate ratio of the quantity of powder which is not strictly true.

P R O P. IX.

Given a piece's impetus, to find the velocity of the ball.

R U L E

R U L E.

Multiply the square root of the impetus by 8, the product is the velocity the ball moves through in a second.

Example.

Suppose the impetus of a piece be 14000 feet, what is the velocity of the ball at the object on the horizon?

The square root of 14000 is 118.3

And 118.3
 $\quad 8$

946.4 feet the ball moves through in a second.

R U L E II.

Multiply the number answering to half the log. of the given impetus by 8, gives the velocity the ball moves in a second.

The log. of 14000

4.14612

Half = 118.3
 $\quad 8$

2.07306

946.4 feet, as above.

P R O P. X.

Given the piece's elevation and the object's distance in feet, to know how long time the ball is going to that object.

R U L E.

As the radius is to the tangent of the piece's elevation, so is the given distance of the object to the square of four times the required number of seconds; this last product divide by 2, and find the number answering to it in the log. tables, a fourth of which number is the time of flight in seconds.

Example.

At an elevation of $38^{\circ} 20'$ I can hit an object a mile and a half from me, required the time of the ball's flight?

As radius

10.00000

To tangent $38^{\circ} 20'$

9.89801

So is 7920 feet ($1\frac{1}{2}$ mile)

3.89872

To the square of 4 times the N^o of seconds 3.79673

Now

Now half of 3.79673 is 1.89836; the number answering to it is 79.14 a fourth of which is $19\frac{3}{4}$ seconds, the time of flight.

But as to the throwing of bombs, the elevation of the mortar is commonly at 45° , then a fourth part of the square root of the distance in feet, will give the number of seconds required.

PROP. XI.

Given the elevation of a mortar 45° , and the time of the bomb's flight to any object, to find the horizontal distance of that object from you.

RULE.

The random of a bomb, when thrown down at an elevation of 45° , is equal to the square of the time multiplied by $16\frac{1}{2}$ feet.

Example.

A mortar elevated at 45° threw a bomb into a town, and it was observed to be 16 seconds in its flight; what is the horizontal distance?

$$\begin{array}{r}
 16 \\
 16 \\
 \hline
 96 \\
 16 \\
 \hline
 256 \\
 16\frac{1}{2} \\
 \hline
 1536 \\
 256 \\
 21 \\
 \hline
 \end{array}$$

Feet 4117, or $823\frac{1}{2}$ paces.

PROP. XII.

Given the greatest random of any piece of cannon, and its absolute force in pounds, to find with what force it will strike an object, at a given distance, whose surface leans backwards any given angle from the perpendicular.

RULE.

As the greatest random is to radius, so is the object's given distance to the sine of double the angle of elevation.

And as radius is to the sine of the angle at the surface of the object, so is the given force to the force required.

Example,

Example.

Admit a piece's greatest random be 9000 feet, and the absolute force of the ball 6952 pounds, with what force will it strike an object 6000 feet distant from you, and whose surface leans backwards from the perpendicular 36° ?

As 9000	3.95424
To radius	10.00000
So is 6000	3.77815
To sine $41^\circ 48'$	9.82391

Whence $\frac{41^\circ 48'}{2} = 20^\circ 54'$, the elevation that the ball strikes the plane of the horizon at, and consequently the surface of the object at an angle of $74^\circ 54'$, for $90^\circ - 36^\circ + 20^\circ 54' = 74^\circ 54'$.

To find the force.

As radius	10.00000
To sine $74^\circ 54'$	9.98474
So is 6952 lb.	3.84210
To 6712 lb. the force required	3.82684

PROP. XIII.

Given a piece's greatest random, and the requisite charge of powder, to find what charge of powder will suffice to strike an object at right angles at a given distance, whose surface reclines any given number of degrees.

RULE.

As the sine of twice the angle of elevation, is to the given object's distance, so is radius to twice the impetus, and as the square root of twice the object's distance is to the square root of twice the impetus now found, so is the given charge of powder to the charge required.

Example.

Admit a cannon's greatest random with a charge of 16 pounds of powder was observed to be 3 miles, what charge will be sufficient to strike an object at right angles, whose surface reclines 36° at a mile and half distance from the piece?

It

It is plain from the question that the elevation must be 36° , to strike it at right angles. Then,

As sine of 72° 9.97820

Is to $1\frac{1}{2}$ mile = 7920 feet 3.89872

So is radius 90° 10.00000

To 8328 feet, double impetus 3.92052

lb. lb.

As $\sqrt{15840} : \sqrt{8328} :: 16 : 11.6$ the charge.

PROJECTIONS ON ASCENTS.

PROP. XIV.

Given the perpendicular height of an object, to find the horizontal distance.

Before this can be answered, the angle which the object makes with the horizon, must be found by some instrument, as a theodolite. Then this

RULE.

As the tangent of the angle found by observation, is to given perpendicular height, so is radius to the horizontal distance required.

Example.

There is a castle situated on a rock 696 feet high, and the angle the castle made with the horizon, was $5^{\circ} 20'$; required my horizontal distance from it?

As tangent $5^{\circ} 20'$	8.97013
To 696	2.84261
So is radius	10.00000
To horizontal dist.=7455	3.87248

PROP. XV.

Given the horizontal distance of an object situated on a rock, and the angle the object makes with the horizon, to find the perpendicular altitude.

RULE.

As radius, to the horizontal distance, so is the tangent of the angle which the object makes with the horizon, to the perpendicular height of the object required.

Example.

There is a castle situated on a rock, whose horizontal distance from me is 7455 feet, and it makes an angle with the horizon= $5^{\circ} 20'$; required the perpendicular height?

As radius	10.00000
To 7455	3.87244
So is tangent $5^{\circ} 20'$	8.97013
To perp. height=696	2.84257

PROP.

PROP. XVI.

Given the greatest random of any piece on the plane of the horizon, to find the greatest random of the said piece, on a plane of a given ascent.

RULE.

To the natural sine of the given ascent add radius; then say, as that sum is to radius, so is the greatest random on the plane of the horizon to the greatest random required.

Example.

Suppose a cannon's greatest random on the plane of the horizon be 7980 feet, what is her greatest random on that plane, which is $8^{\circ} 40'$ above the level of the piece?

Here the natural sine of $8^{\circ} 40'$ is .1506, and radius = 1, then 1.1506 is the sum, but 1.15 is near enough for practice.

As 1.15 : 1 :: 7980 : 6939 feet, the greatest random required.

PROP. XVII.

Given the greatest random on a plane of a given ascent, to find the greatest random on the plane of the horizon.

RULE.

As radius is to the sum of the natural sine of the plane's ascent and radius; so is the greatest random given, to the random on the plane of the horizon required.

Example.

There is a plane which is $8^{\circ} 40'$ above the level of my piece, the greatest random of my piece on that plane was observed to be 6939 feet, required the greatest random of the said piece on the plane of the horizon?

As 1 : 1.15 :: 6939 : 7980 the random required.

Or thus.

To log.	1.15	0.06069
Add log.	6939	3.84131
Log.	7980	3.90200

P R O P. XVIII.

Given the piece's elevation and its horizontal random for that elevation, to find the random of a plane of a given ascent.

R U L E.

From the natural tangent of the piece's elevation, take the natural tangent of the plane's elevation, and find the angle corresponding to that difference; then add that angle to the plane's elevation, and say, as the sine of that sum is to the given horizontal random, so is the angle corresponding to the difference, to the random required.

Example.

A cannon's horizontal random at an elevation of $50^{\circ} 6'$ was found to be 5820 feet, what is her random at the same elevation above the horizon, on that plane whose ascent is $10^{\circ} 20'$?

From natural tangent $50^{\circ} 06' = 1.19598$

Take natural tangent $10 \quad 20 = 0.18233$

The angle corresponding is $\underline{45 \quad 23} = 1.01365$ diff.

To $45^{\circ} 23'$

Add $10 \quad 20$

Sum $\underline{55 \quad 43}$

As sine of $55^{\circ} 43'$

$\underline{9.91711}$

Is to log. 5820

$\underline{3.76492}$

So is sine $45^{\circ} 23'$

$\underline{9.85237}$

To 5014 the random required

$\underline{3.70018}$

Corollary. Having the angle of obliquity of the plane, you have the elevation for the utmost random of any piece, by adding half the angle of obliquity to 45° .

P R O P. XIX.

Given a piece's random at any given elevation, on the plane of the horizon, to find her random at another given elevation, on the plane of a given ascent.

R U L E.

As the sine of twice the given random's elevation is to the given random, so is twice the sine of the other elevation, to her horizontal random.

Then

Then from the natural tangent of the other given elevation, take the natural tangent of the plane's ascent, and see what angle answers to their difference, the sum of these two taken from 180° leaves a third angle.

As sine of this third angle is to the horizontal random, so is the sine answering to the difference to the random required.

Example.

At an elevation of $50^\circ 6'$ I observed a cannon's random on the plane of the horizon was 5820 feet; pray what is her random at $60^\circ 6'$ elevation above the horizon, on a plane whose ascent is $10^\circ 20'$?

As twice sine of $50^\circ 6'$ 9.99308

Is to 5820 feet 3.76492

So is twice the sine of $60^\circ 6'$ 9.93665

To 5111 feet, the horizontal random 3.70849

From natural tangent $60^\circ 06' = 1.73905$

Take natural tangent of $10^\circ 20' = .18233$

Remains $\angle$ 57 17 = 1.55672

Add 10 20

Take 67 37

From 180 0

112^\circ 23' its sup. = 67^\circ 37'.

As sine $67^\circ 37'$ 9.96598

Is to 5111 3.70849

So is sine $57^\circ 17'$ 9.92497

To 4650 feet, the random required 3.66748

PROP. XX.

Given the angle of direction, and random on the plane of the horizon, to find how high a cannon can strike an object on a perpendicular, whose horizontal distance is also known.

RULE.

As the tangent of the angle of direction is to radius, so is the random on the plane of the horizon to a fourth number,

R 2

and

and as this fourth number is to the object's horizontal distance, so is the difference of the distances, to the height required.

Example.

The horizontal random of a cannon at an elevation of $50^{\circ} 6'$ was observed to be 5820 feet, how high will the said cannon strike an object, with the same direction on a perpendicular, whose horizontal distance is 5000 feet?

As tangent $50^{\circ} 6'$	10.07772
Is to radius 90°	10.00000
So is 5820	3.76492
To 4866 a fourth number	3.68720

Then from 5820 take 5000, leaves 820, difference of the distances.

And $4866 : 5000 :: 820 : 843$ Feet *ferè*, the height required.

P R O P. XXI.

Given a piece's impetus, and the distance of an object upon a plane, of a given elevation, to find the direction to hit that object.

R U L E.

As radius is to the object's distance, so is the sine of the plane's elevation, to the perpendicular height of the object; from twice the impetus take the object's height, and note the remainder.

To and from this remainder add and subtract the object's distance, to find the sum and difference.

To the logarithm of the sum add the logarithm of the difference, and find the number belonging to half this sum.

Add and subtract that number to and from twice the impetus, and you will have a greater and a less number, then say

As the height of the object

To the $\left\{ \begin{array}{l} \text{less} \\ \text{greater} \end{array} \right\}$ number,

So tangent, plane's elevation

To tangent $\left\{ \begin{array}{l} \text{less} \\ \text{greater} \end{array} \right\}$ elevation above the horizon.

Example.

Example.

Admit a cannon's impetus be 5820 feet, required at what directions must the piece be elevated, so as to hit a castle standing upon a hill, whose height is $8^{\circ} 30'$ above the level of the piece, and direct distance 7000 feet.

As radius .	10.00000
Is to 7000	3.84509
So is sine of $8^{\circ} 30'$	9.16970
To object's height = 1035	3.01479

From twice impetus = 11640
Take object's height = 1035

Remainder = 10605
Add distance = 7000

Sum = 17605

Difference = 3605

To log. 17605 4.24563
Add log. 3605 3.55690
Sum = 7.80253

The number to it is $7966\frac{1}{2}$ = 3.90126 half sum
To twice the impetus = 11640
Add and subtract $7966\frac{1}{2}$
Greater number = $19606\frac{1}{2}$ its log. = 4.29240
Less number = $3673\frac{1}{2}$ its log. = 3.56507

As 1035 3.01494

Is to $3673\frac{1}{2}$ 3.56507
So is tangent of $8^{\circ} 30'$ 9.17449

To tangenr of $27^{\circ} 57'$ 9.72462

The lower elevation.

And as 1035 3.01494

Is to $19606\frac{1}{2}$ 4.29240

So is tangent $8^{\circ} 30'$ 9.17449

To $70^{\circ} 33'$ the higher elevation = 10.45195
above the horizon.

P R O P. XXII.

Given the horizontal distance, and the perpendicular height of any object, to find the least impetus, that can possibly reach that object.

R U L E.

To the square root of the sum of the squares of the horizontal distance, and object's perpendicular height, add the the object's perpendicular altitude; half of this sum is the impetus.

Example.

Suppose the perpendicular height of a castle was 690 feet, and the horizontal distance 5820 feet; required the least impetus to reach the castle?

$$\text{The square of } 5820 = 33872400$$

$$\text{The square of } 690 = 476100$$

$$\text{The square root of this sum } 34348500 \text{ is } 5860 \text{ and}$$

$$58 \frac{60+600}{2} = \frac{6550}{2} = 3275, \text{ the impetus required.}$$

P R O P. XXIII.

The angle of elevation and the greatest random of a piece being given, to find at what proper distance it ought to be planted to hit an object, whose height above the piece's level is also known.

R U L E.

As radius to the sine of double the given elevation, so is half the greatest random to half the horizontal random, at the same elevation.

And as radius is to the co-tangent of the elevation, so is twice the perpendicular height of the object to a fourth number.

From half the horizontal random take that fourth number, and note the remainder, then a mean proportional between the remainder and half the horizontal random, added to the same half random, gives the distance required.

Example.

Admit the greatest random of a twenty-four pounder was 11640 feet what distance may the same be planted to hit a battery 312 feet above the level of the piece at the elevation of 42° ?

As

As radius 10.00000

Is to the sine of double elevation 84° 9.99761

So is half the greatest random 5820 3.76492

To half horizontal random = 5788 3.76253

As radius 10.00000

Is to co-tangent 42° 10.04556

So is twice the object's height = 624 2.79518

To a fourth number = 693 2.84074

From half the horizontal random = 5788

Take the fourth number = 693

Remainder 5095

To find a mean proportional between the remainder 5095, and half the horizontal random 5788, proceed thus :

To log. 5095 3.70714

Add log. 5788 3.76252

2)7.46966

The mean proportional 3.73483 is 5430 being the number answering to the log. 3.73483.

To half the horizontal random = 5788

Add the mean proportional = 5430

The sum is the distance required 11218 feet, or 2244 paces nearly

PROP. XXIV.

Given a piece's elevation and the random it makes at that elevation, on a plane of a given ascent, to find the direction to hit an object at a given distance, upon the horizon.

RULE.

From the natural tangent of the given elevation, take the natural tangent of the plane's ascent, and note what angle corresponds to this difference in the tables of the natural tangents, the angle last found added to the plane's ascent, and the sum taken from 180° leaves a *third angle*.

R 4

Then

Then as the sine of the angle corresponding to the difference, is to the given random, so is the sine of the *third angle* to the horizontal random at the said given elevation.

And as the horizontal random now found, is to the given horizontal random, so is the sine of double the given elevation to the sine of double the angle of direction sought, half whereof is the direction required.

Example.

Suppose at an elevation of $35^{\circ} 40'$ on a plane whose ascent is $10^{\circ} 30'$ the random was 5280 feet, it is required to find what direction will answer my purpose, so as to strike an object whose horizontal distance is 5090 feet?

From natural tangent of $35^{\circ} 40' = .71769$

Take natural tangent of $10^{\circ} 30' = .18533$

The angle corresponding is $28^{\circ} 2' = .53236$

To $28^{\circ} 02'$
Add $10 \quad 30$

Sum $38 \quad 32$

From $180^{\circ} 00'$
Take $38 \quad 32$

Remains $141 \quad 28$ third angle.

As sine $28^{\circ} 2'$ 9.67208

Is to 5820 3.76492

So is sine $141^{\circ} 28'$ 9.79446

To horizontal random = 7715 3.88730

As 7715 3.88730

Is to 5820 3.76492

So is double elevation $71^{\circ} 20'$ 9.97653

To sine $45^{\circ} 37'$ double the angle 9.85415

Or $134^{\circ} 23'$

Half whereof is $22^{\circ} 48\frac{1}{2}'$

Or $67 \quad 11\frac{1}{2}'$

} the directions required.

PROP. XXV.

Given the horizontal distance, the impetus of the piece, and the object's angle of elevation above the level of the piece, to find the elevation of the piece to hit the object.

RULE.

As radius is to twice the impetus, so is the tangent of the object's elevation to a fourth number; add this fourth number to the given horizontal distance, and note the sum.

Then as twice the impetus is to the co-sine of the object's elevation, so is the sum to the co-sine of an angle, which added to the angle between the zenith and object, gives the double of the complements of the elevation, to hit the object.

Example.

There is a castle whose horizontal distance is 6000 feet, which was observed to stand $10^{\circ} 20'$ above the level of the piece, whose impetus was known to be 4500 feet, required the cannon's elevation to hit the castle?

As radius	10.00000
To twice the impetus 9000	3.95424
So is the tangent of $10^{\circ} 20'$	9.26086
To a fourth number 1641	3.21510

Then $6000 + 1641 = 7641$ the sum.

As twice impetus 9000	3.95424
Is to the co-sine of $10^{\circ} 20'$	9.99289
So is the sum 7641	3.88315
To co-sine of $33^{\circ} 22'$	9.92180

The angle included between the zenith and object is $79^{\circ} 40'$, for $90^{\circ} - 10^{\circ} 20' = 79^{\circ} 40'$.

To $79^{\circ} 40'$	From $90^{\circ} 00'$
Add 33. 22	Take 56 31
2)113 02	Remains 33 29 the lower elevation,
Comp. elev. 56 31	consequently the higher elevation
	$66^{\circ} 50'$ to hit it.

PROP.

P R O P. XXVI.

Given the horizontal random at a known elevation, to find the elevation of the same piece, when planted at a given height above the horizon, so that it may throw a ball to the greatest distance possible, without altering the charge of powder.

R U L E.

1. Multiply the given horizontal distance in feet by $32\frac{1}{8}$ feet, divide this product by the natural sine of double the given angle of elevation, the square root of this quotient is the velocity.

2. Multiply four times the height of the piece by $16\frac{1}{2}$, and to the product add the square of the velocity, then say, as the square root of this sum is to the velocity, so is radius to the tangent of the angle of elevation.

3. As the sine of double elevation of the given horizontal random is to radius, so is the given random to the greatest required.

Example.

At an elevation of 30° with a given charge of powder, the random of a piece on the plane of the horizon was 1500 yards, it is required to find the elevation of the same piece when planted at 44 yards above the level of the horizon, so that (the charge of powder continuing the same) the ball may fall at the greatest distance possible, on a horizontal plane, which is also here required.

$$\begin{array}{rcl} & \text{yards} & \text{feet} \\ \text{1st, } & 1500 \times 3 = 4500 & \times 32\frac{1}{8} = 144750. \\ & \text{nat. sine} & \end{array}$$

And $.86602$ 144750 $(167143.94$ the square of the velocity, whose square root is 408.83 feet, the velocity.

2d, $132 \times 4 = 528 \times 16\frac{1}{2} = 8492$, and 167143.94 added to 8492 , the sum is 175635.94 , whose square root is 419.08 .

$\text{veloc.} \quad \text{rad.}$
Then as $419.08 : 408.83 :: 1 : .97554$ the natural tangent of $44^\circ 18'$, the elevation required.

$\text{sine} \quad \text{rad.}$
3d, As $60^\circ : 1 :: 1500 : 1730.05$ the greatest amplitude.

P R O P.

PROP. XXVII.

The greatest horizontal random of a cannon being known, to find the greatest random and direction, when the said cannon is planted on a plane above the horizontal plane, whose height is also known.

RULE.

1. From the square of the sum of the greatest horizontal random, and the plane's height above the plane of the horizon, take the square of the plane's height above the plane of the horizon, the square root of the remainder is the random required.

2. Then as the random now found, is to the random given, so is radius to the tangent of elevation.

Example.

A cannon planted on an eminence 200 feet above the horizontal plane, whose greatest random on the horizon is known to be 6000 feet, it is required to find the greatest random from that eminence, and the direction to reach it?

First, $6000 + 200 = 6200$ the sum, and 38440000 is the square of it, and 200 squared is 40000. Then,

From 38440000
Take 40000

Remains 38400000 whose square root is 6196, the random required.

2. Then,

As random found 6196 3.79211

Is to the random given 6000 3.77815

So is radius 10.00000

To tangent $44^{\circ} 5'$, the direction 9.98604

PROP. XXVIII.

Given the utmost horizontal random of any piece of artillery, to find at what distance, without altering the elevation or charge of powder, it will be able to strike an object on a plane of a given ascent.

RULE.

Take the difference between one fourth of the given random and the object's elevation, find a mean proportional between

tween the given random and the aforefaid difference, which add to half the given random, the sum will be the distance required.

Example.

If the greatest horizontal random of a piece be 1500 yards, at what distance (supposing the charge of powder and elevation to continue the same) will it be able to strike an object elevated 44 yards above the plane of the horizon?

4) 1500 the greatest random.

375 a fourth part of it.
44 object's elevation.

331 their difference.

1500 multiply by the greatest horizontal random.

1655

331

496500 whose square root is 704.6 yards, the mean proportional.

Then half of the utmost random is 750 yards, and $750 + 704.6$ yards, = 1454.6 yards, the required distance.

Or thus.

To log. 331
Add log. 1500

2.51982

3.17609

2) 5.69591

Answering to 704.6

2.84795

Then 704.6

750.

1454 6 yards as before.

P R O P. XXIX.

Given the angle of elevation of any object above the level of the piece, the horizontal distance, and the piece's elevation, required the impetus to hit the object?

R U L E I.

Find the natural co-sine of the sum of the angles which the line of direction makes with the object and the zenith, and likewise the natural co-sine of their difference; and then,

A

As difference of these co-sines
 To the co-sine of the object's elevation,
 So is half the horizontal distance
 To the height of the perpendicular projection (or impetus.)

Example.

There is an object whose height above the level of the piece is $10^{\circ} 20'$, its horizontal distance is 7920 feet, required the impetus when the elevation of the piece is $41^{\circ} 20'$?

From $41^{\circ} 20'$, ($=\angle CAD$. See *fig. 7*) take $10^{\circ} 20'$ ($=\angle BAD$) remains 31° ($=\angle CAB$.) Also from 90° take $41^{\circ} 20'$ ($=\angle CAD$) leaves $48^{\circ} 40'$ ($=\angle CAZ$) which the line of direction makes with the zenith ZA ; also the sum of the angles is $79^{\circ} 40'$ ($=\angle CAZ + \angle CAB$) its cosine = .17937, and the co-sine of their difference $17^{\circ} 40' = .95283$. The co-sine of the object's elevation $10^{\circ} 20' = .98378$.

From .95283

Take .17937

Diff. cos. = .77346

As .77346

1.88843

To .98378

1.99289

So is 3960

3.59769

To 5037 impetus

3.70215

RULE II.

To radius add the co-sine of the plane's elevation, the arith. com. of the co-sine of the piece's elevation, the arith. com. of the sine of the difference between the piece's elevation and the object's, and the logarithm of a fourth part of the horizontal distance together, a tenth part of the sum is the impetus required.

Example.

There is an object whose height above the level of the piece is $10^{\circ} 20'$, its horizontal distance is 7920 feet, required the impetus when the elevation of the piece is $41^{\circ} 20'$?

The

The difference between $41^{\circ} 20'$ and $10^{\circ} 20' = 31^{\circ}$ of its fine is 9.71184.

To radius 10.00000

Add co-fine $10^{\circ} 20'$ 9.99289

The arith. com. co-fine of $41^{\circ} 20'$ 0.12443

The ar. co. fine ($41^{\circ} 20' - 10^{\circ} 20'$) = 31° 0.28816

The log. of $\frac{7920}{4} = 1980$ 3.29666

To 5037 the impetus 2|3.70214

PROJECTIONS made on DESCENTS.

PROP. XXX.

Given the greatest random, on a plane of a given descent, to find the greatest random on a plane of a given ascent.

RULE.

As the sum of the radius and natural sine of the given ascent, is to the difference between the radius and the natural sine of the given descent, so is the given random to the random required.

Example.

The greatest random of a piece made on a plane whose descent was 10° is 8000 feet, what is her greatest random on a plane which is 5° above the level of the piece?

As 1.08715 : .82636 :: 8000 : 6081, the random required.

Corollary. Given the angle of obliquity, you have the elevation of the utmost random, by subtracting half that angle from 45° .

PROP. XXXI.

Given the greatest horizontal random, to find the greatest random on a plane of a given descent.

RULE.

As radius less the natural sine of the plane's descent is to radius, so is the given horizontal random, to the random required.

Example.

If the greatest horizontal random of a piece be 7960 feet, what is the greatest random on a plane, which is 10° below the level of the piece?

From radius	1
Take natural sine 10°	0.174
Remains	<u>.826</u>

Then, as .826 : 1 :: 7960 : 9634, the random required.
Or you may find the random by the following

RULE.

R U L E.

The complement of half the angle included between the zenith and the descending plane to 90° is the elevation.

Then as tangent of the elevation is to the tangent of the plane's descent, so is the given greatest random to a *fourth* number, which added to the given horizontal random, gives the random required.

Take the above *Example*.

Then $90^\circ + 10^\circ = 100^\circ$, the angle between the zenith and plane, half is 50° , its complement is 40° the elevation.

As tangent 40° 9.92381

Is to the tangent 10° 9.24631

So is 7960 3.90091

To 1673 3.22341

Then to 1673

Add 7960

Sum 9633, the random as above, nearly.

P R O P. XXXII.

Given the greatest random on a plane of a given descent, to find the greatest random on the plane of the horizon.

R U L E.

As radius is to the difference between the radius and natural sine of the plane's descent, so is the given greatest random, to the greatest random on the plane of the horizon required.

Example.

If the greatest random of a piece on a plane whose descent is 10° , is 9634 feet, to find the greatest horizontal random?

As $1 : .8263 :: 9634 : 7960$ feet, the greatest random on the plane of the horizon.

Or thus :

To log. .8263 1.91713

Add log. 9634 3.98380

Sum 7960 3.90093

P R O P.

PROP. XXXIII.

Given the distance of any object on a plane of a given descent, and the piece's impetus, to find the direction to hit that object.

RULE

As radius is to the given distance, so is the sine of the object's angle of depression to a *fourth number*, to which add twice the impetus, and note that *sum*; then as the object's distance is to that *sum*, so is the *fourth number* to a *fifth number*; from the object's given distance take the *fifth number*, and note half the difference.

Then as the impetus is to the half difference, so is radius to the sine of an angle, half the sum of the angle last found, and the object's depression taken from 90° gives the higher elevation, and half their difference the lower elevation.

Example.

What direction must a cannon have to hit an object whose distance is 7400 feet on a plane whose depression is 6° , and impetus 3980 feet?

As radius 10.00000

Is to 7400 3.86923

So is the sine of 6° 9.01923

To 774 a fourth number 2.88846

Then $(3980 \times 2 =) 7960 + 774 = 8734$, the *sum*.

As $7400 : 8734 :: 774 : 913$, a *fifth number*.

And $\frac{7400 - 913}{2} = 3249$ the half difference.

As impetus 3980 3.59988

Is to half difference 3249 3.51175

So is radius 10.00000

To sine $54^\circ 44'$ 9.91187

Then $90 - \frac{54^\circ 44' + 6^\circ}{2} = 59^\circ 38'$, the higher elevation.

And $\frac{54^\circ 44' - 6^\circ}{2} = 24^\circ 22'$, the lower elevation.

S

PROP.

P R O P. XXXIV.

Given a piece's random made at a known elevation on a plane of a given descent, to find the piece's direction to hit an object at a known horizontal distance.

R U L E.

To the natural tangent of the given elevation add the natural tangent of the plane's descent, and see what angle answers to that sum, from which take the plane's descent, the remainder is a *second* angle, the sum of the plane's descent and *second* angle, taken from 180° leaves a third angle.

As sine of the third angle is to the sine of the second angle, so is the descent random to a fourth number; as the fourth number is to the horizontal distance, so is the sine of twice the given elevation to the sine of double the elevation required.

Example.

There is a plane whose descent is 10° , and the random of a piece made on that plane, at an elevation of 42° , was 7200 feet; it is required to find the direction of the same piece, to hit a battery on the plane of the horizon that is 5880 feet from you?

To natural tangent of $42^\circ = .90040$
Add natural tangent of $10^\circ = .17632$

The angle answering is $= 47^\circ 6'$ 1.07672

From $47^\circ 6'$
take $10^\circ 0'$

From 180°
take $47^\circ 6'$

Second $\angle$ $37^\circ 6'$

$132^\circ 54'$ third $\angle$.

As sine $132^\circ 54'$

9.86483

Is to $37^\circ 6'$

9.78046

So is 7200

3.85733

To 5880 a fourth number

3.77296

As 5928

3 77296

Is to 5880

3.76937

So is line $(42^\circ \times 2 =) 84^\circ$ 9.99761To $80^\circ 31'$ 9.99402

Then $\frac{80^\circ 31'}{2} = 40^\circ 15\frac{1}{2}'$, the direction required.

P R O P. XXXV.

Given the greatest impetus of a piece to find the least impetus, to hit an object at a known distance, on a plane of a given descent.

R U L E.

As radius is to the sine of the plane's descent, so is the object's distance to a fourth number; half the difference between the object's distance and the fourth number, is the least impetus required.

Example.

Suppose the greatest impetus of a twenty-four pounder be 6000 feet, it is required to find the least impetus to hit an object whose distance is 4600 feet on a plane $14^\circ 52'$ below the level of the piece?

As radius

10.00000Is to the sine of $14^\circ 52'$

9.40920

So is 4600

3.66275

To 1180 a fourth number

3 07195

Then $\frac{4600 - 1180}{2} = 1710$ the least impetus.

P R O P. XXXVI.

Given the impetus of a piece, placed on a plane whose height above the horizon is also given, to find the elevation to hit an object at a given horizontal distance.

R U L E.

As the object's height above the horizon, is to the horizontal distance, so is radius to the tangent of double the angle of elevation, half whereof is the elevation required.

S 2

Example.

Example.

Admit the impetus of a twenty-four pounder be 6000 feet, planted on a rampart 410 feet high, above the plane of the horizon, it is required to find the elevation of the piece, to hit a battery whose horizontal distance is 5200 feet?

As 410	2.61278
Is to 5200	3.71600
So is radius	10.00000
To tangent of $85^{\circ} 29'$	11.10322

Then $\frac{85^{\circ} 29'}{2} = 42^{\circ} 44\frac{1}{2}'$ the elevation required.

P R O P. XXXVII.

Given the greatest random in feet of any piece of artillery, to find the time of the ball's flight.

R U L E.

Half the square root of one fourth of the greatest random in feet, is equal to the time of flight in seconds.

Example.

Suppose the greatest random of a twenty-four pounder be 15000 feet, to find the time of the ball's flight, in reaching the object?

First, a fourth part of 15000 is = 3750.

And the square root of 3750 = 61.

Therefore $\frac{61}{2} = 30\frac{1}{2}'$ seconds, the time required.

P R O P. XXXVIII.

Given the impetus, piece's elevation, and the elevation of the object above the level of the piece, to find the time of flight.

R U L E.

Take the object's elevation from the piece's elevation, and note the *difference*, and take half the square root of the given impetus in feet; then say, as the co sine of the object's elevation

elevation above the level of the piece, is to half the square root of the impetus, so is the sine of the difference, to the time of flight in seconds.

Example.

There is a plane standing $12^{\circ} 30'$ above the level of my piece, on which is situated a castle, I want to adjust a fuse for a bomb to strike the castle, when the impetus of the piece is 4900 feet, and the elevation is $44^{\circ} 30'$, required the time of flight?

From the piece's elevation	44°	$30'$
Take the object's elevation	12	30
The difference is	32	00

The square root of 4900 is 70, and half $70 = 35$.

As co-sine $12^{\circ} 30'$

9.98958

Is to 35

1.54406

So is the sine of 32°

9.72421

To 19 seconds

1.27869

The foregoing problems seem to be the most material that can occur in the firing of artillery; and by what is already done, may other problems be answered that should happen in practice, for hitting objects, whether on a level surface, or placed above or below the level of the piece.

When any object is to be bombarded on a horizontal plane, engineers generally elevate the piece to 45° , and then increase or diminish the charge of powder, till the object aimed at can be hit; and by this method of proceeding much powder is saved; for at 45° elevation the velocity is less, and consequently requires a less charge of powder for a horizontal random, than at any other elevation; if the piece be levelled below 45° , and the bomb does not fall to the distance required, the piece must be raised a little, but if the bomb falls beyond the object aimed at, it must then be lowered a little, and by these trials the proper elevation may easily be acquired. When you have found the proper elevation, then fix it in that direction, and you will throw very near the object: I say very near, for there often happen circumstances, which alter the flight of the bomb, because

FIG. it is some difficulty to charge a mortar in a hurry, exactly in the same manner, so as to make it correspond with strict mathematical rules.

When any buildings are to be bombarded, it is better to elevate the piece above 45 degrees, than under it, for then the bomb rises to a greater height, and consequently falls with the greater force, and will do more damage; and the greater the charge the better they succeed; but more than sufficient to burst the bomb, cannot produce any advantageous effect.

But when it is required to throw bombs amongst a parcel of men, level the bomb below 45 degrees, for then the bomb will have less force, and will not go so far into the ground, and the splinters occasioned by its explosion will do more execution; but increasing or lessening the charge of powder is better, and then you will have no need to alter the elevation.

The greatest distance to which a bomb of five or six hundred weight can be thrown with the strongest charge, is between two or three miles.

THE problems proposed in the foregoing pages being solved in a practical manner, without shewing the investigation of their several rules from the theory, I thought to have added no more, but to please some part of my readers, who desire to see into the theory of projectiles, I shall here insert from the *Harmonia Mensurarum* of the celebrated Mr. Cotes, an *English* translation of what he has delivered at page 87, concerning the motion of projectiles in a non-resisting medium, viz.

THEOREM.

The curve described by the motion of a body projected obliquely is a parabola, and the velocity of a projectile in any point of the parabola is such, as a body acquires in falling down the fourth part of the parameter belonging to that point taken from the vertex.

3. Suppose the projectile be thrown from A to E, that is, let AE be the line of direction, and ACD be the curve described: let AE be the space which the projectile would describe in any given time, by means of the force impressed, supposing it had no motion downward from the force of gravity. And let AB be the space through which it would descend from A towards the center of the earth in the same time.

time. By completing the parallelogram $EABC$, it will easily appear that the projectile by the composition of motion at the end of the given time will be found in the point C . But AE is as the time in which it is described, consequently $\overline{AE}^2$ or $\overline{BC}^2$ is as the square of the time (by Th. 2. descent of heavy bodies) therefore $\overline{AE}^2$ or $\overline{BC}^2$ is as AB , wherefore the points C of the curve described through which the projectile moves are in the parabola, whose diameter is AB , vertex A , and the parameter belonging to the vertex A is $\frac{\overline{BC}^2}{AB}$ or $\frac{\overline{AE}^2}{EC}$.

The velocity of a projectile in any point A is such as would carry it from A to E in the same time that a body would descend from E to C , and the velocity acquired by this descent from E to C , is to that velocity whereby the length AE would be described in the same time as EC to $\frac{1}{2} AE$ (by Theo. 1. descent of heavy bodies); wherefore the velocity acquired in falling from E to C , is to the velocity acquired in falling down the fourth part of the parameter, namely $\frac{\frac{1}{4}AE^2}{EC}$, in the subduplicate ratio of the length EC to to the length $\frac{\frac{1}{4}AE^2}{EC}$ (by Theo. 2. desc. bodies) that is, as the square root of EC to the square root of $\frac{\frac{1}{4}AE^2}{EC}$, which

square roots are to one another as EC to $\frac{1}{4}AE$, wherefore the velocity of a projectile in the point A , is such as a body would acquire in falling through a fourth part of the parameter belonging to the vertex A . Q. E. D.

Corollary 1. If a projectile be thrown according to the direction AE , and the parameter of the parabola which a projectile describes belonging to the vertex A be equal to $\frac{AE^2}{EC}$, this parabola will pass the point C .

Corollary 2. In the same or different parabolas, the parameters belonging to as many vertices, are to one another in the duplicate ratio of the velocities, wherewith projectiles are carried in those vertices (by The. 2. desc. bodies.)

Corollary 3. Wherefore in any parabolas whatsoever, if the *impetus's* are equal, or the motion of the same projectile, or equal velocities of different projectiles, the parameters

FIG. will always be equal, and *vice versa*, howsoever different the
3. direction of the motion shall be in those places.

PROBLEM.

Given the velocity wherewith a projectile is thrown from any given place, and also the position of a mark, to find the direction to hit that mark.

The velocity being given there will be given the parameter of the parabola which the projectile describes. For it is manifest by the above theorem, if a body descends so far whilst its given velocity in falling be acquired, the height by this descent will be equal to a fourth part of the parameter.

Let C be the mark situated in the given right line AC, at
4. A, the place from whence the projectile is to be thrown, erect AP perpendicular to the horizon, and equal to the parameter found at the point A, (for the parameter is given, since the velocity wherewith the projectile is cast from the point A is given; for it is equal to four times the height from which a body must fall in order to acquire that height.) Let AP be bisected by the line KH, cutting it perpendicularly in G, at A erect AK perpendicular to AC, and continue it till it meets in K, from K as a center, and with the distance KA, describe the circle AHP. Lastly, let the right line BCEI be drawn perpendicular to the horizontal line AB, so as to pass through the mark C, and if possible to cut the circle in E and I; AE and AI will be the directions sought, in either of which the projectile being thrown with the given velocity will hit the mark. Q. E. D.

For if PE, PI be joined by reason of the similar triangles PAE, AEC, PA will be equal to $\frac{AE^2}{EC}$, for PA : AE :: AE · EC, and PA × EC = AE × AE, therefore PA = $\frac{AE^2}{EC}$. Likewise by reason of the similar triangles PAI,

AIC, PA will be equal to $\frac{AI^2}{IC}$, for PA : AI :: AI : IC,

wherefore, since PA is equal to the parameter at the point A of the parabola which the projectile describes, it is evident

dent by Corollary 1. of the above Theorem, that this parabola will pass through the mark C. FIG. 4

Corollary 1. It is manifest that the right line AH bisects the angle CAP, which measures the visible distance between the mark and the zenith.

Corollary 2. When the points E, I, coincide in H, or the two directions AE, AI coincide in one line AH, then AC the distance of the mark becomes AK, a *maximum*, that is the greatest distance or random that can be passed through with that given velocity.

Corollary 3. Any two directions AE, AI either of which may be made to the same mark, are equally distant from the direction AH, that is, the angles EAH, IAH are equal.

Corollary 4. The horizontal distance of the mark AC or AB, is as the sine of double the angle of elevation EAC or IAC. For if the right line CEI meet the right line GH in F; AC or GF will be equal to the sine of the arc AE or AI, which measures double the angle EAC or IAC. 5

Corollary 5. But Ax the greatest distance of the mark situated in the horizontal line $A\beta$ is equal to AG half of the parameter, and consequently is given, and this mark x may be hit, when HAx the angle of elevation, is half a right one, or 45 degrees.

Corollary 6. If a projectile be thrown according to the direction of AE, the altitude of the highest point of the parabola which the projectile describes above the horizon, will be equal to $\frac{1}{4} EC$. For if AC be bisected at T, and TR be drawn parallel to CE, cutting AE in R; AC will be an ordinate to TR, the axis of the parabola, which its principal vertex will divide into two equal parts in V; wherefore the altitude TV is equal to $\frac{1}{2} TR$ or $\frac{1}{4} CE$; but when CE is the versed sine of the arch AE, or of double the angle CAE, it is manifest that TV, the altitude of the projectile, is the versed sine of double the angle of elevation.

Corollary 7. But the greatest altitude of a projectile thrown according to the direction AP, perpendicular to the horizon, will be equal to $\frac{1}{4}$ of the parameter AP, and consequently is given; for in this case AE, EC, coincide with the perpendicular direction AP, the altitude TV, or $\frac{1}{4} EC$, will now become $\frac{1}{4} AP$.

Corollary 8. The time wherein a projectile, thrown according to the direction AE, will be carried from A to C, is

FIG. is as the chord AE or as $\frac{1}{2}AE$, that is, as the sine of the angle of the elevation EAC.

5. *Corollary 9.* But the greatest time when the projectile is thrown according to the direction AP, which is perpendicular to the horizon, is the same as a body will descend from P to A, through the whole parameter, and consequently is given. For in this case the projectile ascends in a right line to the altitude $\frac{1}{2}AP$, that is, will rise to a fourth part of the parameter, and then descends from the same altitude; and the times of ascent and descent taken together, make twice the time of the descent alone.

Since the writing of the above, Mr. *Emerson's Principles of Mechanics* came to hand, where I find at page 33 to page 37, he has elegantly treated the subject of projectiles, and seems to have rendered that subject quite complete, by adding a new proposition to the other, from which he deduces two general scholia, which answer all the usual questions in the doctrine of projectiles; all the foregoing problems I compared with his scholia, and the answers exactly agreed, though quite different in the method of solution.

The first scholium is deduced from his general proposition of horizontal distances of projections, made with any velocities, and at any elevations; where, for brevity's sake, he puts

v = velocity of the projectile, measured by the space it passes thro' in time 1.

6. f = descent of a body by gravity in the same time.

x = AE, the horizontal distance, or amplitude.

s = sine } of the elevation VAE.

c = co-sine }

A = sine } of twice the elevation.

B = vers. sine }

h = height of the perpendicular projection with the velocity v .

Let

a = altitude of projection.

t = time of flight.

SCHOLIUM.

Then $h = \frac{v^2}{4f}$, whence

$$\text{Horizontal distance } x = \frac{vvc}{f} = \frac{vvA}{2f} = 4sch = 2Ah.$$

Altitude

$$\text{Altitude of projection } a = \frac{vvs}{4f} = \frac{vvB}{8f} = ssh = \frac{1}{4}Bb.$$

$$\text{Time of flight } t = \frac{vs}{f} = 2s\sqrt{\frac{h}{f}}.$$

From the *data* of any question in horizontal distances of projections, may the unknown quantities be found by those that are known by an easy reduction. Whence

$$v = 2\sqrt{fb} = \sqrt{\frac{fx}{sc}} = \sqrt{\frac{2fx}{A}} = \sqrt{\frac{2Afb}{sc}} = \frac{2\sqrt{fa}}{s} \\ = \sqrt{\frac{8af}{B}} = \frac{ft}{s}.$$

$$A = 2sc = \frac{x}{2h} = \frac{vvs}{2fb} = \frac{2fx}{vv}.$$

$$B = \frac{8fa}{vv} = \frac{8fssh}{vv}.$$

$$s = \sqrt{\frac{a}{b}} = \frac{ft}{v} = \frac{t\sqrt{f}}{2\sqrt{b}}.$$

$$b = \frac{vv}{4f} = \frac{x}{2A} = \frac{a}{ss}.$$

Here follow some examples shewing the use of the foregoing scholium.

Example 1.

Given the greatest random = 6840 feet, required the piece's elevation to hit an object a mile off?

Here $\frac{6840}{2} = 3420 = b$, and $x = 5280$ feet.

Then $A = \frac{x}{2b} = .77193 = 50^\circ 31'$, double the elevation,

half is $= 25^\circ 15\frac{1}{2}'$, elevation required. Or $2sc = \frac{x}{2b} = .77193$

$= A$, the sine of double the elevation as above. See *Prob. I.* Now by having the angle of direction given, you have a the altitude of projection;

For $a = ssh$ by the scholium = 623, nearly.

Example 2.

Given the greatest random of a cannon = 6840 feet to find what random the said piece makes at an elevation of $25^\circ 15\frac{1}{2}'$?

Here

Here $b=3420$; $A=\text{fine } 50^{\circ} 31'=.7718$ twice the elevation; $x=\text{random required}$.

Then $x=2Ab=5280$ feet. See *Prob. II.*

Example 3.

Given the distance of an object a mile $=5280$ feet, and the piece's elevation $=64^{\circ} 44\frac{1}{2}'$; to find the greatest random, and consequently the perpendicular shot?

Here $x=5280$, $A=129^{\circ} 29'=.77193$ twice the elevation.

Then $2b=\frac{x}{A}=6840$ feet, the greatest random. And $b=3420$. See *Prob. III.*

Example 4.

Given the altitude of projection $=623$ feet, and the perpendicular projection $=3420$ feet, to find the horizontal distance?

Here $b=3420$, $a=623$, $x=\text{distance required}$.

Then $ssh=a$, and $s=\sqrt{\frac{a}{b}}=.4267$ the sine of $25^{\circ} 15\frac{1}{2}'$ the elevation.

Now put $A=\text{fine of twice the elevation}=50^{\circ} 31'$; then $2Ab=x=5280$ feet, the distance from the object. See *Prob. V.*

Example 5.

Given the greatest random of a piece $=28000$ feet, to find the ball's velocity?

Here $b=14000$, $f=16$ feet the descent of a heavy body in the first second.

Then $v=2\sqrt{fh}=946.4$ velocity. See *Prob. IX.*

Example 6.

Given the piece's elevation $=38^{\circ} 20'$, and object's distance $=7920$ feet, required the time of the ball's flight?

First sine $38^{\circ} 20'=.62023$

Co-sine $=.78441$

And $x=7920$, $f=16$ feet.

Then $b=\frac{x}{4s}=4070$.

And $t=2s\sqrt{\frac{b}{f}}=19\frac{1}{2}$ seconds. See *Prob. X.*

P R O P.

P R O P. XXXIX.

Let a ball be projected at an angle of 32 degrees elevation above the horizon, and let its horizontal distance be 2400 yards; it is required to determine the greatest altitude it will ascend, the perpendicular projection, time of flight, and velocity?

Here $s = .52991$ nat. sine } of 32° elevation.
 $c = .84804$ co-sine }
 $f = 16\frac{1}{2}$ feet.
 $x = 2400$ yards,

Then $h = \frac{x}{4sc} = 1335.15$ yards the perpendicular shot.

$sh = 374.91$ yards the altitude of projection.

$2\sqrt{\frac{h}{f}} = 9.64$ seconds, time of flight.

$v = 2\sqrt{fh} = 293.06$ yards in a second, the velocity.

It will be needless to proceed any farther, these examples being abundantly sufficient to shew the use of this general scholium.

I shall here subjoin a table of theorems for projections on the plane of the horizon.

First, put D for the angle of direction.

b = tangent of the direction.

v = velocity.

$f = 16\frac{1}{2}$ feet.

x = horizontal distance.

h = perpendicular projection.

a = altitude of projection.

t = time of flight.

s = sine

c = co sine } elevation.

TABLE

A TABLE for horizontal Projections.			
Cases	Given	Required	Theorem.
1	D, b	a, x, t,	$a = ssh$ $x = 4sch$ $t = \frac{s\sqrt{b}}{2.006}$ seconds.
2	D, x	a, b, t	$a = \frac{bx}{4}$ $b = \frac{1}{4}x \times \frac{1}{b} + b$ $t = \frac{\sqrt{bx}}{4.012}$ seconds.
3	b, x	a, b, t	$a = \frac{b}{2} \pm \frac{1}{4}\sqrt{4b^2 - x^2}$ $b = \frac{2b \pm \sqrt{4b^2 - x^2}}{x}$ $t = \frac{\sqrt{2b \pm \sqrt{4bb - xx}}}{4.012}$ seconds.
4	a, b	x, b, t	$x = 4\sqrt{ha - aa}$ $b = \sqrt{\frac{a}{h-a}}$ seconds. $t = \frac{\sqrt{a}}{2.006}$ seconds.
5	t, b	a, x, b	$a = 4.023tt$ $x = 8.024b\sqrt{b - 4.023tt}$ $b = \frac{2.0054t}{\sqrt{b - 4.023tt}}$
6	x, a	b, b, t	$b = \frac{4a}{x}$ $b = a + \frac{xx}{16a}$ $t = \frac{\sqrt{a}}{2.006}$ seconds.

A TABLE

A TABLE of horizontal Projections continued.			
Cases	Given	Required	Theorems.
7	D, a	x, b, t	$x = \frac{4a}{b}$ $b = a + \frac{a}{bb}$ $t = \frac{\sqrt{a}}{2.006} \text{ seconds.}$
8	t, x	a, b, h	$a = 4.023tt$ $b = \frac{16.092tt}{x}$ $h = 4.023tt + \frac{1.1xx}{4.023tt}$
9	D, t	a, x, b	$a = 4.023tt$ $x = \frac{6.092tt}{b}$ $b = 4.023tt \times \frac{1}{bb} + 1.$

FIG.

The other scholium is deduced from his general proposition of the distances of projections made on inclined planes.

Where AB denotes the inclined plane,

AC the direction of the projectile,

AZ the zenith line,

AEB the path of the ball, and let

v = velocity of the projectile in A, measured by the space it describes in the time 1.

f = space described by a descending body in the same time.

s = sine of CAB.

c = sine of CAZ.

z = sine of ZAB.

x = AB the oblique distance or random.

b = height of the perpendicular projection.

SCHOLIUM.

Perpendicular projection $b = \frac{vv}{4f}$. Whence

Length of the projection $x = \frac{scvv}{fzz} = \frac{4sch}{zz}$.

Height

FIG. 7. Height of the projection $a = \frac{svv}{4fz} = \frac{sh}{zz}$.

Time of flight $t = \frac{sv}{fz} = \frac{2s}{z} \sqrt{\frac{h}{f}}$.

Hence by reduction, we have,

$$v = 2\sqrt{fh} = z\sqrt{\frac{fx}{sc}} = 2\sqrt{\frac{sch}{x}} = \frac{2z\sqrt{af}}{s} = \frac{ftz}{s}.$$

$$z = v\sqrt{\frac{sc}{fx}} = 2\sqrt{\frac{sch}{x}} = \frac{sv}{2\sqrt{fa}} = s\sqrt{\frac{h}{a}} = \frac{2s}{t}\sqrt{\frac{h}{f}}$$

$$= 2s\sqrt{\frac{b}{ft}} = \frac{sv}{ft}.$$

$$s = \frac{2z\sqrt{af}}{v} = z\sqrt{\frac{a}{b}} = \frac{tz}{2}\sqrt{\frac{f}{b}} = \frac{ftz}{v}.$$

PROP. XL.

Given the utmost horizontal random to find the greatest random of the same piece, on a plane of a given ascent.

Example.

Admit a cannon's greatest horizontal random be 7980 feet, to find the greatest random of the said piece, upon that plane which is $8^{\circ} 40'$ above the level of the piece?

From $\angle ZAD = 90^{\circ} 00'$

Take $\angle BAD = 8 \quad 40$

$\angle ZAB = 81 \quad 20$

Since the greatest projection upon an inclined plane, is when the line of direction bisects the angle between the plane and the zenith, viz. $\angle ZAB$, see page 176. of Mr. Emerson's Fluxions: therefore $\angle ZAC = \angle CAB = 40^{\circ} 40'$. Then by the scholium

$$s = \text{fine } \angle CAB = 40^{\circ} 40' = .65165$$

$$c = \text{fine } \angle CAZ = 40 \quad 40 = .65165$$

$$z = \text{fine } \angle ZAB = 81 \quad 20 = .98858$$

$x =$ required random.

$$\text{Here } h = \frac{7980}{2} = 3990 \text{ the perpendicular projection.}$$

$$\text{And } x = \frac{4sch}{zz} = 6939 \text{ feet, the random required.}$$

Given

PROP. XLI.

Given the distance of an object upon a plane of a given elevation, and the piece's perpendicular projection, to find the elevation to hit that object.

Example.

Suppose the perpendicular projection be 5820 feet, the object's distance be 7000 feet, on a plane whose ascent is $8^{\circ} 30'$ above the horizontal level, to find the angle of direction to hit that object?

$$\text{From } 90^{\circ} \quad 00' = \angle ZAD$$

$$\text{Take } 8 \quad 30 = \angle BAD$$

$$\hline 81 \quad 30 = \angle ZAB$$

$$s = \text{fine } \angle CAB$$

$$c = \text{fine } \angle CAZ$$

$$z = \text{fine } \angle ZAB = 81^{\circ} 30' = .98901, \text{ the sum of the angles } CAB, CAZ.$$

$$b = 5820$$

$$x = 7000$$

$$\text{Then } \frac{4sch}{zz} = x \text{ by the scholium.}$$

Therefore $sc = \frac{xzz}{4b} = .294114$ the product of the fines of the angles CAB, CAZ. Then by *Prop. 4. Trigonometry*, the angles are found to be $70^{\circ} 33'$, and $27^{\circ} 57'$, the elevation required. See *Prob. 21*.

PROP. XLII.

Given the utmost random on an ascending plane, to find the impetus, height of the projection, time of flight, and velocity.

Example.

Suppose the length of the projection on an ascending plane is 6840 feet, being the utmost random of a cannon, to find the height of the perpendicular projection, the height of the projection, time of flight, and velocity.

T

Here

FIG.

7. Here since $ZAC = CAB$, therefore

$$x = \frac{4ssb}{zz},$$

$$h = \frac{1}{4}x \times \frac{zz}{ss} = x \times \frac{s^2 CAD}{Rad.^2}.$$

$$v = \sqrt{4fb}$$

$$\frac{ssb}{zz} = \frac{1}{4}x, \text{ height of the projection.}$$

$$\frac{2s}{z} \sqrt{\frac{h}{f}} = \sqrt{\frac{x}{f}}, \text{ the time of flight.}$$

As I have been very particular in illustrating the foregoing problems in numbers, I shall not insist any more on this, it being so easy to be understood.

Notwithstanding what has already been advanced on this subject, the knowing the true distance that any piece will throw a bullet at all times, is a matter subject to many uncertainties, since there is such a variety in the trueness of the bore, in the levelling and directing the piece, the nature of the powder taking fire, and the resistance of the air, which last, being a resisting medium, renders the curve of the shot nearer an hyperbola than parabola; and if any person considers the curve as such, as is laid down in the *Principia* of the illustrious Sir Isaac Newton, so as to apply it to practice, would, in my opinion, take upon him a very hard task; for it is one of the most difficult problems in all the doctrine of fluxions, to shew how far a projectile deviates from a parabolic track, by the resistance of the air; for the principles and calculations made use of on this occasion, will in no wise be fitting to the experience and understanding of learners; and what makes it worse still, it can be of no manner of use in the art of gunnery, because it can be reduced to no rules.

The following tables are computed from Sir Isaac Newton's principles, and applied to the motion of projectiles by D. Bernoulli, in a Latin memoir printed in tom. II. page 338, &c. of the *Com. Acad. Petrop.* I thought (says he) no small profit would occur to these speculations, if I reduced to a calculation some of the experiments made by the excellent M. Gunther before some academicians, whence it will appear what stupendous effects the air has upon bodies of near eight thousand times its specific gravity. And from hence the forces

forces of gunpowder may be exactly compared among themselves; and many other useful things deduced. There were abundance of experiments made of divers kinds, among which I shall pick out those that can be serviceable in this affair; such as those made with guns exactly placed in the perpendicular, and observing the quantity of powder, and the time of the globe's ascending and descending. The diameter of the ball was $23\frac{1}{4}$ hundred parts of an *English* foot, and made of iron; its specific gravity to that of air, as 7650 to 1, supposing the air all of the same density; the length of the bore was 7.7 feet, whence the following experiments were made.

Quantity of gunpowder in ounces Holland.	Whole time of ascent & descent.	Height of the globe in resisting air.	Time of ascent in resisting air.	Time of descent in resisting air.	Height to which the globe would ascend in vacuo with the same force.	Time of rising and falling in vacuo, when projected with the same force.
$\frac{1}{2}$	11	486	5.42	5.58	541	11.6
2	34	4550	14.37	19.63	13694	58
4	45	7819	16.84	28.16	58750	121

The following experiments were made with a gun just six feet the length of the bore, and with the same globe.

Ounces of powder.	Number of seconds that the globe staid in the air by observation.	Height to which the globe ascended in feet by calculation.	Time of ascent in seconds by calculation.	Time of descent in seconds by calculation.	Height in feet that the globe would ascend with the same force in vacuo by calculation.	Time of rising and falling in vacuo, with the same force in seconds.
$\frac{1}{2}$	8	257	3.95	4.05	274	8.2
2	20.5	1665	9.74	10.76	2404	24.5
4	28	3187	12.5	15.5	6604	40.5
6	32.5	4304	13.9	18.6	11810	54.3
8	38	5643	15.54	22.46	22394	74

Both these tables were exactly calculated; we shall just deduce some of the principal consequences therefrom. The greatest time of ascent was $16''.84$, and in the next, the greatest time of ascent was $15''.54$, the difference is only $1.3''$, and yet the difference of heights to which the globe would ascend in the resisting air is not to be neglected, as amounting to 2176 feet; from this alone it is plain to any body, that there is a certain time which the globe, though ascending ever so high, can never exceed; and the greatest time for the ascent in our globe is $= 19''.27$, whence is confirmed the following

THEOREM.

The time of ascent and descent taken together, is greater in vacuo than in pleno, the initial velocities being the same in both; likewise it may be infinite in both cases, but yet infinitely greater in vacuo than in pleno.

That the time of ascent and descent of a globe projected upwards with an infinite force in an uniform resisting medium, is infinitely less than when projected upwards with the same force in *vacuo*, though the forces of gravity be the same.

Supposing the forces of explosion, or the first velocities to be equal and infinite, then will also the height to which the globe will arise in *vacuo*, be infinitely greater than the height it can arise to in a resisting medium; and yet both of them is here infinite.

These assertions have been partly demonstrated already in the preceding paragraph; but it is worth notice, that the air which we supposed 7650 times lighter than iron, should exert so great a resistance upon an iron ball, that only staving $45''$ in the air, it should destroy $\frac{1}{2}$ of the height of its ascent by that force.

From the foregoing tables we may also gather some things concerning the force of gunpowder. Here the forces impressed upon balls are as the heights to which the globes would ascend in *vacuo*. In the first table we find these heights to be as 13694 to 541, or nearly as 26 to 1; when the quantities of powder were as 4 to 1. Whence it appears, that the strength of powder in driving a ball, increases far faster than in proportion to the weight of the powder; and I can see no other reason for it than this, that a great part of the inflamed powder flies through the touch-hole, and between the ball and the barrel; and that this useless part of the powder is in a far less ratio, when a great deal is fired, than when only a little.

tle. But this may be collected from experiments ; considering the powder as air exceedingly condensed, that is, that force of air very much condensed increases in a much greater ratio than the density ; for by calculation I found, that air 500 times denser than in its natural state, was at least 6000 times stronger than the elasticity of the common air.

I shall here annex some other experiments made with a mortar.

The diameter of the mortar = 1.05 feet *Englsh.*

diameter of the ball = 1.01 feet.

weight of the ball = 200 pounds *Holland.*

Quantity of powder in pounds <i>Holland</i>	$\frac{1}{2}$	1	2	3	4	5	6
Time in seconds the globe staid in the air.	3	7.5	12	16.5	20	23	25

Thus far D. *Bernoulli* ; as he never meddles with projectiles, I thought his memoir but of small service here, because it is only upon perpendicular shots ; but I thought his tables of experiments might be of some service.

I shall here observe this is the only person, Mr *Robins* says at page 51 of his preface, that has considered the actions of the air on military projectiles, founded on *Newton's* doctrine, but I can see very little in *Bernoulli's* memoir worth reading. For in the first part he lays down several hypotheses concerning the pressure of fluids, and by some experiments he says, he comes at last to this, that the pressure of any column of a fluid is just equal to the weight of it ; at the end he calculates the resistance of a curve or solid in a fluid, in a curve he says it is $= Fl. \frac{p\dot{x}^3}{bs^2}$ and in a solid, that the resistance

is to the resistance of the base as the fluent of $\frac{np\dot{x}\dot{x}^3}{bds^2}$ to $\frac{np\dot{x}^2}{2b}$;

according to our way of notation : but the whole memoir is wrote in the differential method, in an unintelligible manner. Now all this first part is elegantly calculated at page 254 of Mr. *Emerson's Fluxions*. But *Bernoulli* absurdly calls resistance by the name of pressure.

In the second part he spends a deal of time in finding the greatest velocity a descending globe can acquire in a fluid, and comes to the same conclusion as Sir *Isaac Newton*, Cor. 2. Prob. 38. B. II. Then afterwards he proposes to find the resistance of ships in the water, in the finding of which he takes

takes as many things for granted as he thinks fit, but would have the resistance found by experiment of men drawing a ship; all the calculations he here gives, contain nothing but what is common; this part he ends by exploding *Cartesius's* vortices, and the descent of bodies upon this hypothesis.

The whole design of the third and fourth parts, is only to find the motion of bodies ascending and descending in a fluid; and this has been done by a great many people, especially at page 288 of *Emerson's Fluxions*, in a more elegant, shorter, and plainer method; I can see no use any part of this can be of in the art of gunnery, for no body ever shoots perpendicular with great guns.

Mr. *Robins's* book, or *New Principles of Gunnery*, is a very good thing, and the experiments seem to be made with great care; but *Bernoulli* differs from him in several things, for he makes the resistance as the square of the velocity, but Mr. *Robins* makes it more. *Bernoulli* also says, that the force of powder (condensed as air condenses) increases in a far greater ratio than the density; but Mr. *Robins* says, page 7, that its force is as its density. Thus have I given a short account of this memoir of *Bernoulli's*.

Having a little spare room, I shall insert here a problem with its solution, for the entertainment of my readers, to determine the velocity of a bullet, though of no material use at all in the art of gunnery.

P R O B L E M XXXVIII.

Given the dimensions of any piece of artillery, the density of its ball, and the quantity of its charge, to determine the velocity which the ball will acquire from the explosion, supposing the elasticity of the powder at the first instant of its firing to be given?

S O L U T I O N.

It has been proved by Mr. *Robins*, that the quantity of the pressure exerted by gunpowder at the moment of its explosion, has an elasticity $999\frac{1}{3}$, or in round numbers 1000 times greater than common air; and since common air by its elasticity exerts a pressure on any given surface equal to the weight of the incumbent atmosphere with which it is in equilibrio, the pressure exerted by fired powder before it has dilated itself, is 1000 times greater than the pressure of the atmosphere. Now in order to solve the problem, the two following principles must be assumed.

I. That

I. That as soon as the bullet is got out of the piece, the FIG. action of the powder on the bullet ceases.

II. That all the powder of the charge is fired, and converted into an elastic fluid, before the bullet is sensibly moved from its place. These being premised,

Let AB be the axis of any piece of artillery, A the breech, 8. B the muzzle, DC the diameter of the bore, DEGC a part of the cavity filled with powder.

Let $AF=a$, $AM=x$, $AB=b$, and v =velocity of the ball at M, $s=16\frac{1}{2}$ feet, $c=3.1416$, d =diameter of the bore, or of the ball, in inches, w =weight of the ball in pounds; then since by experiments the pressure of the atmosphere upon a square inch is 14 pounds, therefore $\frac{14000 acdd}{4}$ =force of the powder at F, and

$$\frac{1}{a} : \frac{1}{x} :: \frac{14000cdd}{4} : \frac{14000acd}{4x} = \text{force of the powder at}$$

M; now by *Schol.* p. 13, of Mr. *Emerson's Mechanics*, $v \propto F \dot{x}$, and in a falling body the velocities generated are as the times, or as spaces uniformly described in these times; therefore $2s$ (space) : $2s$ (velocity) : : $\dot{x}$ (space) : $\dot{v}$ =velocity generated by a falling body in the time of describing $\dot{x}$; therefore in the case of a falling body, writing $2s$, $\dot{x}$, w , for v , $\dot{v}$, F , and in case of the ball, writing $\frac{14000acd}{4x}$ for F , in the

general proportion ($v \dot{v} \propto F \dot{x}$) it will $2s \dot{x} : w \dot{x} :: v \dot{v} : \frac{14000acd}{4x} \dot{x}$, therefore $v \dot{v} = \frac{14000 ac d s \dot{x}}{2 w x}$, and the fluent is

$$\frac{v v}{2} = \frac{14000 ac d s}{2 w} \times \log. x. \text{ But in F, } x=a; \text{ and corrected it}$$

$$\text{is } v v = \frac{14000 ac d s}{w} \times \log. x - \log. a = \frac{14000 ac d s}{w} \log. \frac{x}{a}.$$

And when $x=b$, $v = \sqrt{\frac{14000 ac d s}{w} \times \log. \frac{b}{a}}$, the velocity of the ball at the mouth of the cannon.

PROBLEM XXXIX.

FIG. 127. Suppose the uniform force of gravity to tend every where perpendicularly to the plane of the horizon VB , to determine the curve VPB , which a projectile describes in an uniform medium, which resisteth in any multiply ratio of the velocity?

This being a very intricate problem, I shall not trouble the reader with the fluxionary process, as it is already done in Vol. II. of my *Miscellanea Curiosa Mathematica*, but only lay down the following corollaries, which are deduced from a tedious calculation in fluxions.

Cor. 1. If the resistance of the medium be small, as is commonly experienced by an iron cannon ball projected through the air with a sufficient velocity, the curve VPB which the ball describes in a resisting medium, does not differ much from the conic parabola Vpb , which it would describe by the same uniform force of gravity g or 1 , acting on it. Yet because the resistance diminishes the velocity of the projection, CP the ordinate to the curve VPB , in a resisting medium, will be something less than Cp the ordinate to the curve Vpb of the conic parabola.

Put the abscissa $VC=x$, ordinate $CP=y$, ordinate $Cp=z$, amplitude $VB=b$, then $Cb=b-x$. But by the property of the parabola, the rectangle under the abscissas VC and Cb , is equal to the rectangle of the ordinate Cp into some given quantity, suppose m ; that is $VC \times Cb$ or $bx - xx = zm$, consequently $z = \frac{bx}{m} - \frac{xx}{m}$; therefore when the ordinate CP or y is a little less than Cp or z , we may put $y = \frac{bx}{m} - \frac{xx}{m} - ex^3$; then this equation in which e is small, will express the nature of the curve VPB very nearly. Put b and e for $\frac{b}{m}$ and $\frac{1}{m}$ respectively, then the equation to the curve becomes $y = bx - cxx - ex^3$. Now to determine the coefficients b , c , and e , take the first, second, and third fluxions of the equation, &c. and make $\dot{x}$ constant, whence we have $\dot{x} : \dot{y} :: 1 : b$; and when C coincides with V , it will be $\dot{x} : \dot{y} :: \text{rad. } VQ : \text{tangent } QS$ of the angle of projection TVQ ; wherefore if radius $= 1$, b will be the tangent of the angle of projection or elevation, and therefore when this angle is given,

given, then b is given. Now if v denote the velocity with which a body is projected from V , and f the altitude from which a body in falling by the constant force of gravity g acting on it, acquires that velocity v , by falling in a non-resisting space, then $2f = \frac{1+bb}{2c}$, and $c = \frac{1+bb}{4f}$, where

b is the tangent of the angle of projection, and $\text{rad.} = 1$. Therefore when b and f are given, c will be given; consequent-

ly $e = \frac{c\sqrt{1+bb}}{3a} = \frac{1+bb \times \sqrt{1+bb}}{12af}$; now for c and e in

the assumed equation to the curve, put their values just now found, then the equation $y = bx - cx^2 - ex^3$, becomes $y = bx$

$-\frac{1+bb}{4f}xx - \frac{1+bb}{12af}x^3$, where a is the altitude from which

a body by the constant force of gravity acting on it, would fall in a non-resisting medium, to acquire the greatest velocity. Likewise a may be found by experiment. For if a body be projected from V under a given angle TVB , with a given velocity in a supposed medium, and VB the amplitude of projection be observed, which let be denoted by A , then in the equation to the curve VPB , instead of x put A , and for y put a , because the ordinate CP or y vanishes in B ,

then $a = bA - \frac{1+bb}{4f}AA - \frac{1+bb}{12af}A^3$, whence $a = \frac{1+bb}{12fb - 1+bb \times 3A}AA$.

Cor. 2. To find the amplitude VB , make $y = a$, then $\frac{1+bb}{12af}xx + \frac{1+bb}{4f}x = b$, whence the amplitude VB or

$$x = -\frac{3a}{2\sqrt{1+bb}} + \sqrt{\frac{9a^2}{4+4bb} + \frac{12afb}{1+bb}}$$

Cor. 3. To find DG , the greatest altitude of projection; take the fluxion of the equation to the curve VPB , and make $y = a$, &c. Then VD or $x = -\frac{a}{\sqrt{1+bb}} +$

$$\sqrt{\frac{aa}{1+bb} + \frac{4afb}{1+bb}}$$

the equation to the curve, you will obtain y a *maximum*, or DG for the greatest altitude of projection.

Cor. 4. To determine b , the tangent of the angle TVB, under which a body projected with a given velocity, shall pass through the given point P. Instead of x and y in the equation to the curve, put the given quantities $VC=p$, and

$CP=q$; then the equation becomes $q = bp - \frac{\sqrt{1+bb}}{4f} pp - \frac{1+bb}{12af} p^3$. If the density of the medium be infinitely small,

a will be infinite, and therefore $q = bp - \frac{1+bb}{4f} pp$, the value of b the tangent may be found by this equation, which let be denoted by n , then in the above equation, put $1+bb$ for $1+nn$ for $1+bb^{\frac{1}{2}}$, whence $q = bp - \frac{1+bb}{4f} pp - \frac{1+bb\sqrt{1+nn}}{12af} p^3$; which being but of two dimensions will easily give the value of b , very nearly.

Cor. 5. Given the velocity of the projection, to find the angle of elevation, so that the piece may throw a ball the farthest distance possible. If in the equation *Cor. 2*, in which x denotes any amplitude VB, the tangent b be made variable, and the fluxions taken, and putting $\dot{x}=0$, &c. then $4f \times 1 + 2bb^{\frac{1}{2}} = 3ab \times 1 - bb\sqrt{1+bb}$; but since the tangent of the angle of elevation is b , rad. = 1, then $\sqrt{1+bb}$ is the secant; put s = sine of the angle of elevation for the greatest random, then by trigonometry $\sqrt{1+bb} : b :: 1 : s$, then $1+bb : bb :: 1^2 : ss$, and dividing, $1 : bb :: 1 - ss : ss$, whence $b = \frac{s}{\sqrt{1-ss}}$; now instead of b substitute

$\frac{s}{\sqrt{1-ss}}$ in the equation now found, and as we shall have

$$4f \times \frac{1-3ss}{1-ss}^2 = 3as \times \frac{1-2ss}{1-ss}^2, \text{ that is, } 4f \times 1 - 3ss = 3as$$

$\times 1 - 2ss$, from which equation s the sine of the required angle will be had. Or s may be found by approximation. In the above equation put $3as = 3a\sqrt{\frac{1}{2}}$, for if the curve were described in a non-resisting medium, the angle TVB would be half a right one, or 45 degrees, and therefore its sine = $\sqrt{\frac{1}{2}}$, rad. = 1. And therefore in a medium very rare,

s is $= \sqrt{\frac{1}{2}}$ nearly, consequently the equation will be $4f \times 1 - 3s^2 = 1 - 2ss \times 3a\sqrt{\frac{1}{2}}$, which will be easily solved like an equation of two dimensions, and hence s is found something less than $\sqrt{\frac{1}{2}}$, and consequently the angle of elevation to the greatest random in a resisting medium, as the air, is something less than the elevation of 45 degrees.

Cor. 6. If the medium be something more dense, the equation to the curve must be assumed $y = bx - \frac{1+bb}{4f}xx - \frac{1+bb}{12af}x^3 - bx^4$; to find the value of b , put c for $\frac{1+bb}{4f}$, and e for $\frac{1+bb}{12af}$, then the equation is $y = bx - cx^2 - ex^3 - bx^4$, taking the first, second, and third fluxions, and making $\dot{x}$ constant at *Cor. 1*, we get $e = \frac{c\sqrt{1+bb}}{3a} = \frac{1+b}{12af}$, and $b = \frac{1+bb^2}{48a^2f} - \frac{b \cdot 1+bb^{\frac{3}{2}}}{96aff}$; whence the assumed equation will be $y = bx - \frac{1+bb}{4f}xx - \frac{1+bb^{\frac{3}{2}}}{12af}x^3 - \frac{1+bb^{\frac{3}{2}}}{48a^2f}x^4 + \frac{b \cdot 1+bb^{\frac{3}{2}}}{96aff}x^4$.

Cor. 7. If the resistance of the uniform medium be supposed partly constant, and partly proportional to the square of the velocity, the curve VPB may be defined very nearly; for if the whole resistance be small, the equation to the curve VPB may be supposed to be $y = bx - cx^2 - ex^3$. Where b the tangent of the angle of elevation is found as in *Cor. 1*, rad. $= 1$, $c = \frac{1+bb}{4f}$, and f = altitude from which a body in fall-

ing, by the constant force of gravity acting on it, in a non-resisting medium, acquires the velocity of the projection;

whence n above $= -\frac{1+bb}{2ac} + \frac{3c\sqrt{1+bb}}{2cc}$ and $e = \frac{n \cdot 1+bb^{\frac{3}{2}}}{24ff}$,

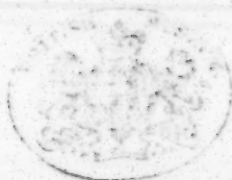
$+ \frac{1+bb^{\frac{3}{2}}}{12af}$, consequently the assumed equation becomes $y =$

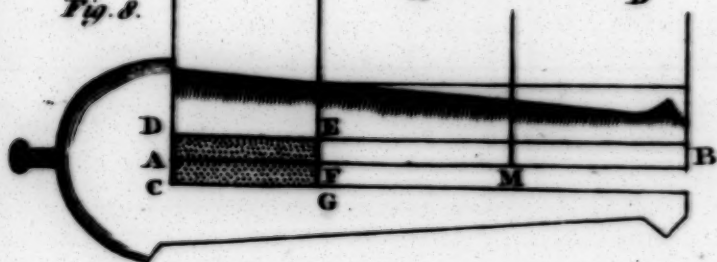
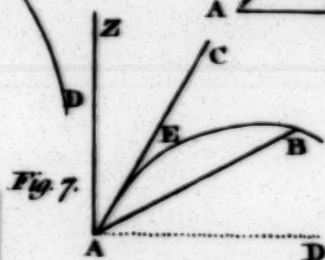
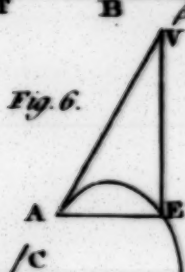
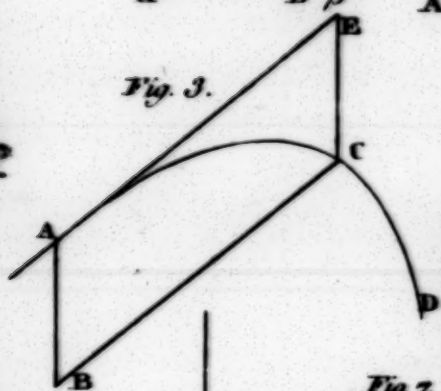
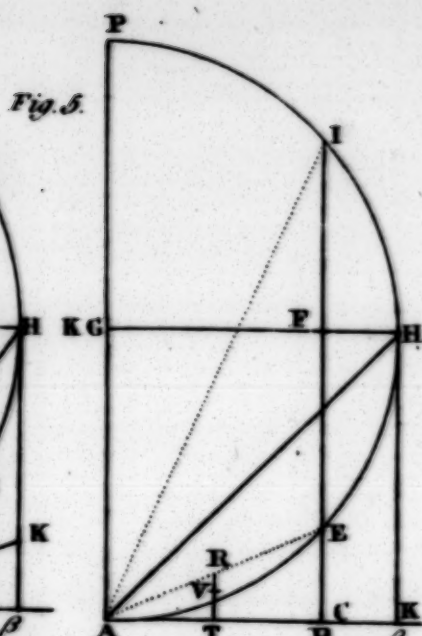
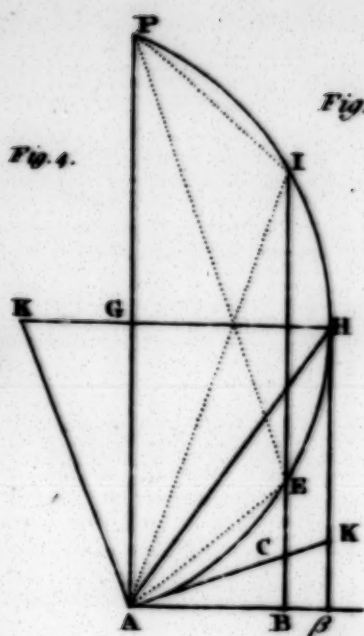
$bx - \frac{1+bb}{4f}xx - \frac{1+bb^{\frac{3}{2}}}{12af}x^3 - \frac{n \cdot 1+bb^{\frac{3}{2}}}{24ff}x^3$, and a and n , may be determined from phenomena, as *Cor. 1*. above.

The

The above Corollaries are the results of a very intricate calculation in fluxions for finding the proper requisites of cannon balls projected in a resisting medium, as the air ; but since the conclusions come out so compounded, they can be of no use at all in practical gunnery ; for the equations fully evince how useless such abstruse calculations are, and how perplex the description of the curve is ; so that it can scarcely be accommodated to any use whatever.

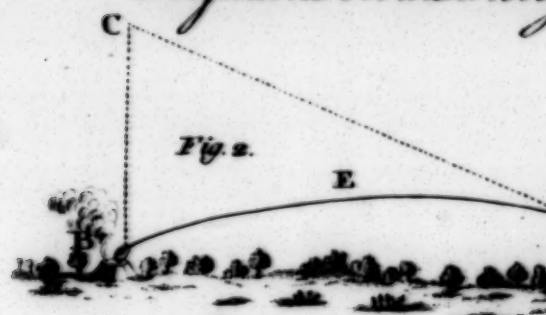
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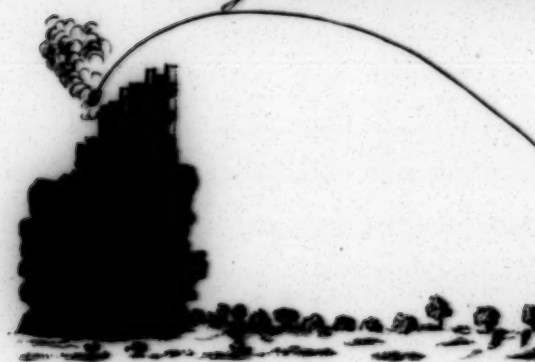


At the End

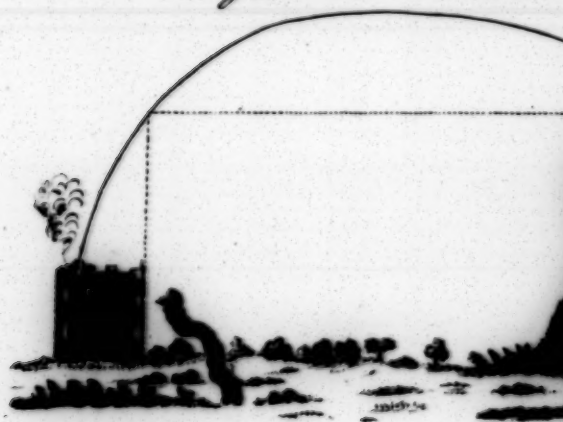
Projections on the Horizontal



Projections on Ascending



Projections on Descending



Ascents

Descents

